Dynamical Chiral Symmetry Breaking and Confinement:

Its Interrelation and Effects on the Hadron Mass Spectrum

Mario Schröck
in collaboration with L.Ya. Glozman and C.B. Lang

Rome, February 20, 2014

Outline

- Motivation and introduction
- Mesons under Dirac low-mode truncation
- Effects on the quark propagator
- Mesons: excited states
- Baryons: ground and excited states
- Summary

Motivation

- can confinement persist in a world without dynamical chiral symmetry breaking (DχSB)?
- which patterns exhibits the hadron spectrum in a chirally symmetric world?
- how important is DχSB for the mass of light hadrons?
- which role plays DχSB for the dynamics of quarks?

Reminder: chiral symmetry

- the QCD Lagrangian with two massless quark flavors is invariant under

$$\mathrm{SU}(2)_A \times \mathrm{SU}(2)_V \times \mathrm{U}(1)_A \times \mathrm{U}(1)_V$$

- $\mathrm{U}(1)_V$ conserves the baryon number
- $\mathrm{SU}(2)_V$ is the isospin symmetry $(m_N \approx m_P)$
- $\mathrm{SU}(2)_A$ is broken by the dynamics of QCD
- $\mathrm{U}(1)_A$ is broken dynamically and explicitly by the quantization of QCD (axial anomaly)

Chiral symmetry on the lattice

- a chirally symmetric Dirac operator must obey

$$\{D,\, \gamma_5\} = 0$$

- No-go theorem: it is impossible to have a (naively) chirally invariant, doubler-free, local and translational invariant discretization of fermions on the lattice

[Nielsen, Ninomiya Phys. Lett. B **105** (1981) 219]

- way out: replace continuum condition with lattice version to obtain an exact formulation of chiral symmetry on the lattice (GW equation):

$$\{D,\, \gamma_5\} = aD\gamma_5 D$$

[Ginsparg, Wilson, Phys. Rev. D **25** (1982) 2649]

The CI Dirac operator

- the chirally improved (CI) Dirac operator is an approximate solution to the GW equation
- it is obtained by expanding the most general Dirac operator in a basis of simple operators

$$D(x,y) = \sum_{i=1}^{16} c_{xy}^{(i)}(U)\Gamma_i + m_0$$

- inserting this into the GW eq. then turns into a system of coupled quadratic equations for the expansion coefficients $c_{xy}^{(i)}(U)$
- this expansion provides for a natural cutoff that turns the quadratic equations into a simple finite system.

[Gattringer, Phys. Rev. D **63** (2001) 114501]

Eigenvalues of the Dirac operator

- the difference of left- and right-handed zero modes of the Dirac operator accounts for the *topological charge* which is responsible for the axial anomaly

[Atiyah, Singer, Ann. Math. **93** (1971) 139]

- the spectrum of non-GW fermions exhibits purely real modes which would be the zero modes

- the density of the smallest nonzero eigenvalues is related to the chiral condensate

$$\langle \overline{\psi} \psi \rangle = -\pi \rho(0)$$

[Banks, Casher, Nucl. Phys. B **169** (1980) 103]

“Unbreaking” chiral symmetry

- we subtract the Dirac low-mode contribution from the valence quark propagators

$$S_{\mathrm{red}(k)} = S_{\mathrm{full}} - \sum_{i=1}^{k} \mu_i^{-1} \left|w_i\right\rangle \left\langle w_i\right| \gamma_5$$

- $\mu_i, \ \left|w_i\right\rangle$ are the eigenvalues and vectors of the hermitian Dirac operator $D_5 = \gamma_5 D$ and k denotes the truncation level
- this truncation corresponds to removing the chiral condensate of the valence quark sector by hand
- in the following we are going to perform a hadron spectroscopy with the truncated quark propagators

Hadron spectroscopy

- on the lattice we study Euclidean correlation functions

$$\langle\, O(t)\,\overline{O}(0)\,\rangle = \sum_j \langle 0|\,\hat{O}\,|j\rangle\,\langle j|\,\hat{O}^\dagger\,|0\rangle\,\mathrm{e}^{-tE_j}$$

$$= A\,\mathrm{e}^{-tE_0}\left(1+\mathcal{O}(\mathrm{e}^{-t\Delta E})\right)$$

- where O is an interpolating field with the quantum numbers of the state one is interested in, e.g., a pion:

$$O_\pi(n) = \overline{d}(n)\gamma_5 u(n)$$

- projection to zero momentum allows the identification of the exponential with the effective mass $m_{\mathrm{eff}}(t)$

The variational analysis

- we collect different interpolators O_i describing the same state and define the cross correlation matrix

$$C_{ij}(t) \equiv \langle\, O_i(t)\, \overline{O}_j(0)\, \rangle$$

- solving the generalized eigenvalue problem

$$C(t)\vec{v} = \lambda(t) C(t_0)\vec{v}$$

- gives an estimate for the energies

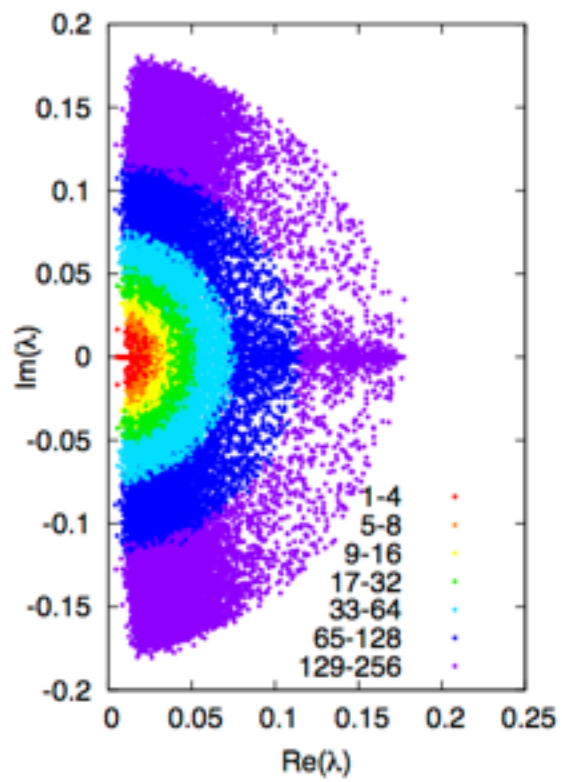
$$\lambda_k(t) \sim \mathrm{e}^{-tE_k}\left(1 + \mathcal{O}(\mathrm{e}^{-t\Delta E_k})\right)$$

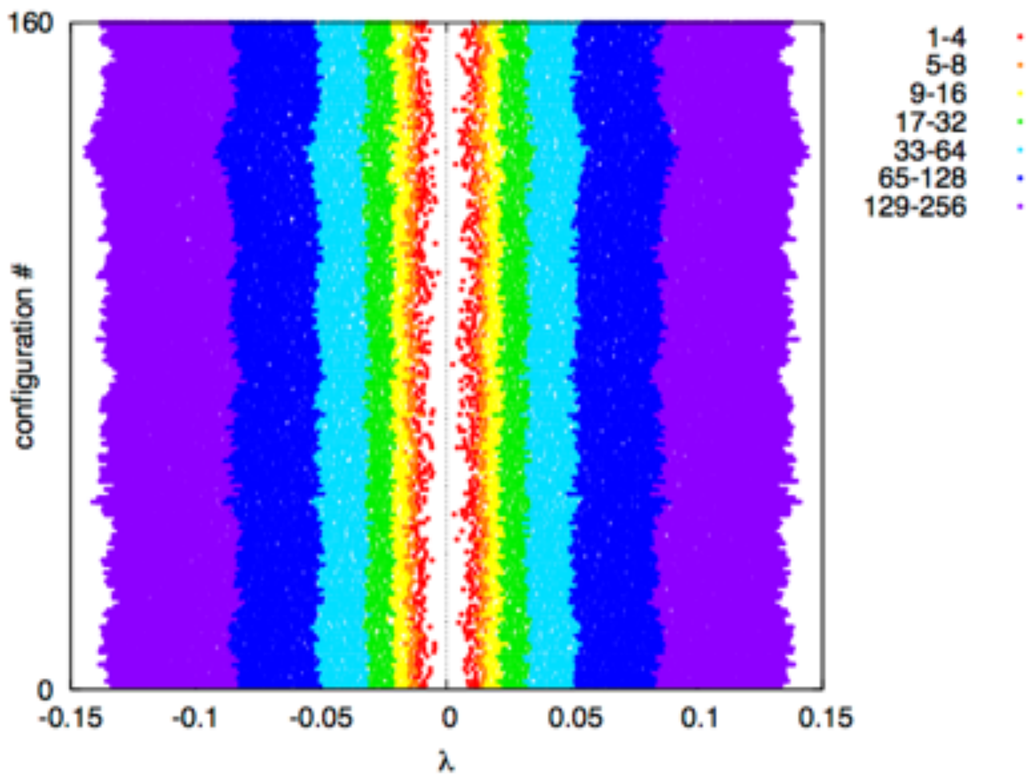
- the eigenvectors indicate the overlap of different states

[Lüscher, Wolf, Nucl. Phys. B **339** (1990) 222]

The setup

- we adopt 161 gauge field configurations with two flavors of degenerate CI fermions [Gattringer et al., PRD **79** (2009) 054501]
- pion mass $m_\pi = 322(5)\,\mathrm{MeV}$
- lattice size $16^3 \times 32$, lattice spacing $a = 0.144(1)\,\mathrm{fm}$
- $L \cdot m_\pi \approx 3.75$
- Jacobi smeared "narrow" quark sources





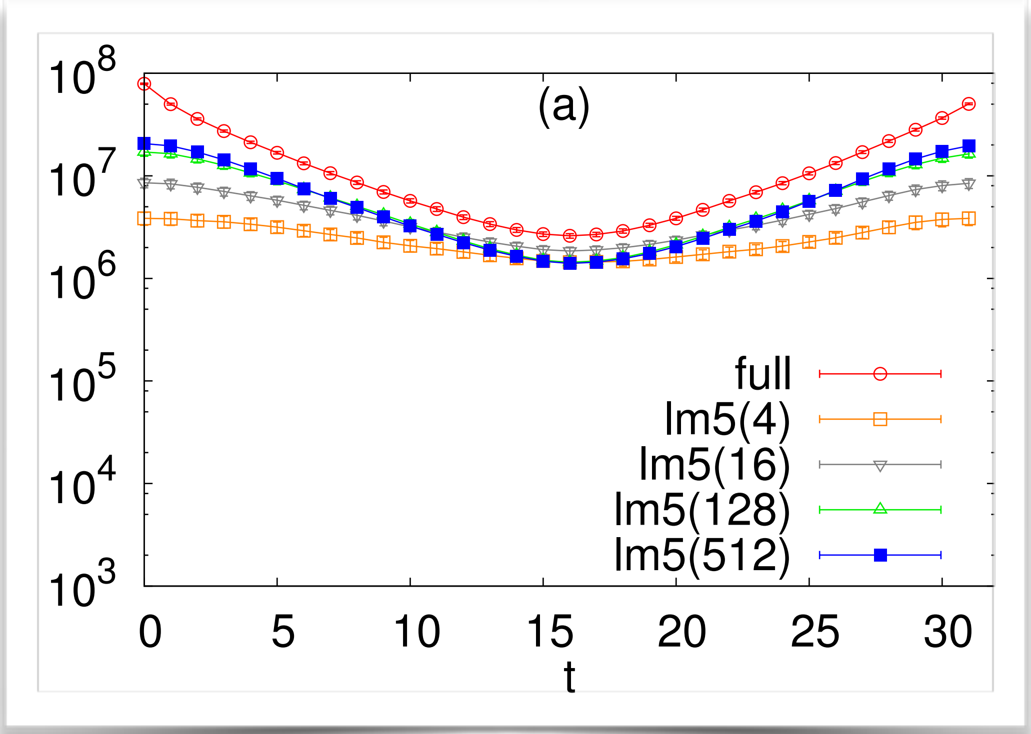
Mesons under low-mode truncation

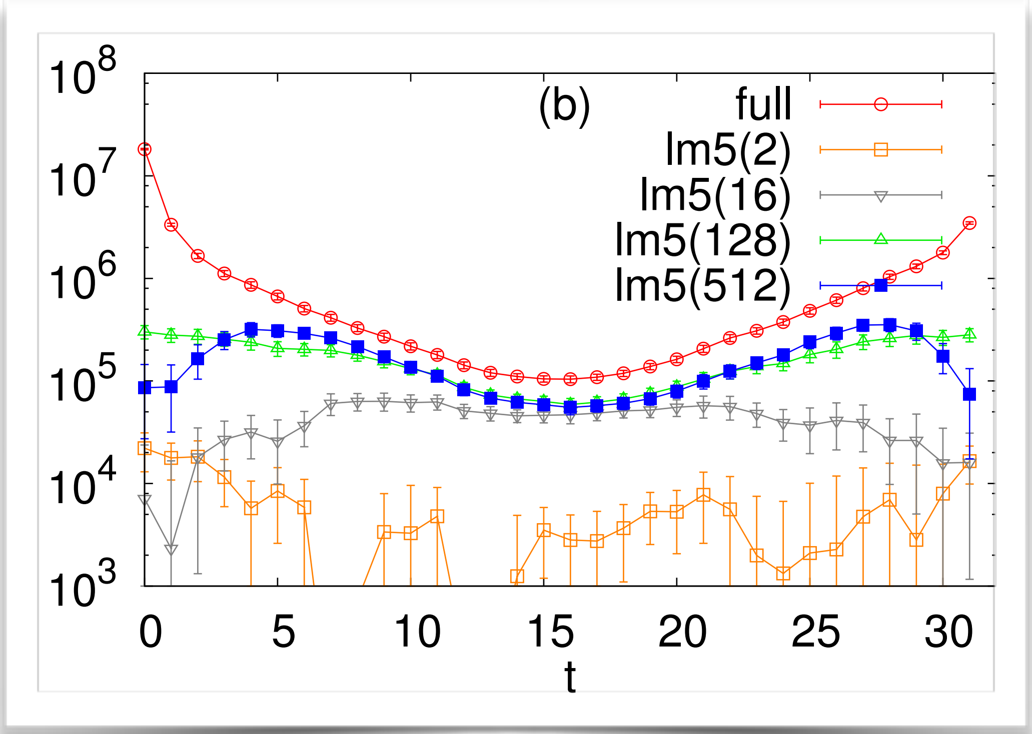
- we restrict ourselves to the study of isovector mesons (no need for disconnected diagrams)
- the following Dirac low-mode truncated meson correlators will be investigated:

