Lattice Quark Propagator in Coulomb Gauge

Renormalization and the influence of sea quarks

Mario Schröck

Advisors: Hugo Reinhardt, Giuseppe Burgio, Markus Quandt

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UNI GRAZ

Eberhard Karls Universität Tübingen

Outline

Two-point functions and path integrals

- the quark propagator is the fermionic two-point function of QCD

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- we can explicitly integrate out the quark field dependence

$$S(x,y) = \frac{\int \mathcal{D}U\ D^{-1}[U]\,(x,y)\ \mathrm{e}^{-\ln\det D[U]-S_G[U]}}{\int \mathcal{D}U\ \ \mathrm{e}^{-\ln\det D[U]-S_G[U]}}$$

Gauge (non-)invariance

- while the Lagrangian $L = \overline{\psi} D \psi$ is gauge invariant, the Dirac operator alone is not and neither is its inverse
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$$S^{(0)}(p)^{-1} = i\gamma_i p_i + i\gamma_4 p_4 + M$$

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- How changes the free propagator when the interactions with the gluon fields are turned on?

The interacting quark propagator

- ansatz for the *interacting* inverse propagator

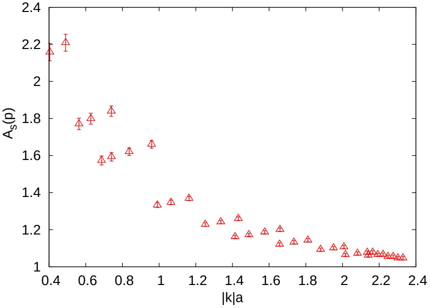
$$S^{-1}(p) = i\gamma_i p_i\, A_s(p) + i\gamma_4 p_4\, A_t(p) + i\gamma_i p_i \gamma_4 p_4\, A_d(p) + B_m(p)$$

  here we introduced the dimensionless real scalar functions

  1. $A_s(p)$ spatial
  2. $A_t(p)$ temporal
  3. $A_d(p)$ mixed
  4. $B_m(p)$ massive

- we extract these functions from the (staggered) lattice quark propagator using the Asqtad action, cf. Landau gauge: [Bowman, Heller, Williams, 2002]

Dressing functions in the quenched approximation



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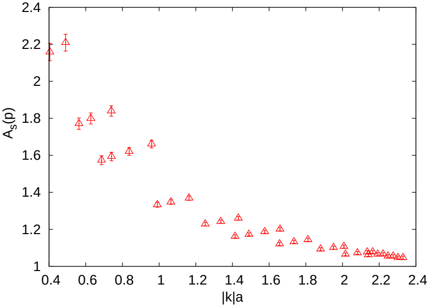

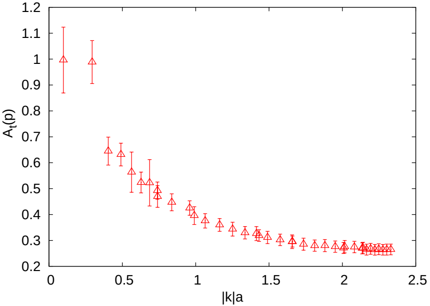

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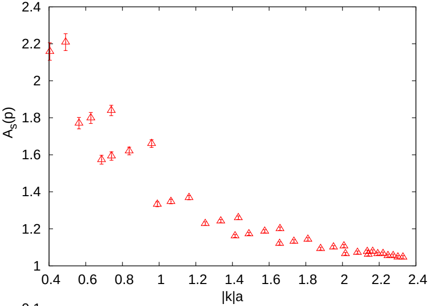

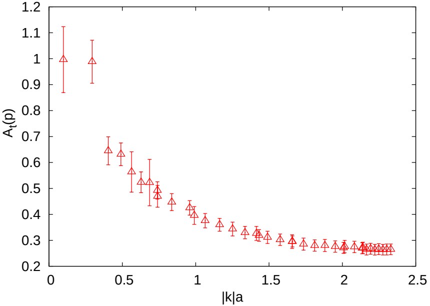

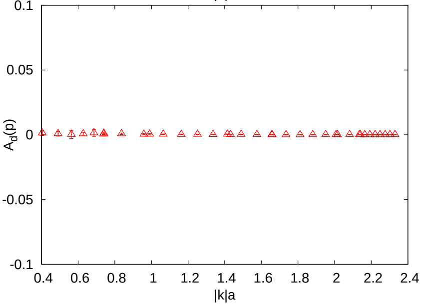

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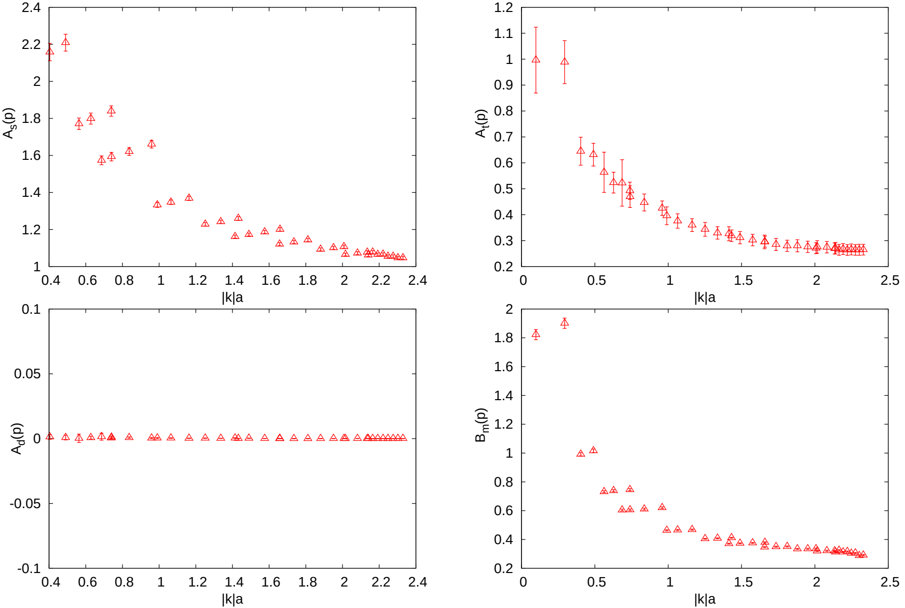

Regularization

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- we want to relate the regularized propagator to the renormalized propagator via a renormalization constant Z_2 (MOM scheme)

$$S(\mu; p) = Z_2^{-1}(\mu; a)\, S^{\text{reg}}(a; p),$$

a is the cutoff, μ a renormalization point which has to be chosen.

Renormalization- and mass function

- ansatz for the renormalized propagator

$$S(\mu;p) = Z(\mu;p)\Big(i\gamma_i k_i + i\gamma_4 k_4 \alpha(p) + M(p)\Big)^{-1},$$

with

$$Z(\mu;p) = A_s^{-1}(\mu;p), \quad \alpha(p) = \frac{A_t(\mu;p)}{A_s(\mu;p)}, \quad M(p) = \frac{B_m(\mu;p)}{A_s(\mu;p)a}.$$

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- the MOM scheme has the renormalization point b.c.

$$Z(\mu; \mu) = 1, \quad \alpha(\mu) = c(\mu), \quad M(\mu) = m(\mu),$$

for a large enough μ, $m(\mu)$ becomes the running quark mass and $c(\mu)$ is expected to go to one.

Renormalization- and mass function

- the regularized propagator

$$S^{\text{reg}}(a;p) = Z(a;p)\Big(i\gamma_i k_i + i\gamma_4 k_4 \alpha(p) + M(p)\Big)^{-1},$$

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- the renormalized quark propagator is independent of a, thus

$$Z(a';p) = \left(\frac{Z_2(\mu;a')}{Z_2(\mu;a)}\right) Z(a;p),$$

- and $\alpha(p)$, $M(p)$ are expected to be independent of the scale a (and of the renormalization point μ)

Quenched gauge configurations with different scales:

β	N_{config}	$L^3 \times T$	a	La	Ta
5.7	94	$16^3 \times 32$	0.170 fm	2.72 fm	5.44 fm
6.0	108	$16^3 \times 32$	0.093 fm	1.49 fm	2.98 fm
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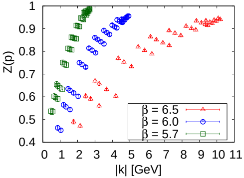
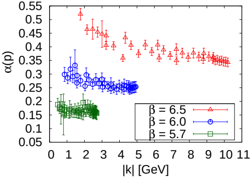
Residual gauge fixing 1