$$\rho\,(1^{--}) \quad \xleftrightarrow{\mathrm{SU}(2)_A} \quad a_1\,(1^{++})$$

$$\pi\,(0^{-+}) \quad \xleftrightarrow{\mathrm{U}(1)_A} \quad a_0\,(0^{++})$$

Pion low-modes only

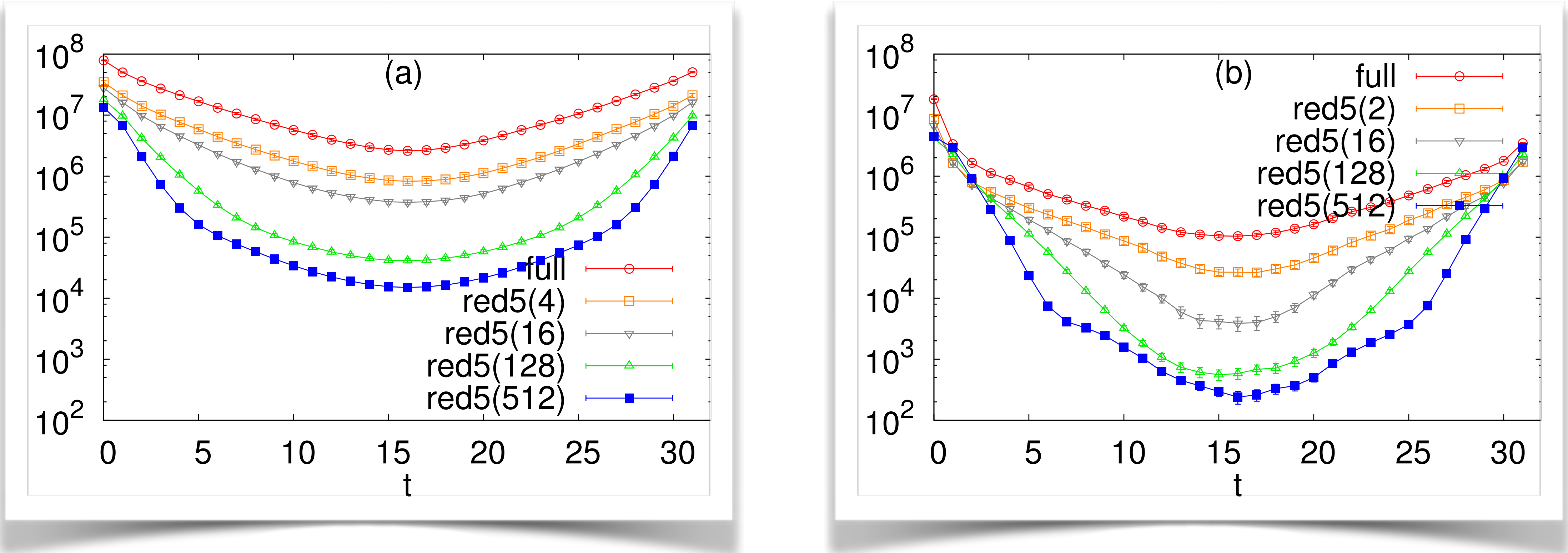




[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode contribution to the correlators for the $J^{PC} = 0^{-+}$ sector in comparison to the correlators from full propagators
- interpolators: (a) $\bar{u}\gamma_5 d$ (b) $\bar{u}\gamma_4\gamma_5 d$

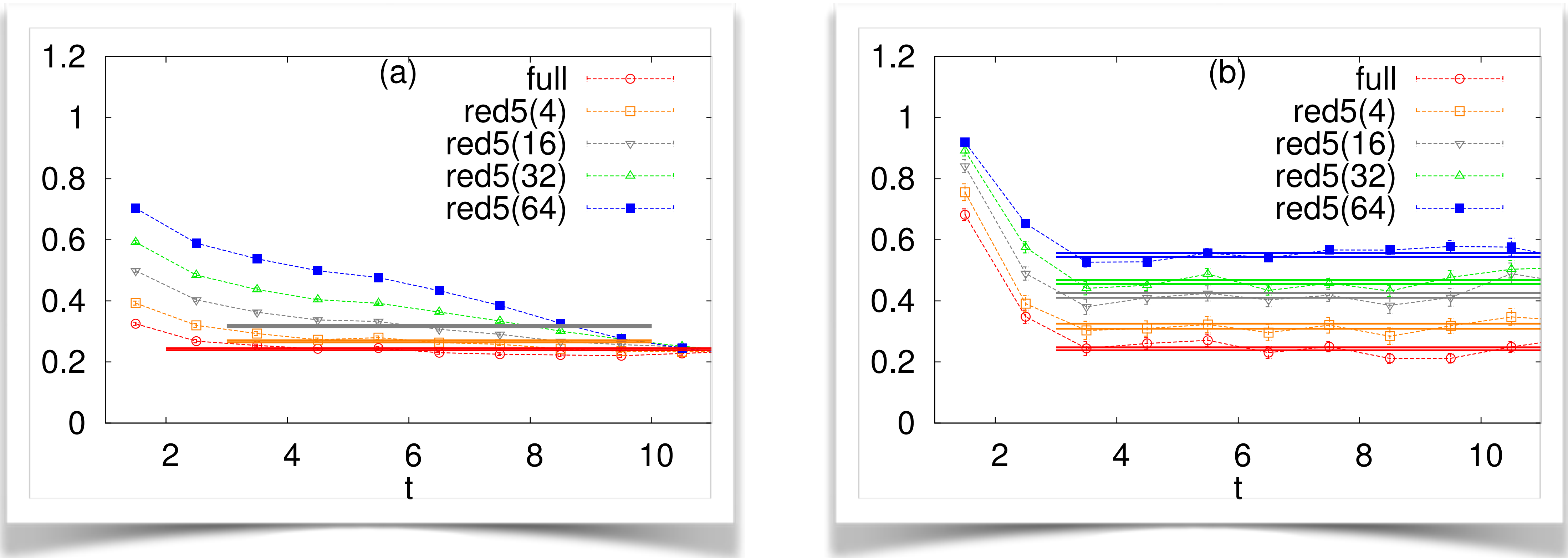
Pion without low-modes



[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode truncated correlators of the $J^{PC} = 0^{-+}$ sector in comparison to the correlators from full propagators
- interpolators: (a) $\bar{u}\gamma_5 d$ (b) $\bar{u}\gamma_4\gamma_5 d$

Pion without low-modes

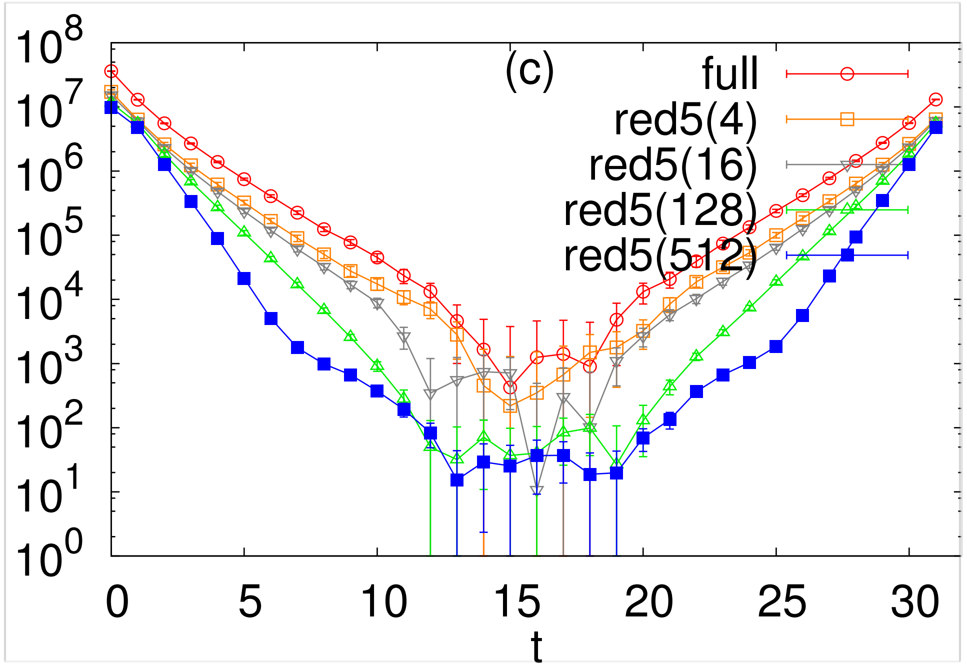


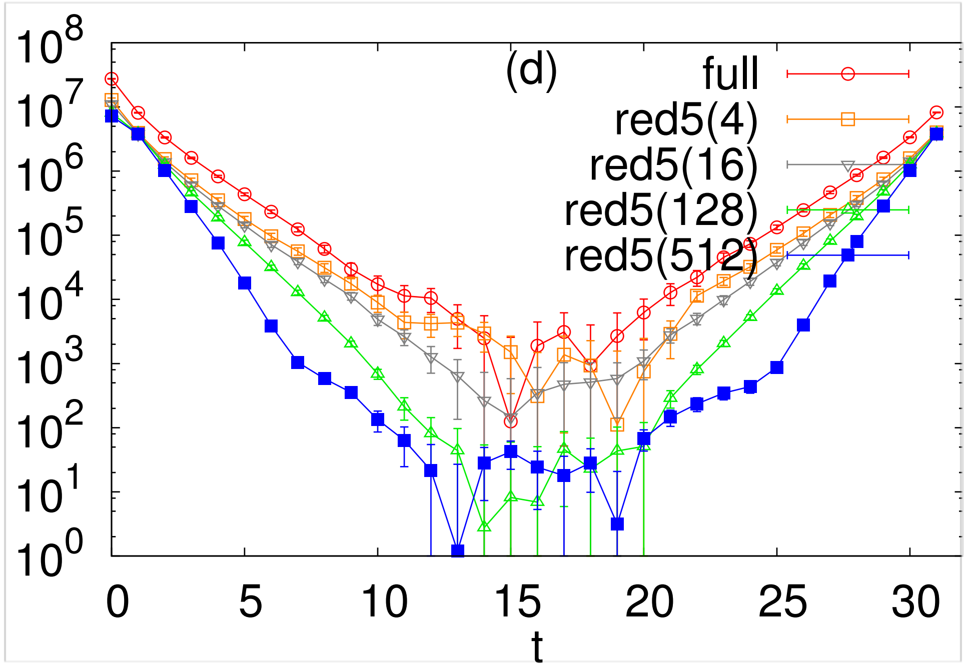
[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode truncated effective masses of the $J^{PC} = 0^{-+}$ sector in comparison to the eff. masses from full propagators

- interpolators: (a) $\bar{u}\gamma_5 d$ (b) $\bar{u}\gamma_4\gamma_5 d$

Rho without low-modes

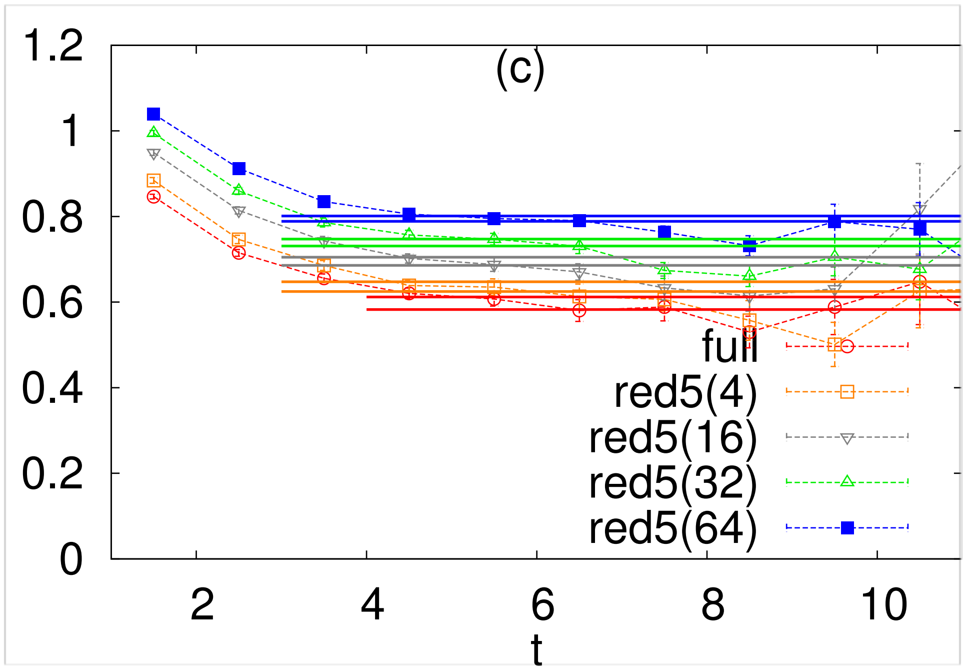


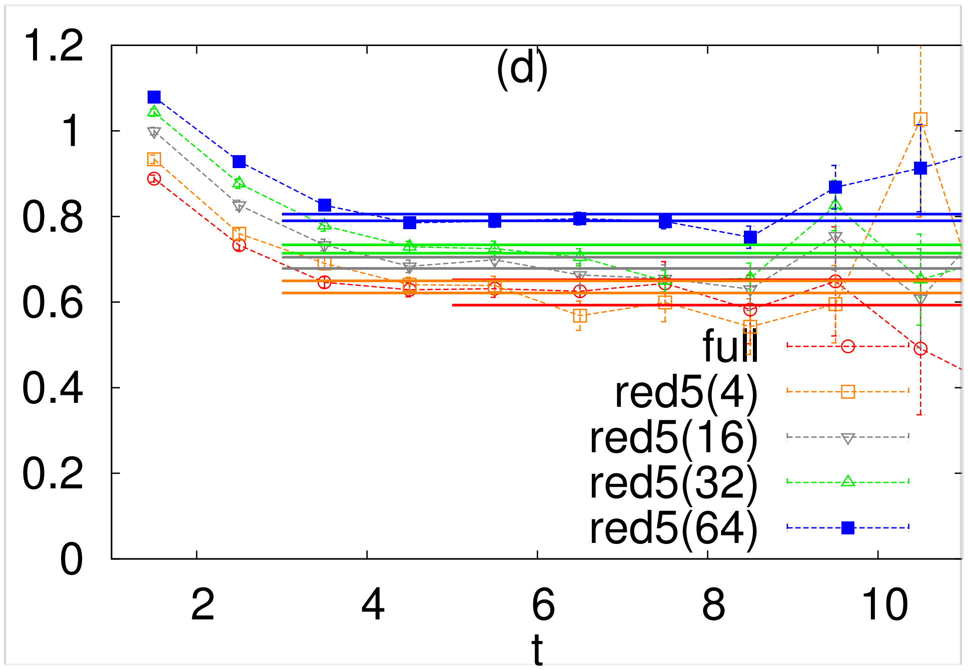


[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode truncated correlators of the $J^{PC} = 1^{--}$ sector in comparison to the correlators from full propagators
- interpolators: (c) $\bar{u}\gamma_i d$ (d) $\bar{u}\gamma_4\gamma_i d$

Rho without low-modes

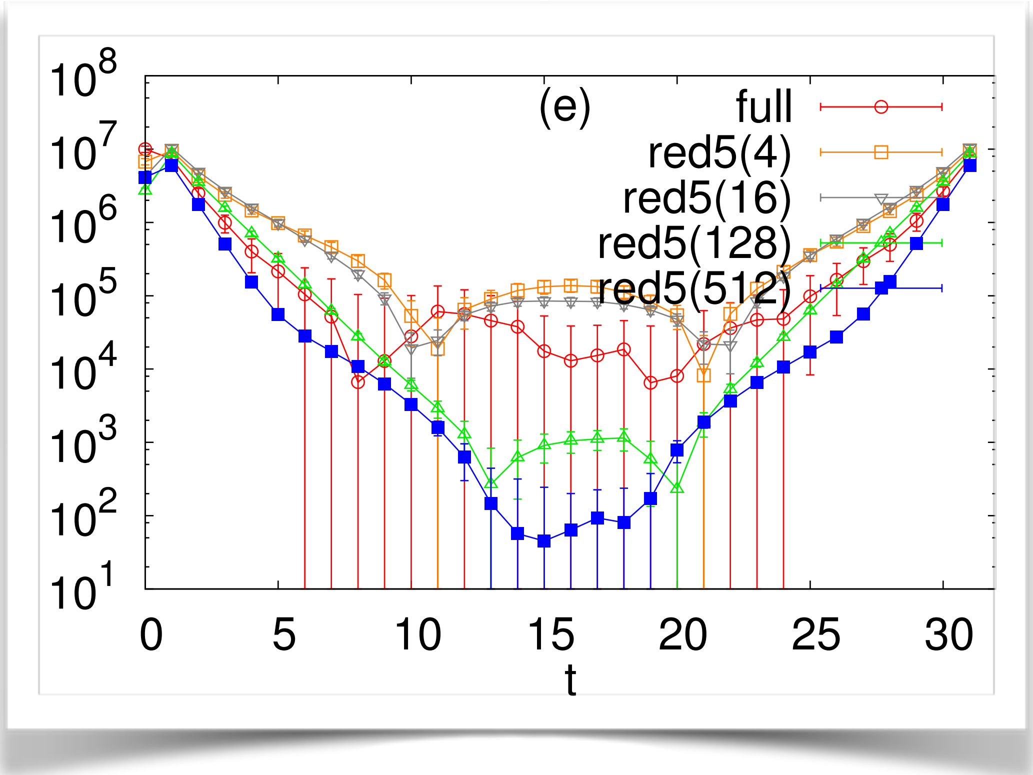


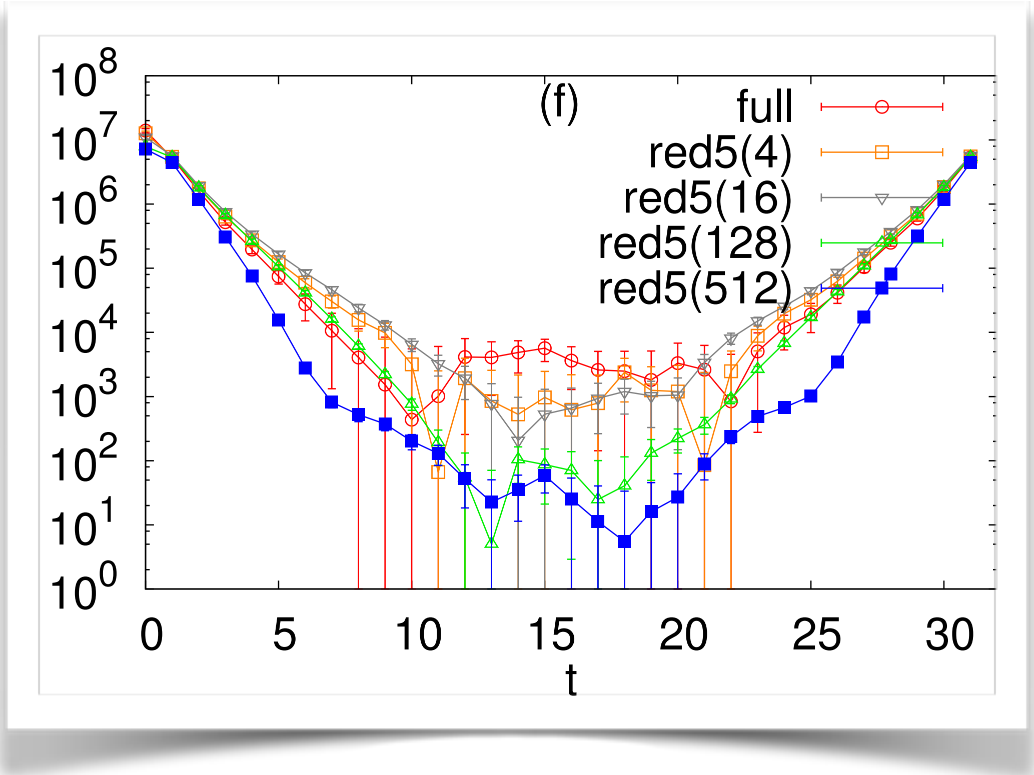


[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode truncated effective masses of the $J^{PC} = 1^{--}$ sector in comparison to the eff. masses from full propagators
- interpolators: (c) $\bar{u}\gamma_i d$ (d) $\bar{u}\gamma_4\gamma_i d$

a_0 and a_1 without low-modes

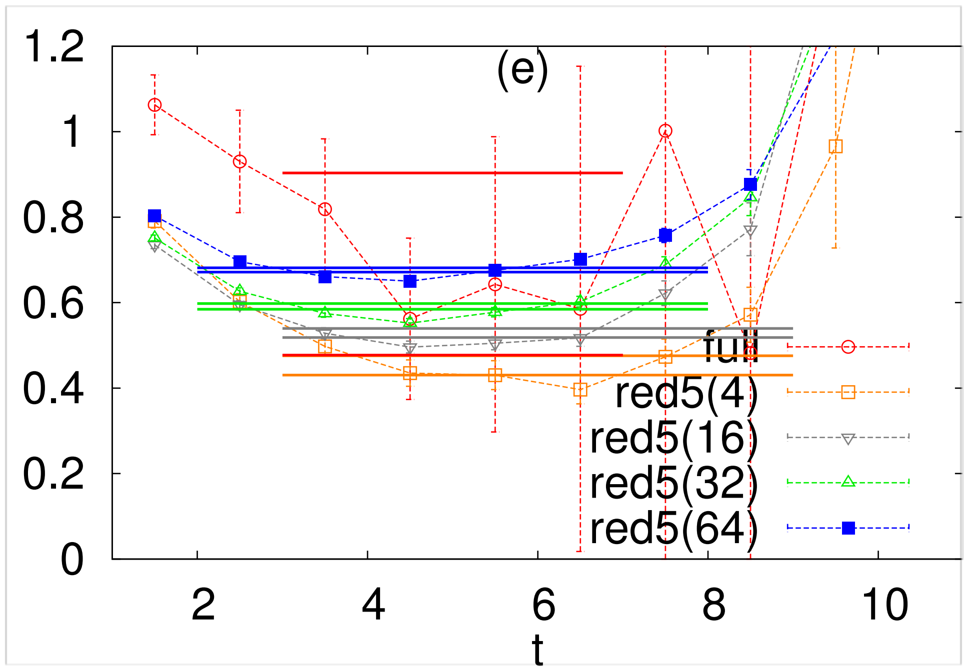


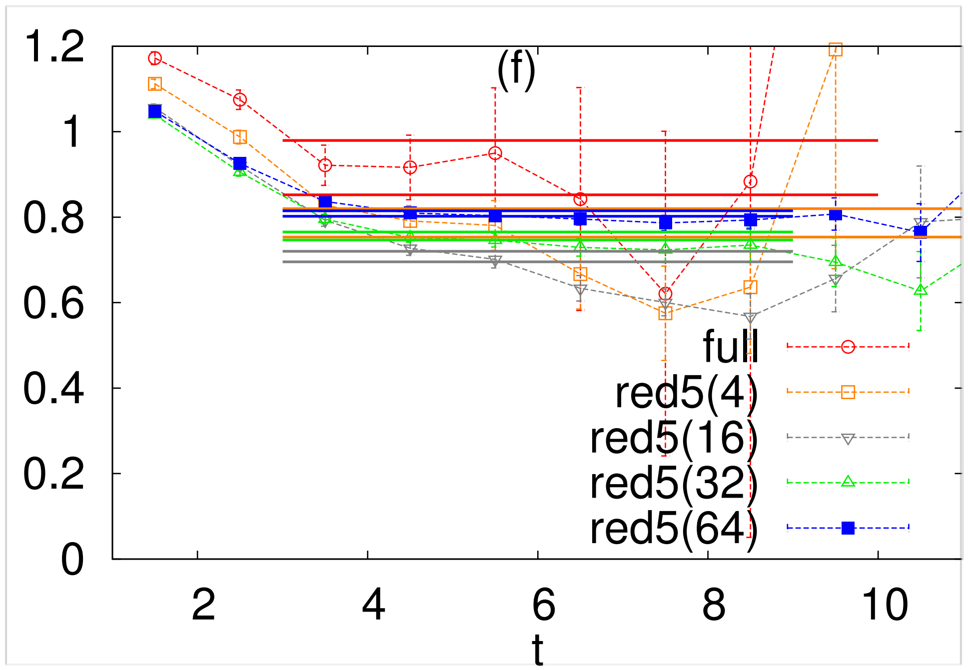


[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode truncated correlators of the $J^{PC} = 0^{++}, 1^{++}$ sector in comparison to the correlators from full propagators
- interpolators: (e) $\bar{u}d$ (f) $\bar{u}\gamma_i\gamma_5 d$