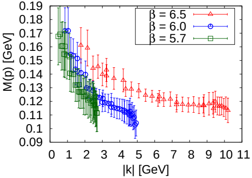
$Z(p) = \frac{1}{A_s(p)}$



$\alpha(p) = \frac{A_t(p)}{A_s(p)}$



$M(p) = \frac{B_m(p)}{A_s(p)a}$



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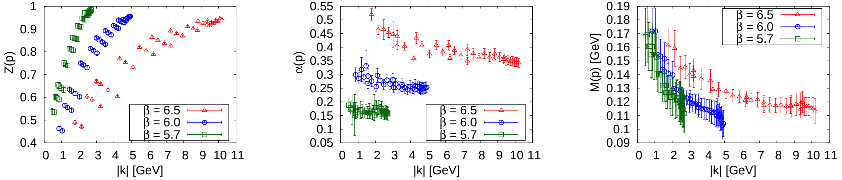
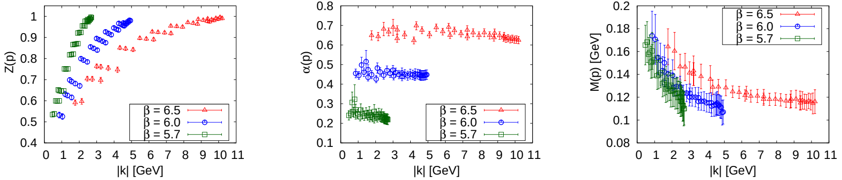


Residual gauge fixing 2



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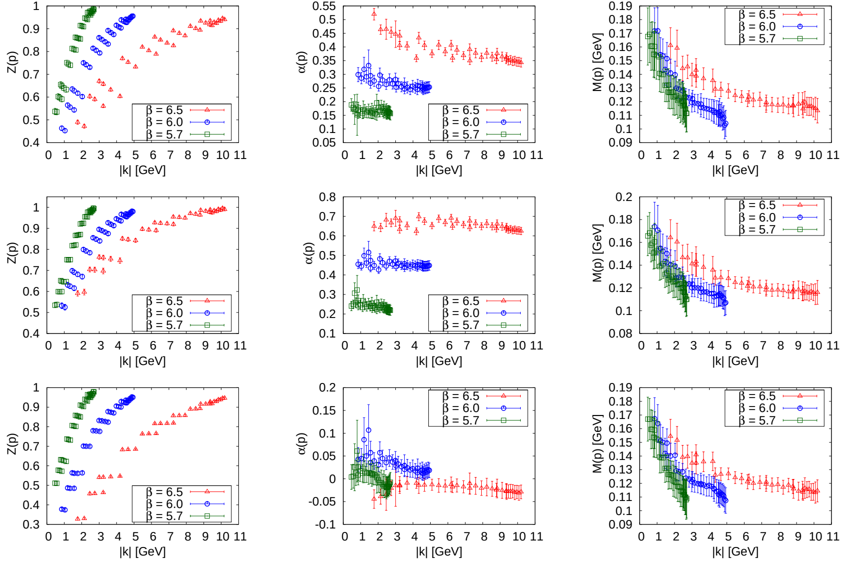
Residual gauge fixing 1			
Residual gauge fixing 2			
No residual gauge fixing			



Static propagator

- we integrate out the energy k_4 to obtain the static propagator

$$S^{-1}(\mathbf{p}) = i\gamma_i k_i a \int_{-\pi}^{\pi} \frac{\mathrm{d}p_4}{2\pi} A_s(p) + \int_{-\pi}^{\pi} \frac{\mathrm{d}p_4}{2\pi} B_m(p)$$

Note that the temporal function $A_t(p)$ is even and thus the product $k_4 A_t(p)$, integrated from $-\pi$ to π, vanishes.

- from the static propagator we extract the static renormalization function $Z(\mathbf{p})$ and the static mass function $M(\mathbf{p})$,

$$Z(\mathbf{p}) = \left(\int_{-\pi}^{\pi} \frac{\mathrm{d}p_4}{2\pi} A_s(p) \right)^{-1}, \quad M(\mathbf{p}) = \frac{\int_{-\pi}^{\pi} \frac{\mathrm{d}p_4}{2\pi} B_m(p)}{\int_{-\pi}^{\pi} \frac{\mathrm{d}p_4}{2\pi} A_s(p)}.$$

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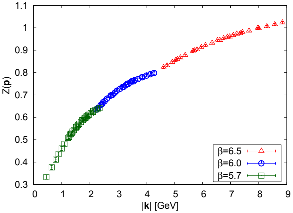