a_0 and a_1 without low-modes

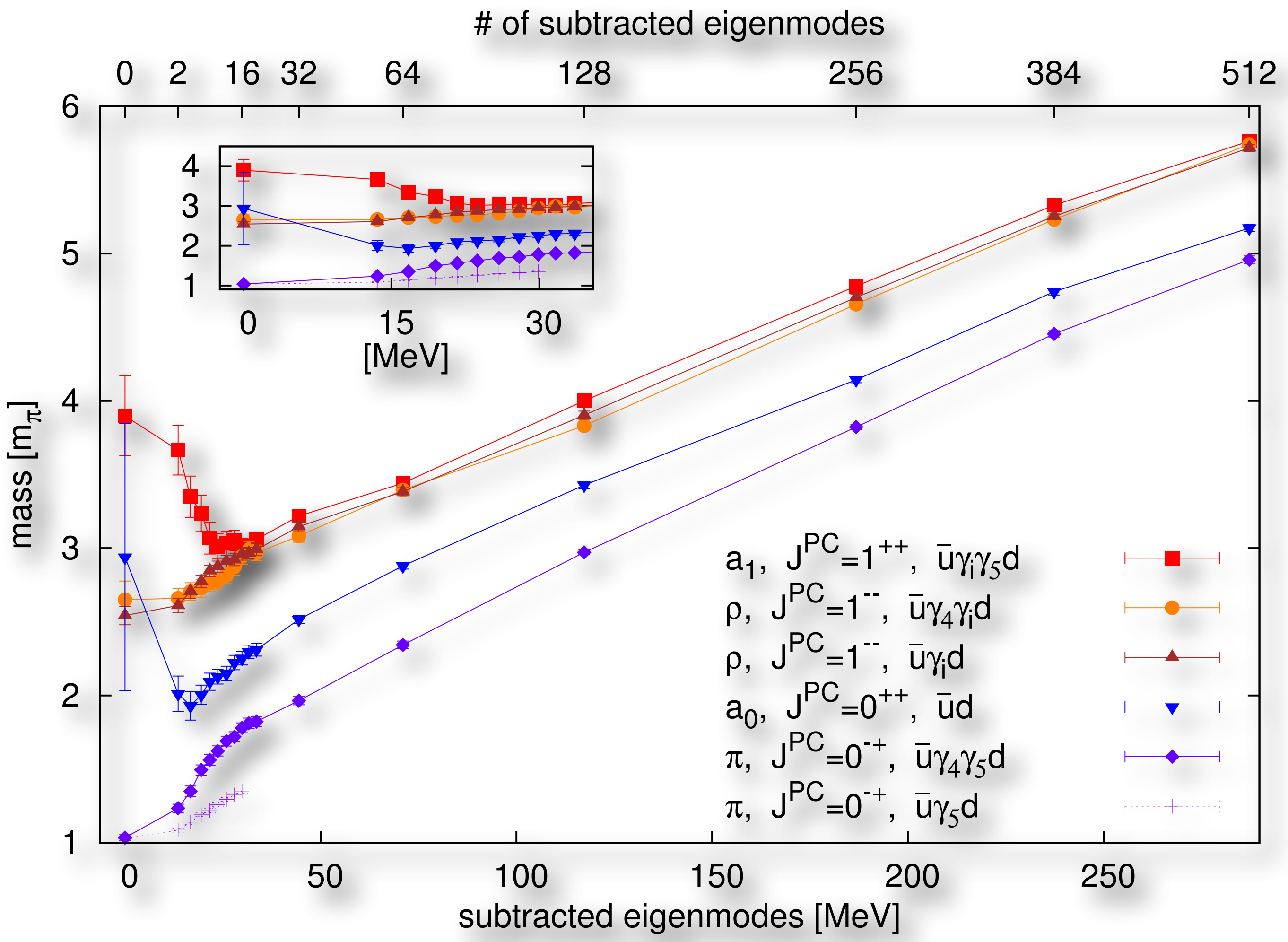




[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode truncated effective masses of the $J^{PC} = 0^{++}, 1^{++}$ sector in comparison to the eff. masses from full propagators
- interpolators: (e) $\bar{u}d$ (f) $\bar{u}\gamma_i\gamma_5 d$

Meson mass evolution



[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- degeneracy of rho and a_1: restoration of the chiral symmetry
- fate of $\mathrm{U}(1)_A$ unclear
- mesons masses are growing with the truncation level

Quark propagator

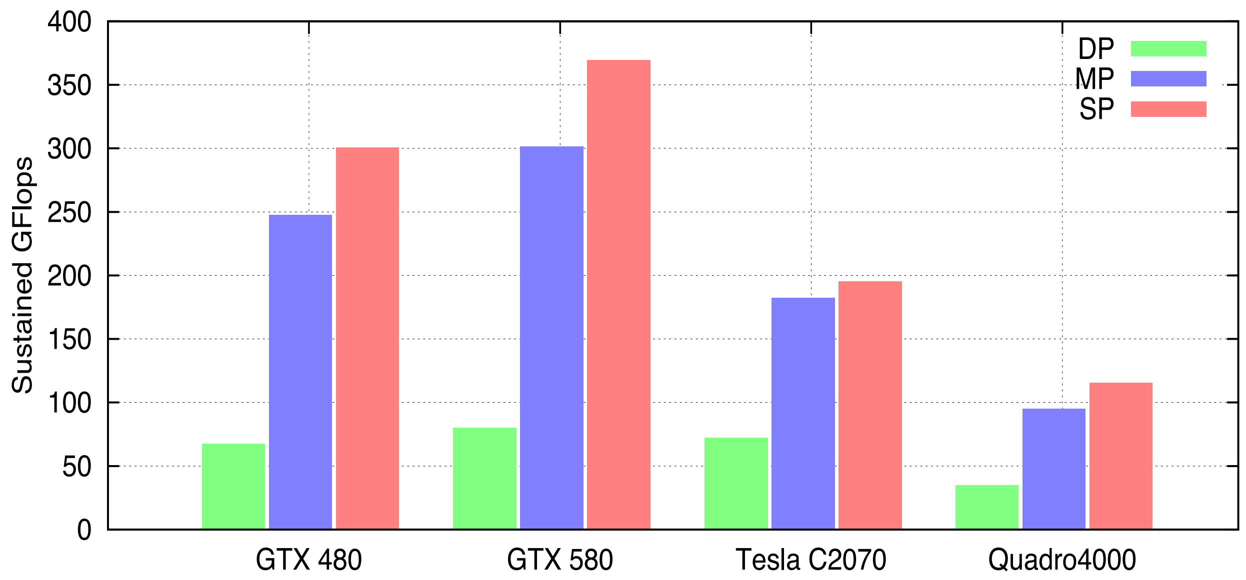
- we study the (Landau gauge) quark propagator to shed light on the origin of the large meson mass upon Dirac low-mode reduction
- the renormalized quark propagator has the form

$$S(\mu;p^2) = \left(i\not{p}A(\mu;p^2) + B(\mu;p^2)\right)^{-1} = \frac{Z(\mu;p^2)}{i\not{p} + M(p^2)}$$

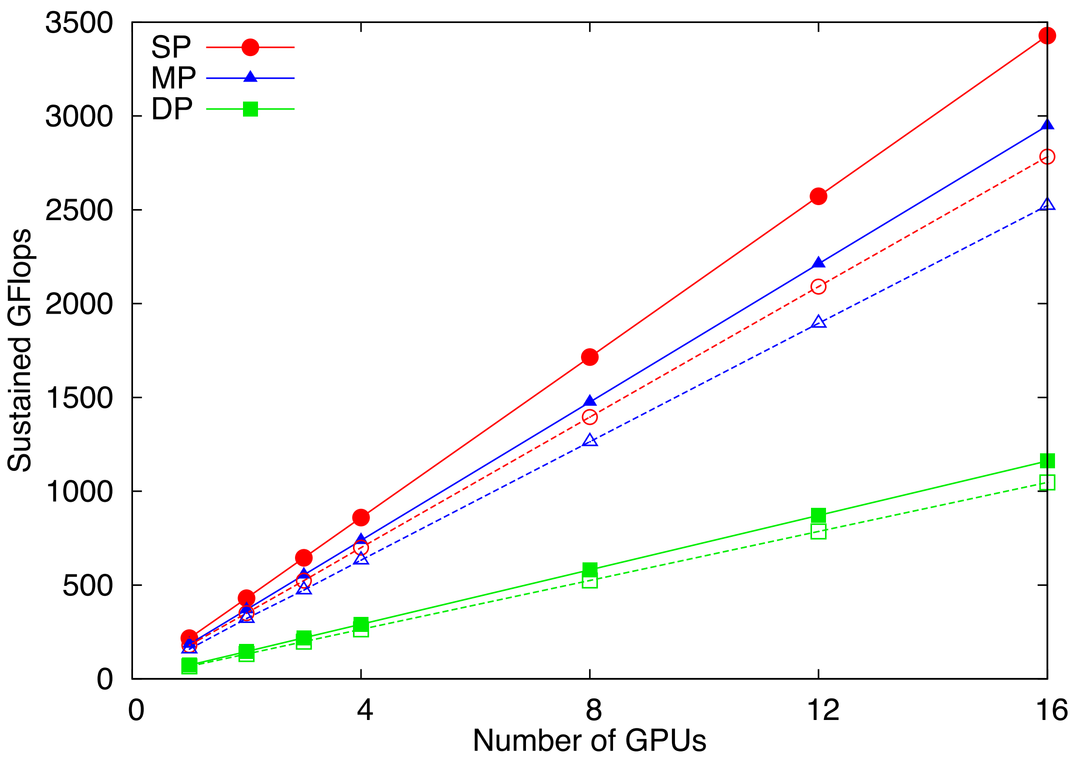
- we extract the wavefunction renormalization function $Z(\mu;p^2)$ and the mass function $M(p^2)$ from the lattice and study their evolution under low-mode truncation

Lattice gauge fixing on GPUs

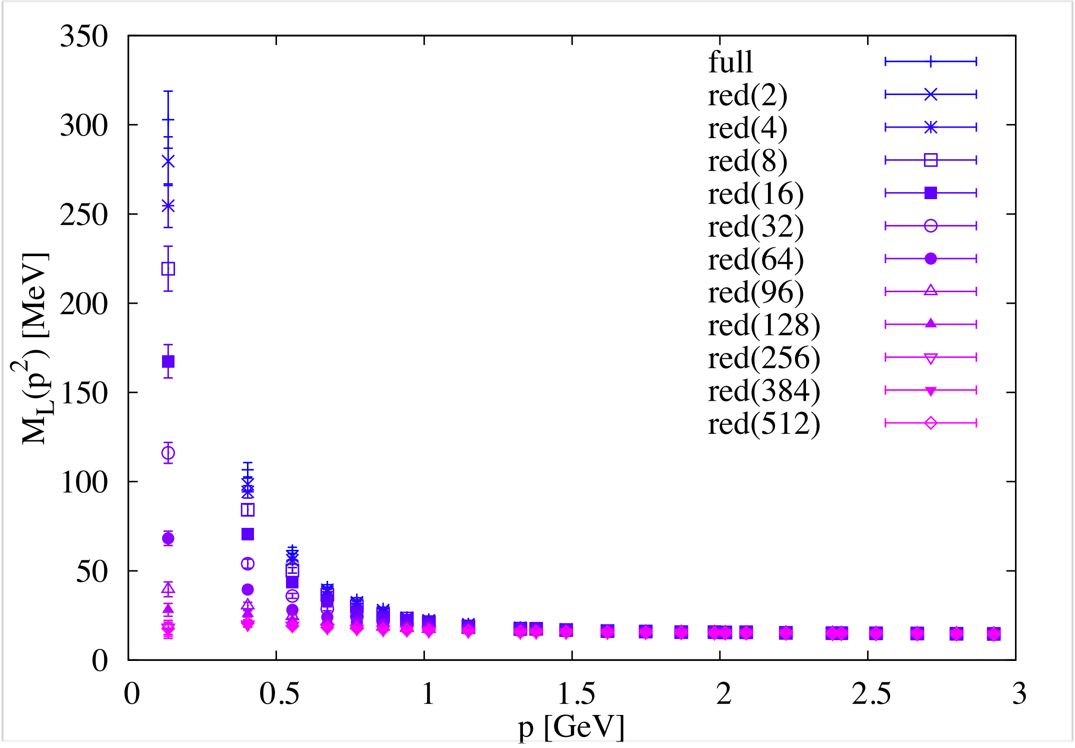
- our *cuLGT* code supports Landau, Coulomb and maximally Abelian gauge fixing on multi-GPUs
- the "mephisto" cluster: five nodes, each node four NVIDIA Tesla C2070 GPUs and two Intel Xeon Six-Core Westmere CPUs @ 2.67GHz with *FermiQCD*
- code performance: one GPU $\sim$ 470 CPU cores

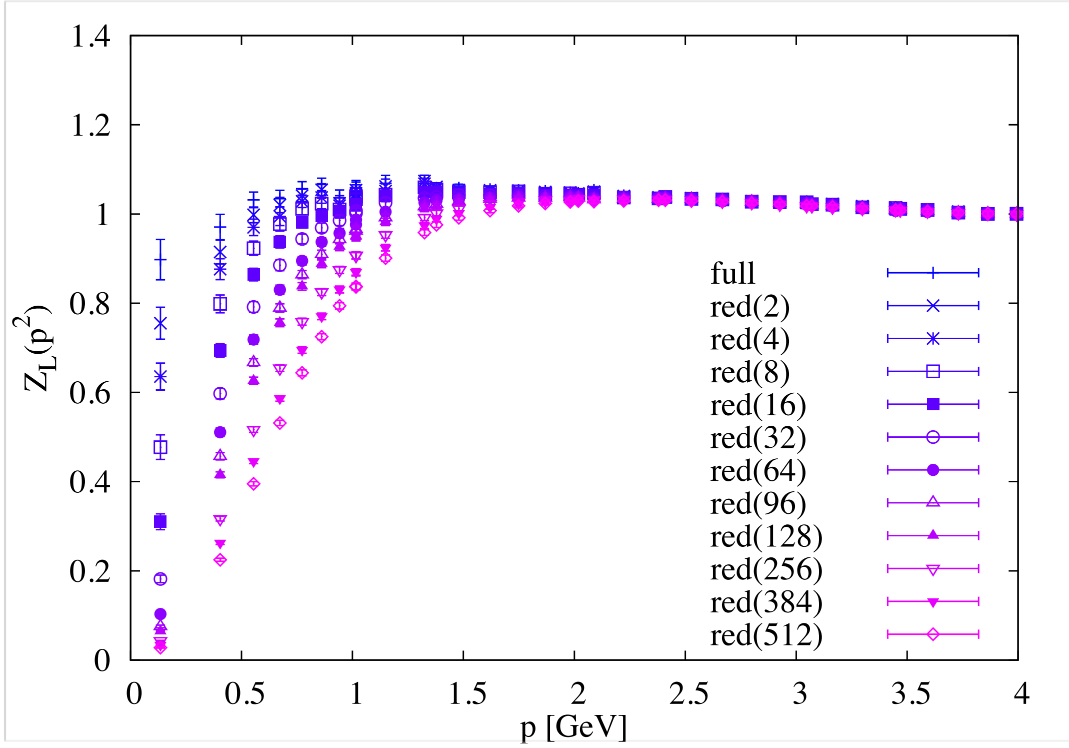


[M.S., H. Vogt, Comp. Phys. Commun. **184** (2013) 1907-1919]



Truncated quark propagator





[M.S., Phys. Lett. B **711** (2012) 217-224]

- flattening of $M(p^2) \iff$ vanishing of $\langle\,\overline{\psi}\psi\,\rangle$
- $Z(p^2)\big|_{p\ll 1} \to 0 \iff S(p^2)\big|_{p\ll 1} \to 0$:

  suppression of low momentum quarks

Dirac modes and quark momenta

- the eigenvalues of the free Dirac operator can be derived analytically

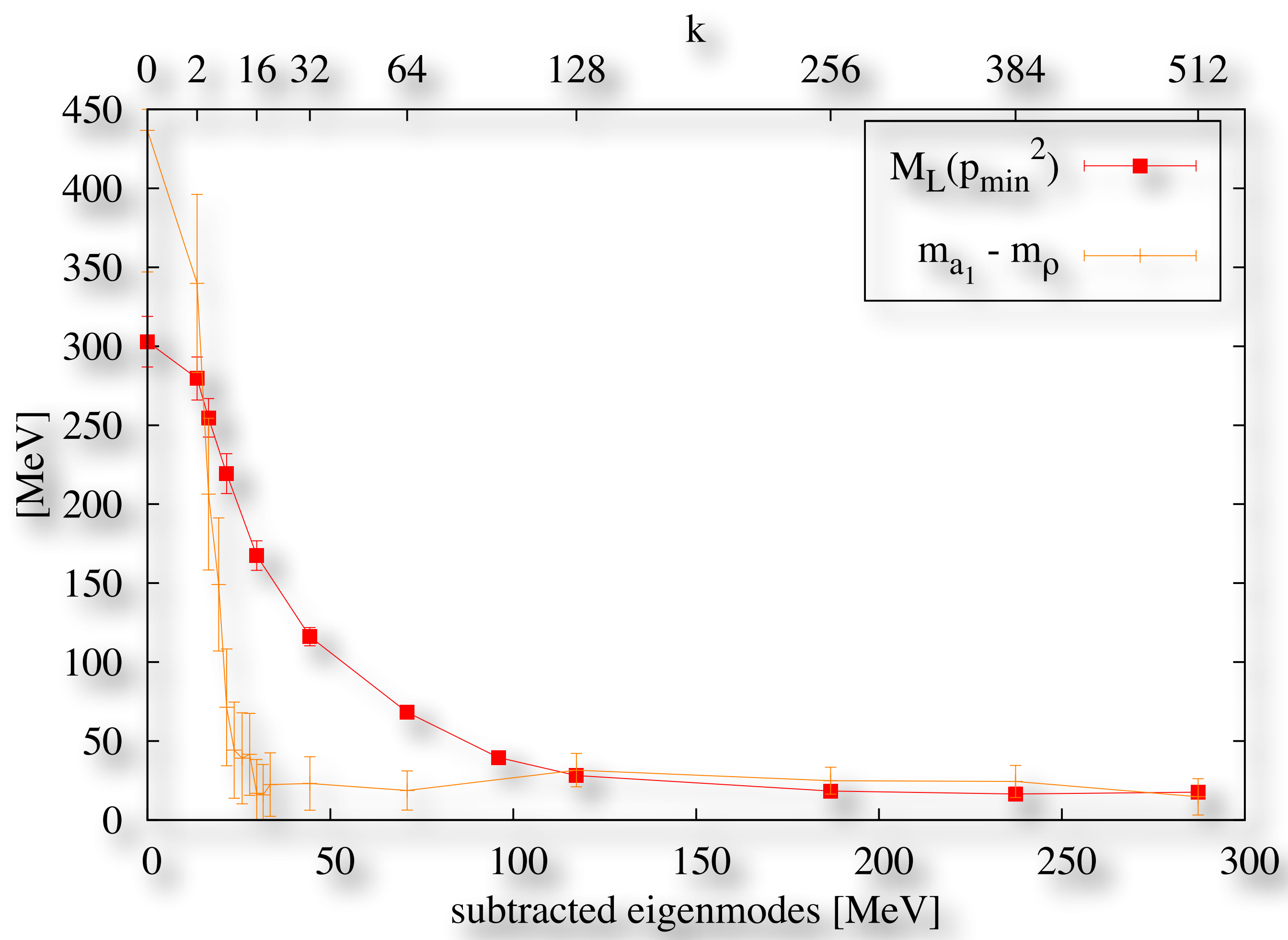
$$\lambda = s \pm i\,|k|$$

- where $s(p)$ denotes the scalar part of the Dirac operator and $k(p)$ are the lattice momenta
- setting the small eigenvalues to zero makes the low momentum states imaginary and thus unphysical

Increased quark momenta

i. explains growing of meson masses

ii. chiral restoration in mesons is partially *effective:* compare chiral restoration in mesons with vanishing of the chiral condensate:



[M.S., Phys. Lett. B **711** (2012) 217-224]

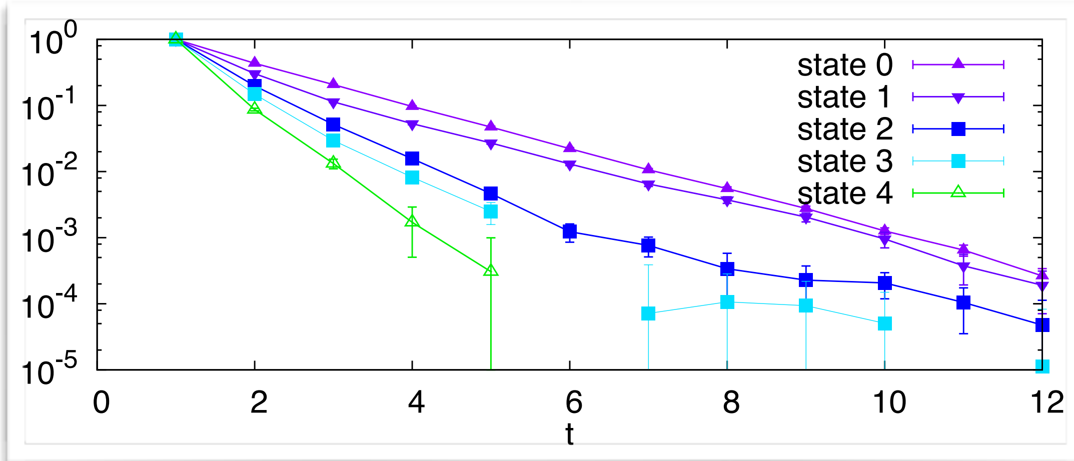
Variational analysis: mesons

- we extend our study by adopting different quark source smearings (Jacobi smearing "wide" and "narrow" and a derivative source)
- the variational method than allows the extraction of excited states
- derivative source crucial for tensor meson b_1, which would-be connected via

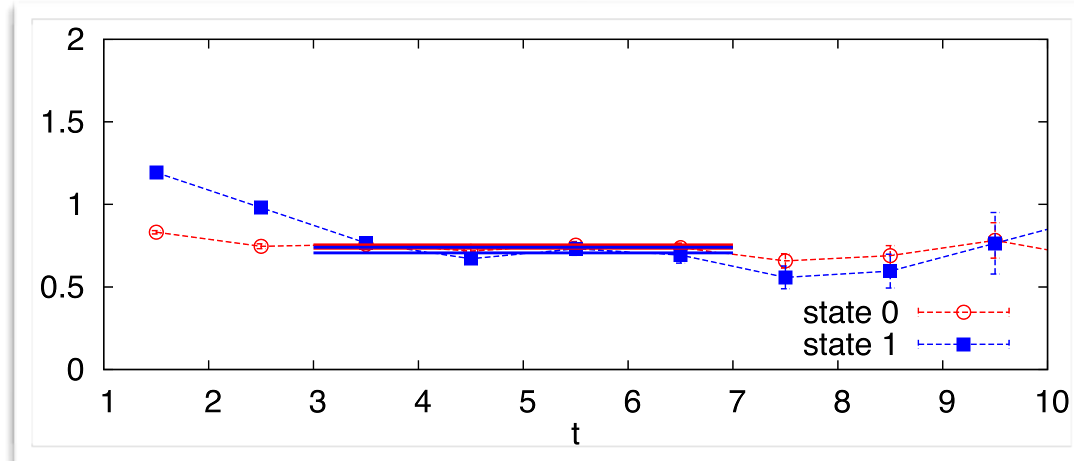
$$b_1\,(1^{+-}) \xleftrightarrow{\quad \mathrm{U}(1)_A \quad} \rho\,(1^{--})$$