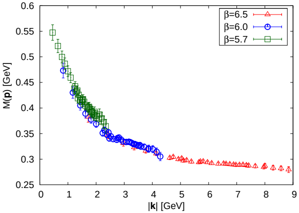

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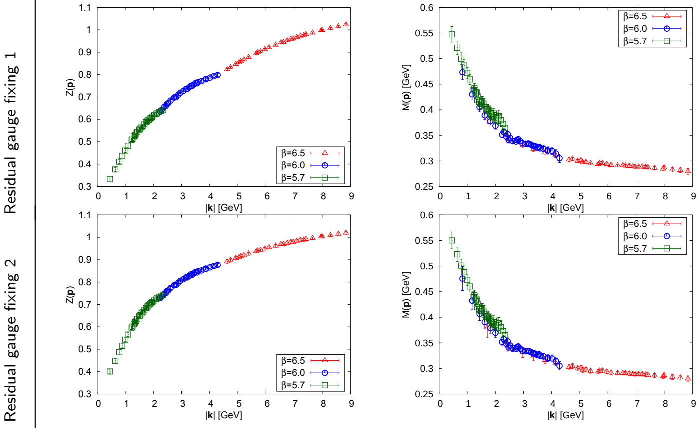

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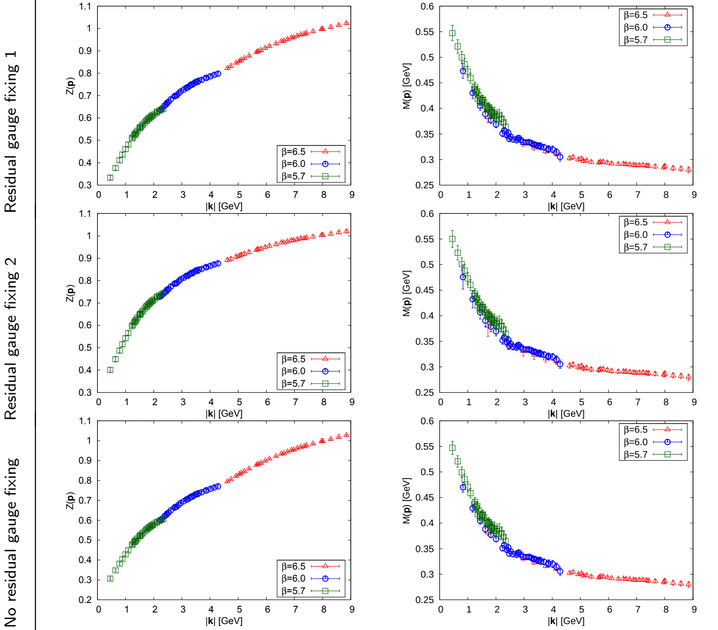

MILC configurations

N_f	$m_{u/d}$	m_s	N_{config}	$L^3 \times T$	a	La	Ta
quenched	∞	∞	79	$20^3 \times 64$	0.125 fm	2.5 fm	8 fm
2+1	11 MeV	79 MeV	88	$20^3 \times 64$	0.125 fm	2.5 fm	8 fm

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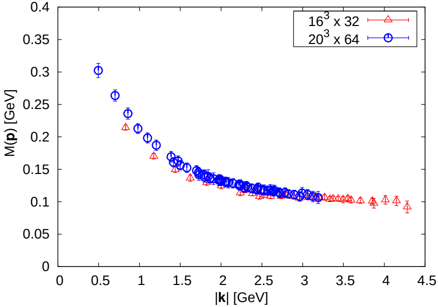

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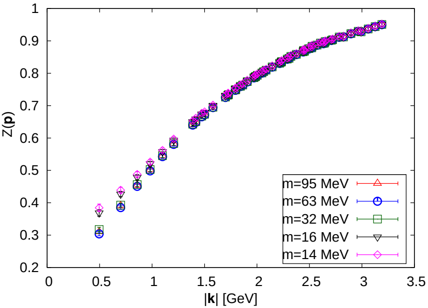
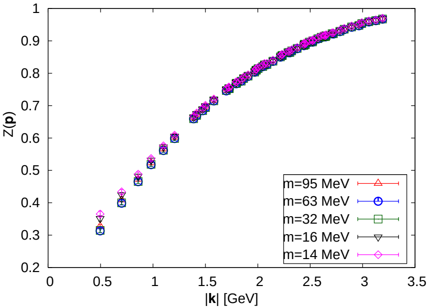
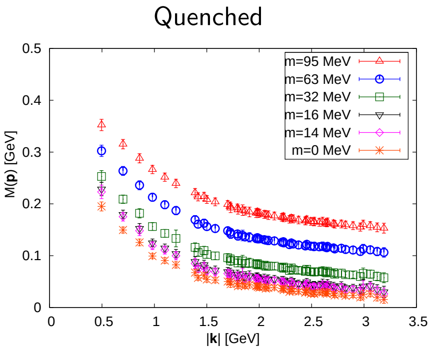