Truncation $k = 64$ of $\rho\ (1^{--})$

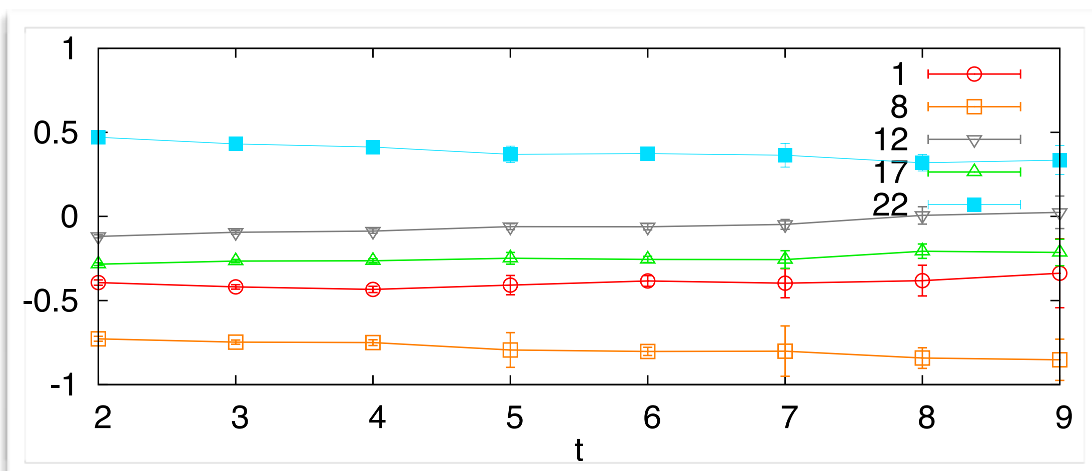
Correlators (eigenvalues) of all states



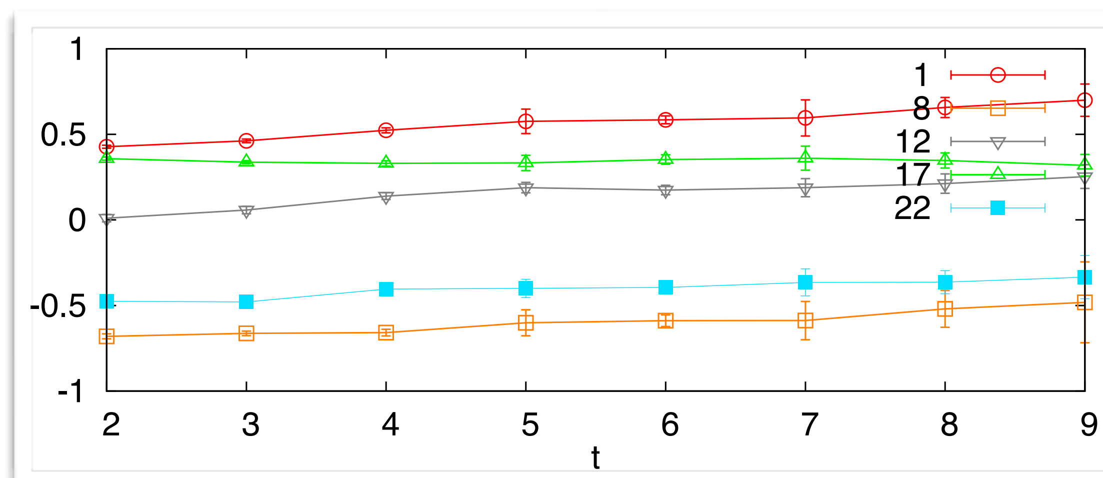
Effective masses of lowest two states



Eigenvectors of ground state



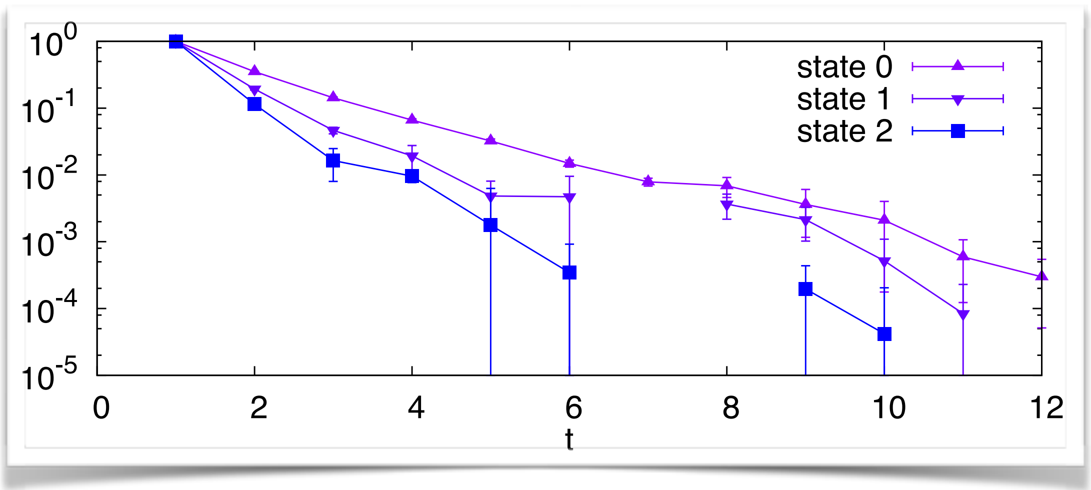
Eigenvectors of first excited state



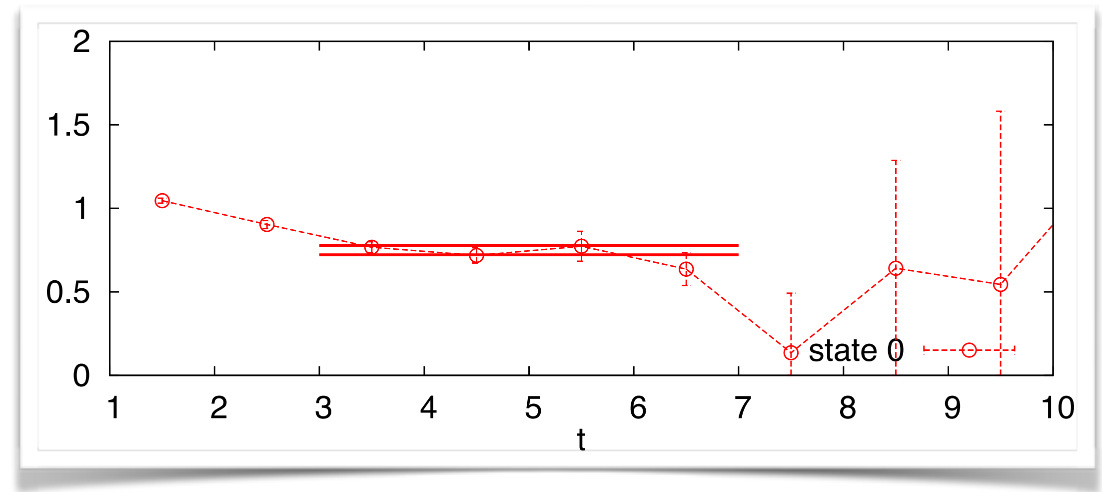
[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Low-mode truncated $a_1(1^{++})$

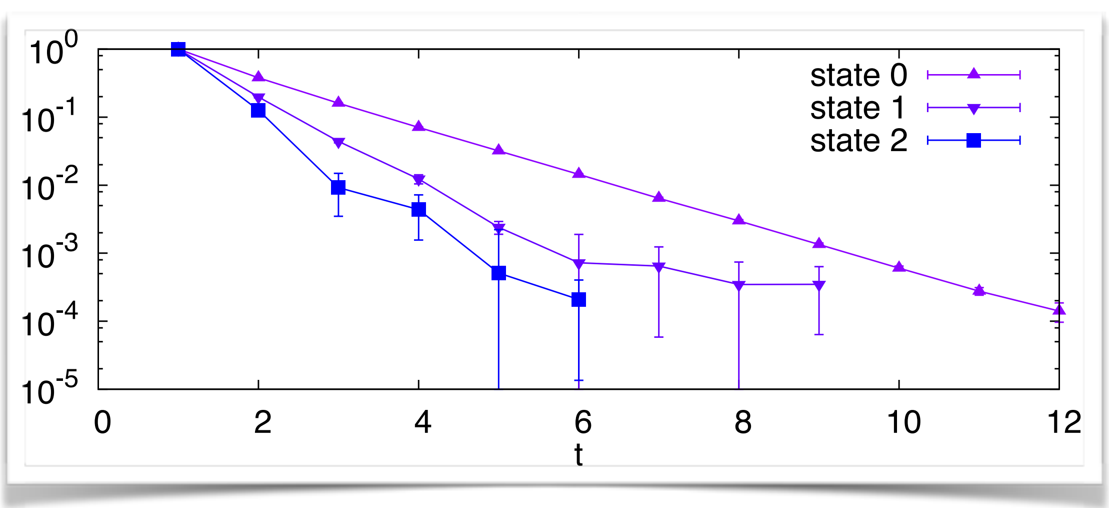
Correlators (eigenvalues), k=4



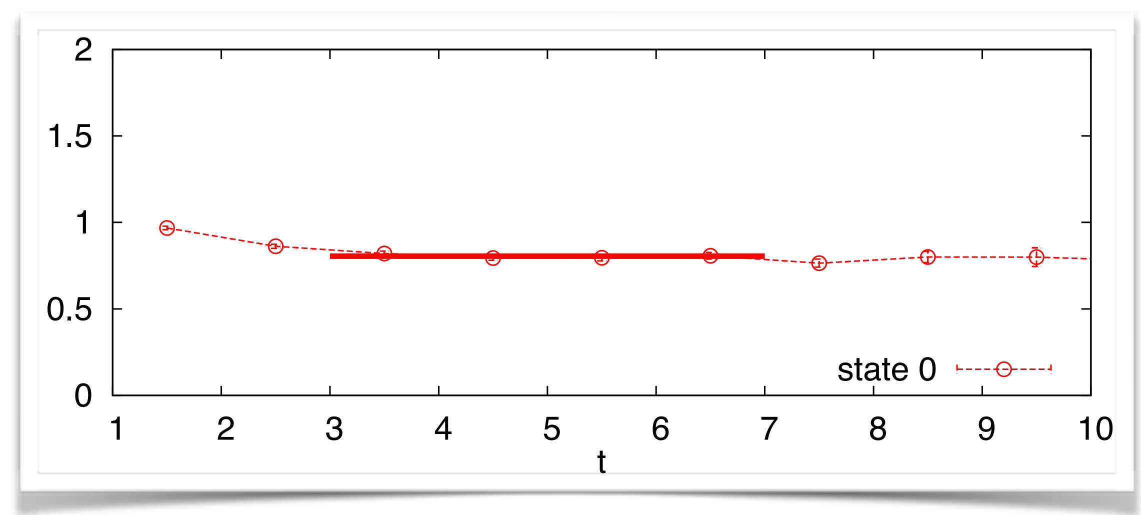
Effective masses of ground state



Correlators (eigenvalues), k=64



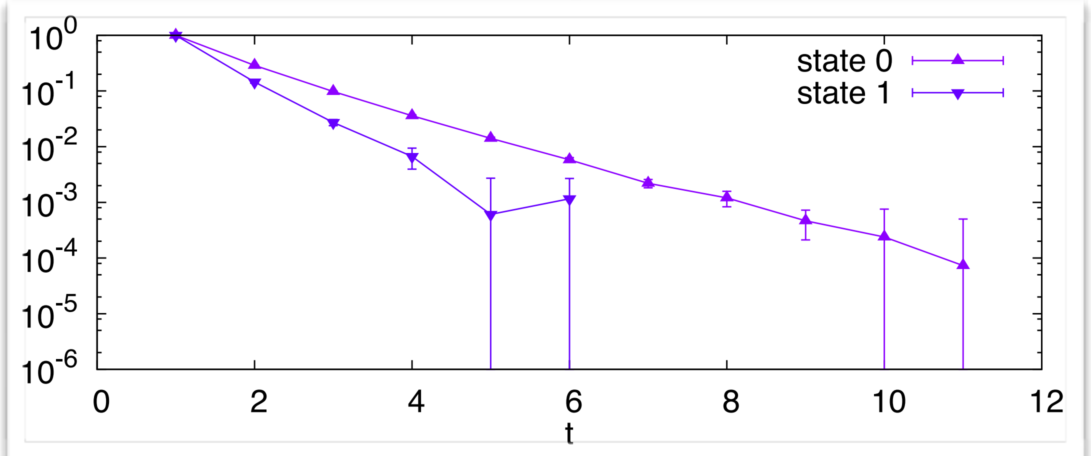
Effective masses of ground state



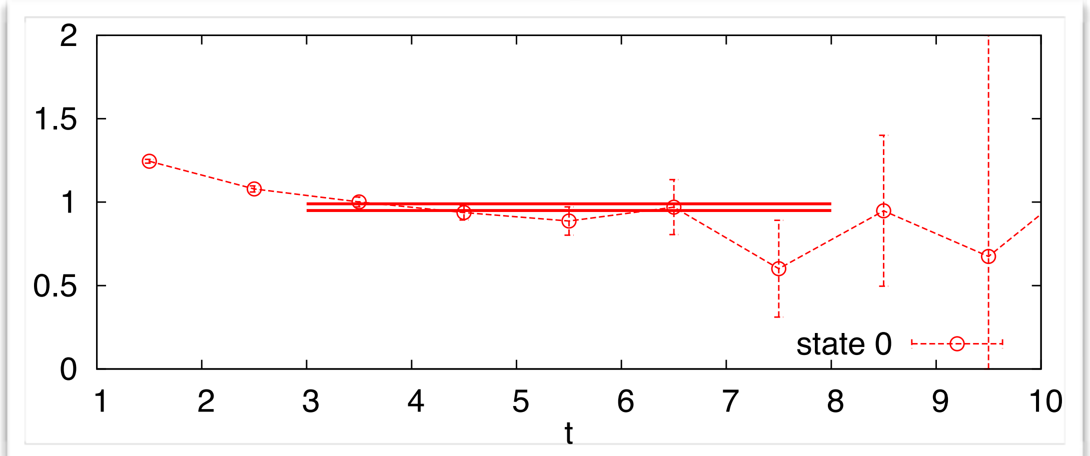
[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Low-mode truncated $b_1(1^{+-})$

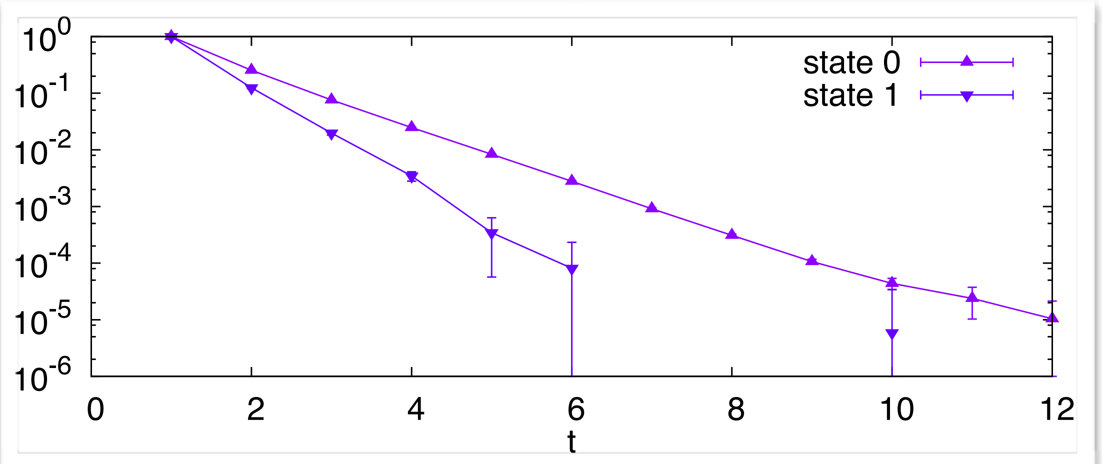
Correlators (eigenvalues), k=2



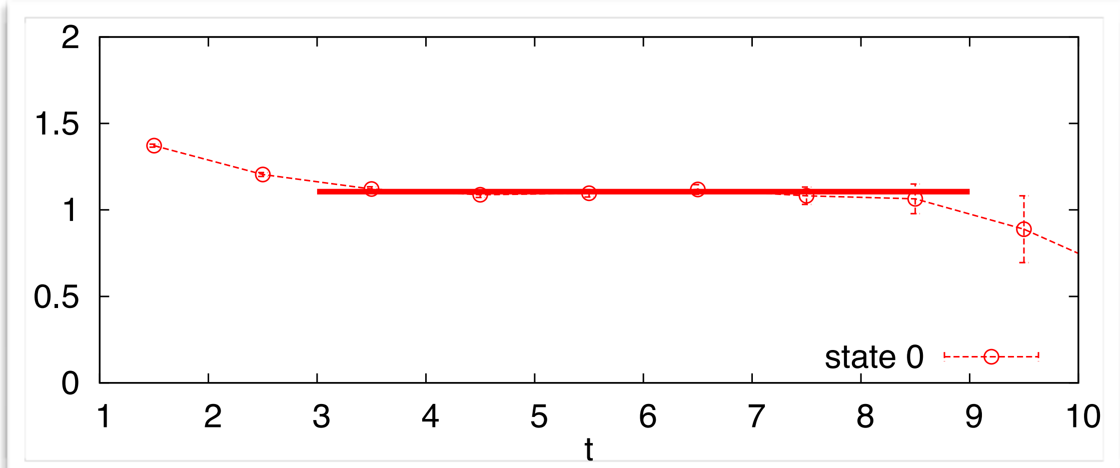
Effective masses of ground state



Correlators (eigenvalues), k=128

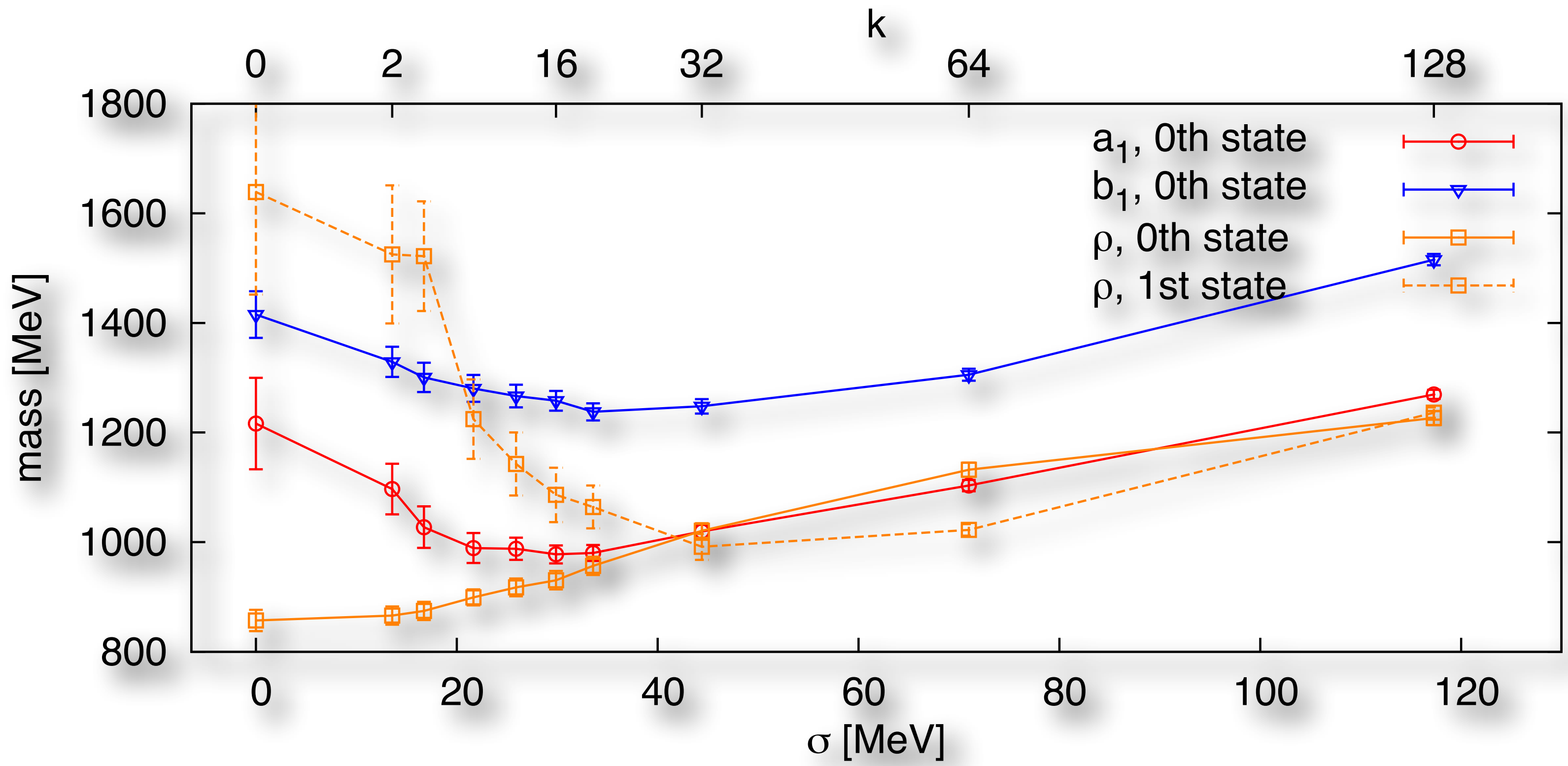


Effective masses of ground state



[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

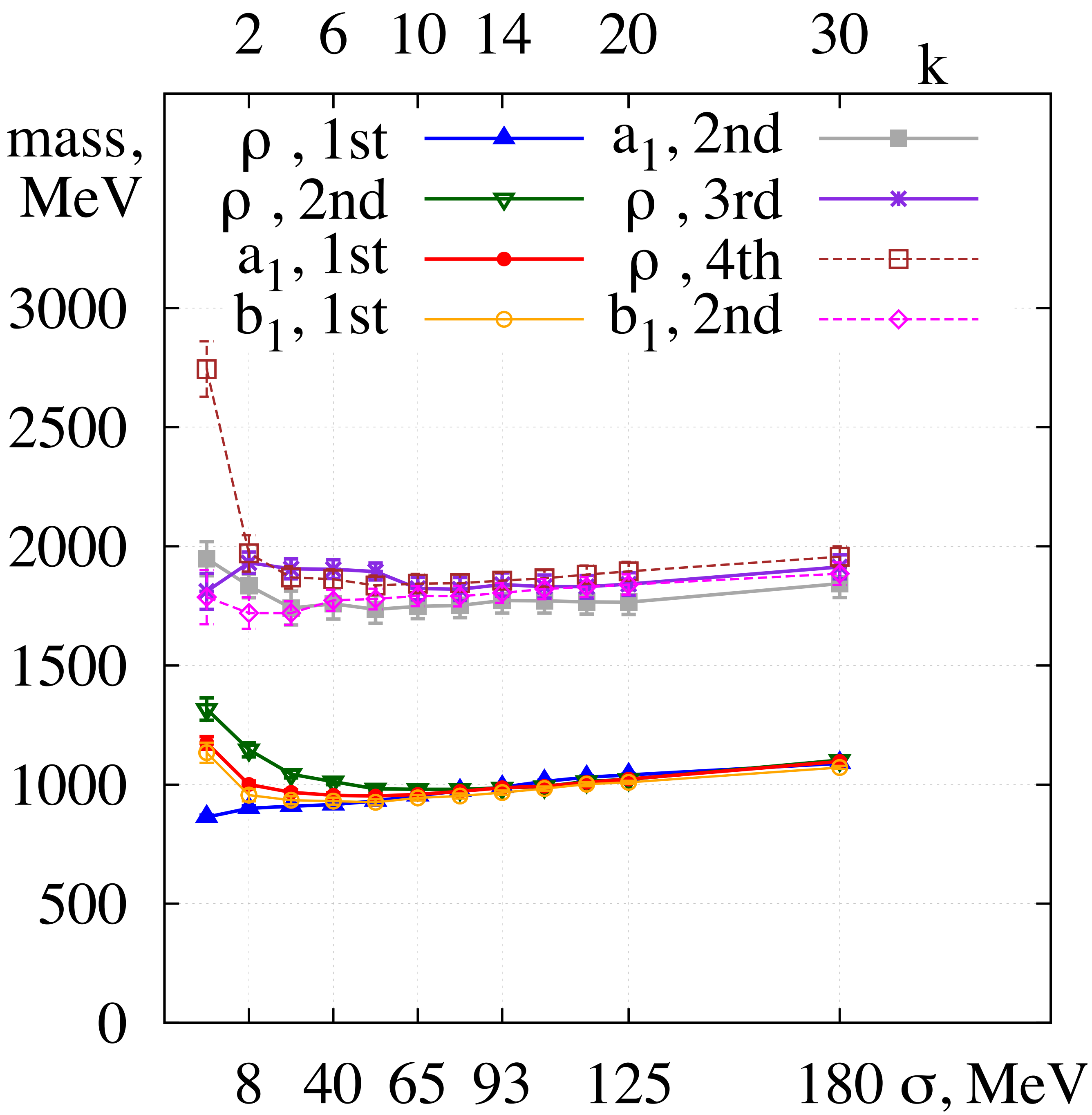
Meson mass evolution



[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

- degeneracy of two lowest rho states
- b_1 mass remains larger then rho mass: single flavor axial symmetry remains broken

Newest overlap results



[M. Denissenya, L.Ya. Glozman, C.B. Lang, arXiv:1402.1887]

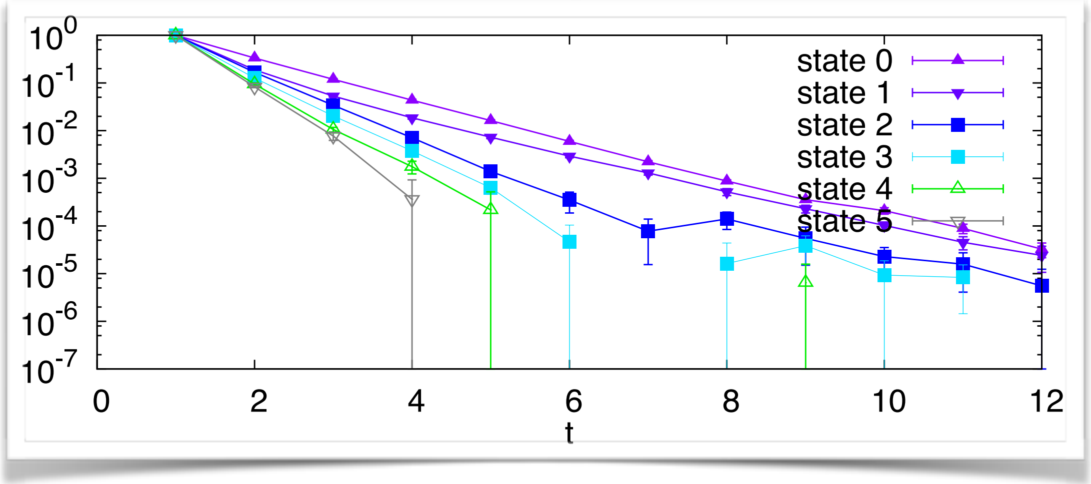
- exact chiral symmetry on the lattice
- in contrast to CI results: reveals restoration of $\mathrm{U}(1)_A$

Variational analysis: baryons

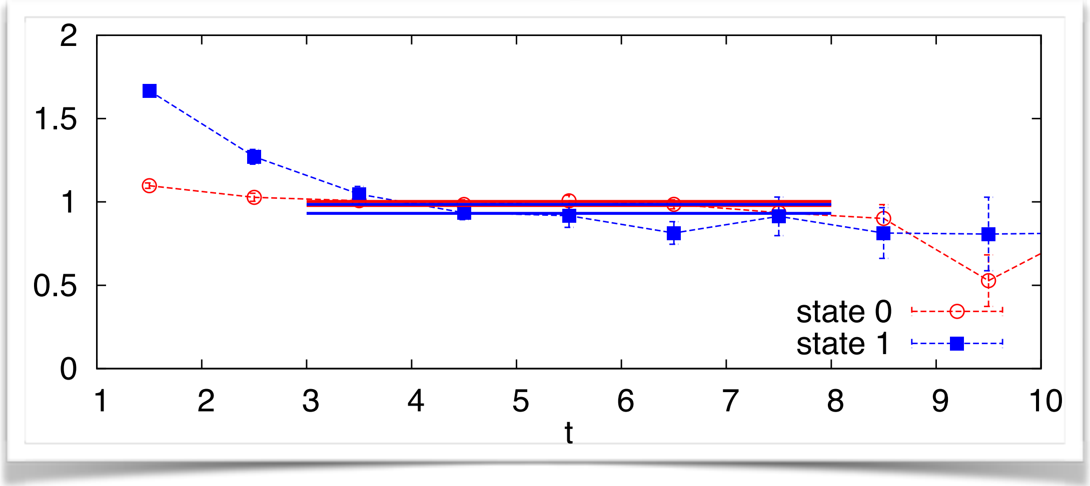
- we study the nucleon and Δ ground and first excited state of positive and negative parity
- can we find parity doubling?
- what happens to the N-Δ splitting?