Dynamical




Quenched
M(p) [GeV]
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m=63 MeV
m=32 MeV
m=16 MeV
m=14 MeV
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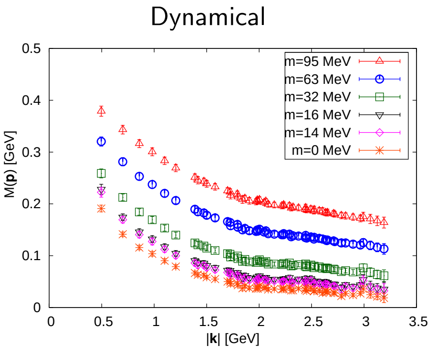
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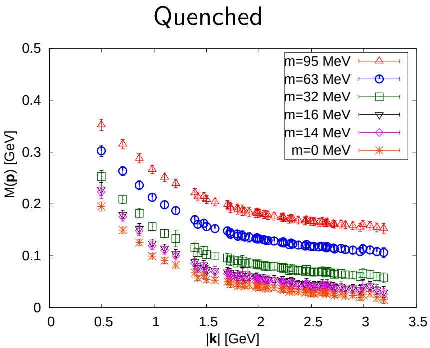

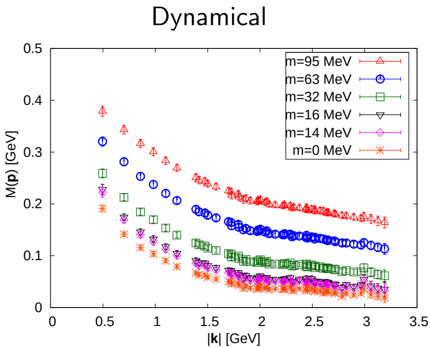

$$M(\mathbf{p}, m) = \frac{m_\chi(m)}{1 + b\frac{\mathbf{k}^2}{\Lambda^2}\log\left(\mathrm{e} + \frac{\mathbf{k}^2}{\Lambda^2}\right)^{-\gamma}} + \frac{m_r(m)}{\log\left(\mathrm{e} + \frac{\mathbf{k}^2}{\Lambda^2}\right)^{\gamma}}$$

constituent quark mass: $M(0, m) = m_\chi(m) + m_r(m)$

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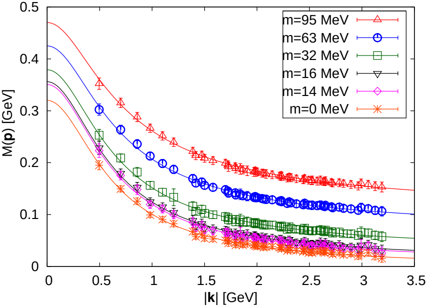

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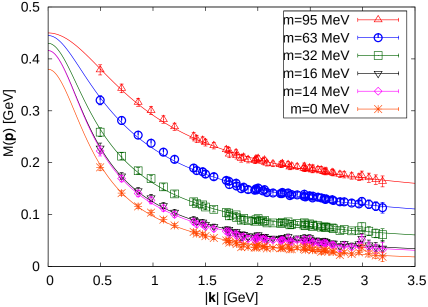

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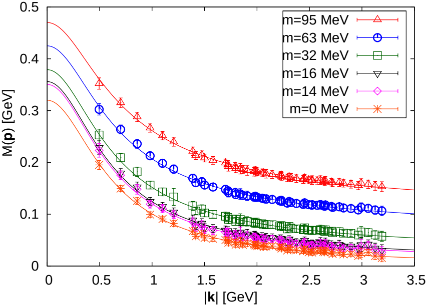

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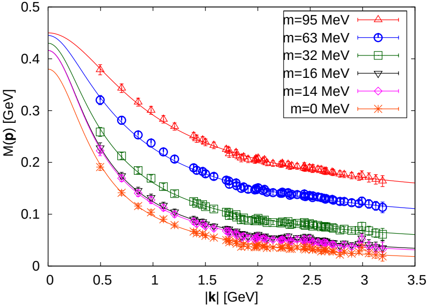

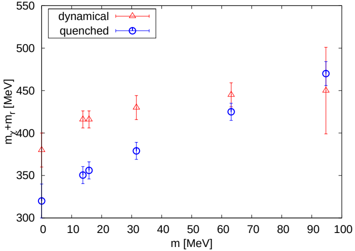

Summary

- we calculated the dressing functions of the Coulomb gauge quark propagator on the lattice
- a speculative tensorial component of the interacting Coulomb gauge quark propagator does not exist
- the static Coulomb gauge quark propagator is renormalizable
- for the lightest of our valence quark masses, more mass is dynamically generated in full QCD then in the quenched approximation
- dynamical quarks screen mass generation for the heavier bare valence quark masses