Truncation $k = 20$ of $N(1/2^+)$

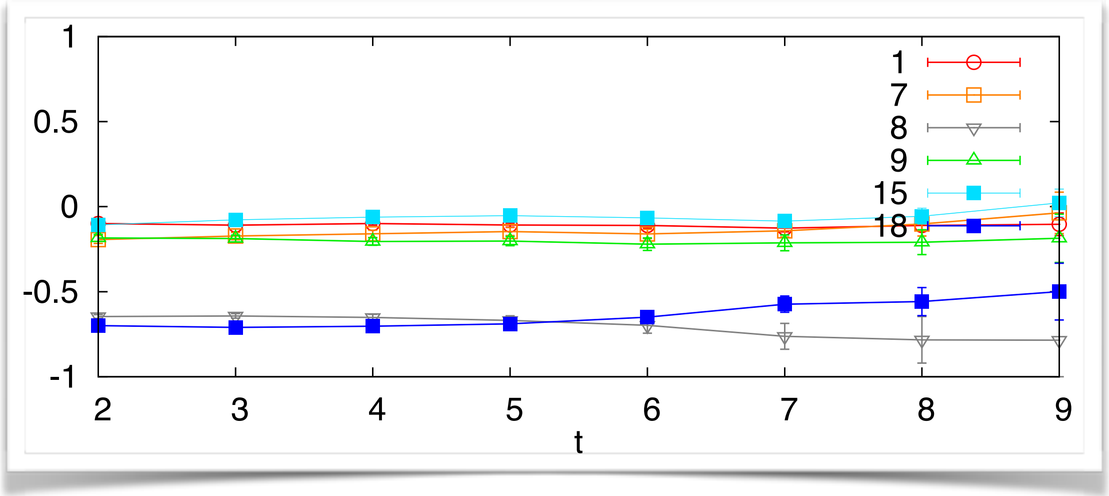
Correlators (eigenvalues) of all states



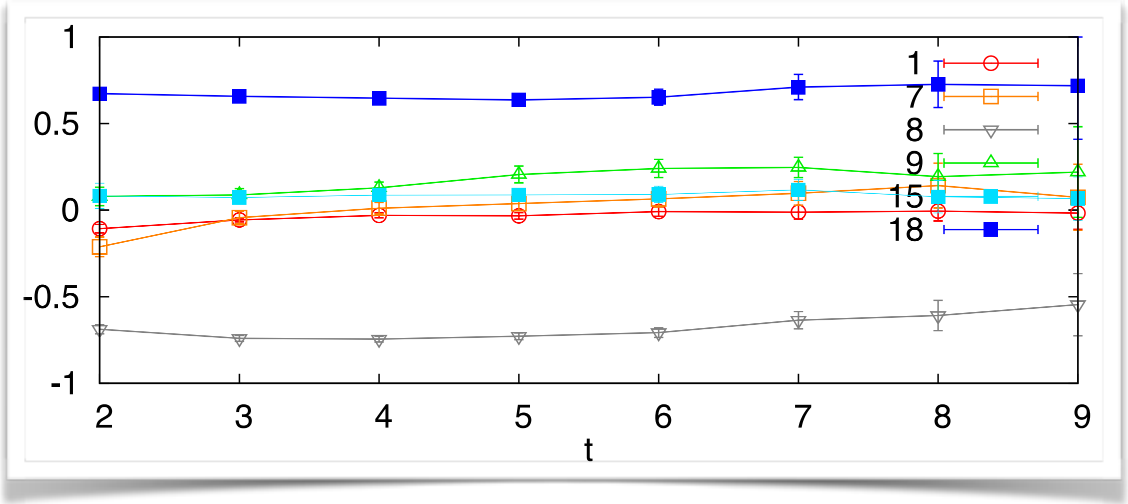
Effective masses of lowest two states



Eigenvectors of ground state



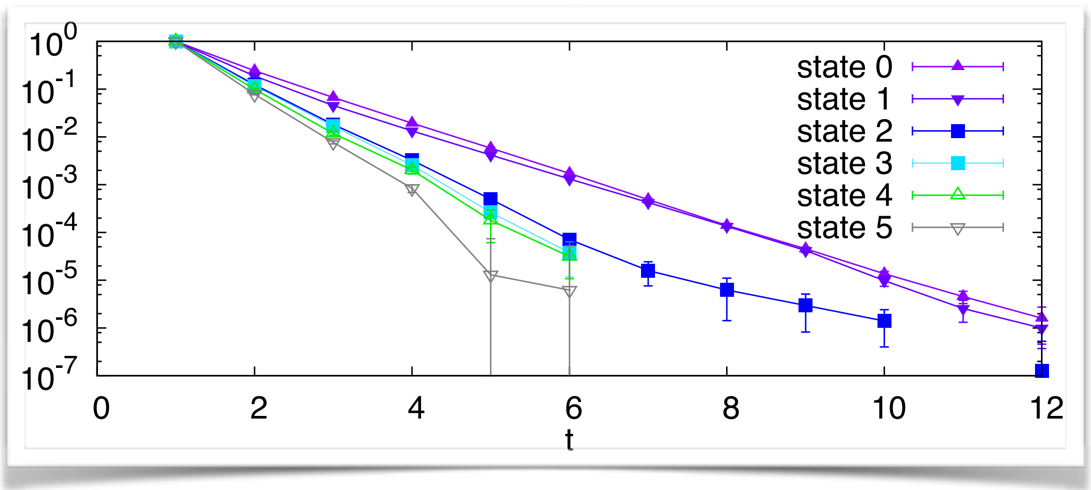
Eigenvectors of first excited state



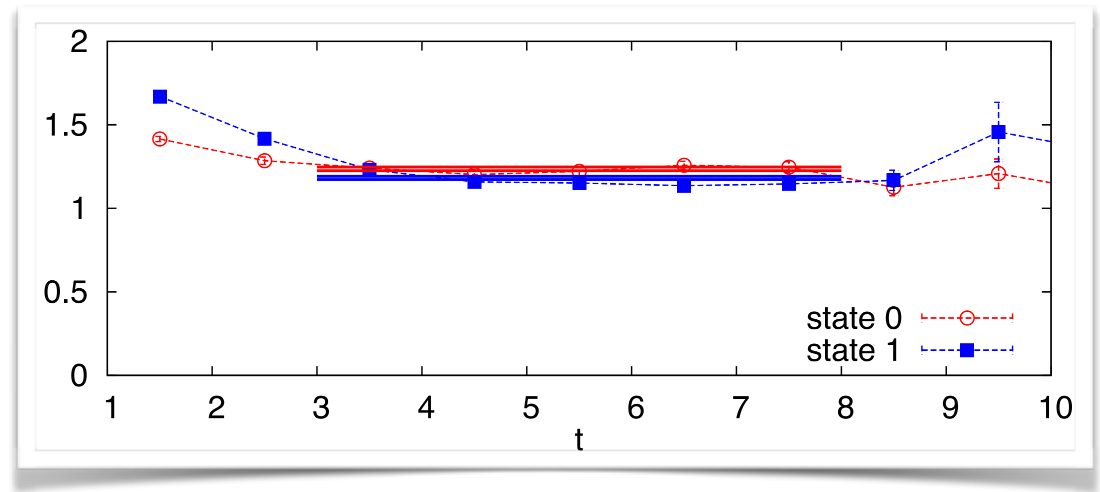
[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Truncation $k = 64$ of $N(1/2^-)$

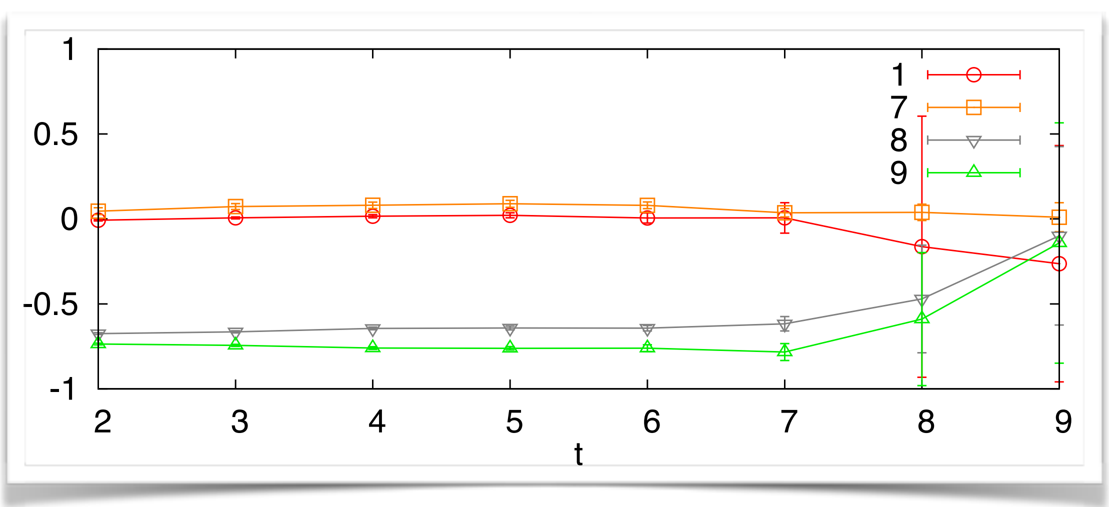
Correlators (eigenvalues) of all states



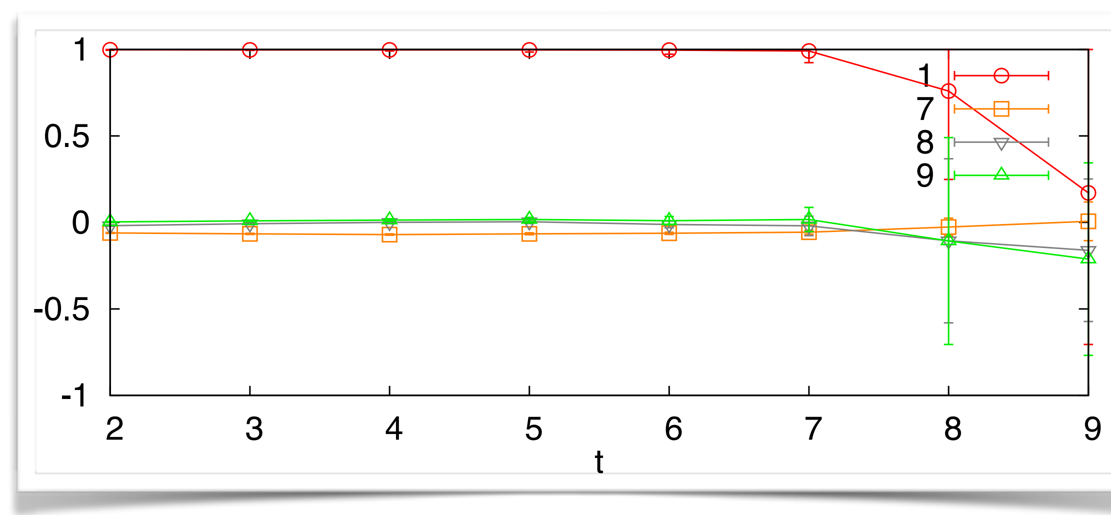
Effective masses of lowest two states



Eigenvectors of ground state



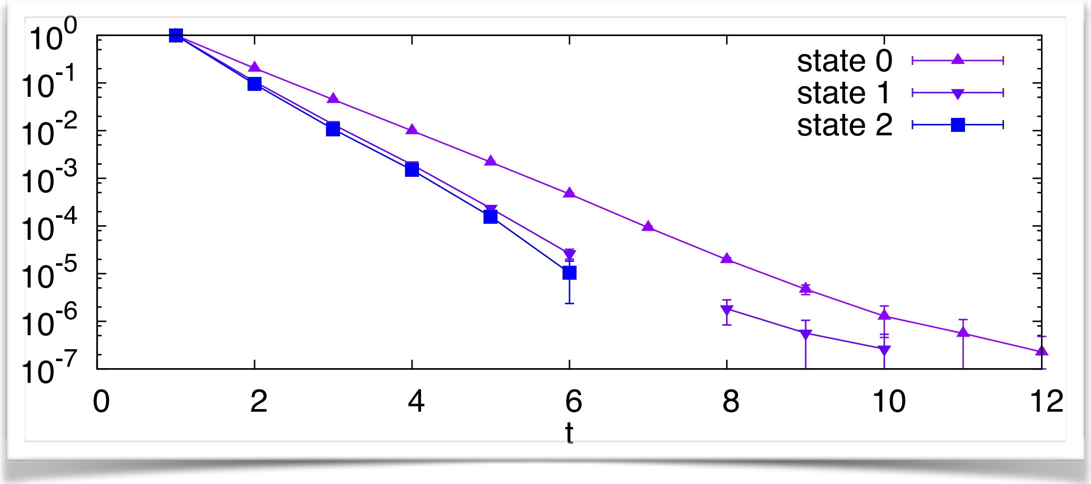
Eigenvectors of first excited state



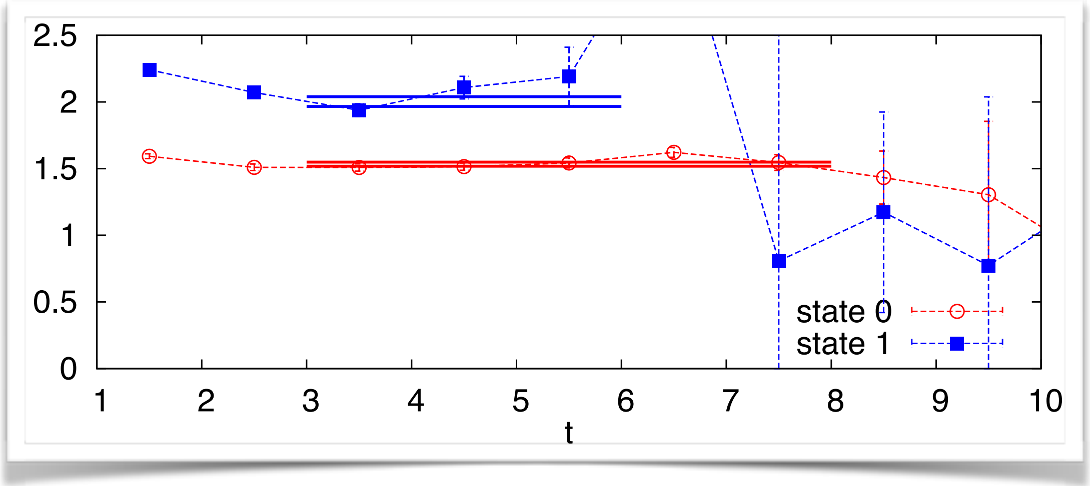
[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Truncation $k = 128$ of $\Delta(1/2^+)$

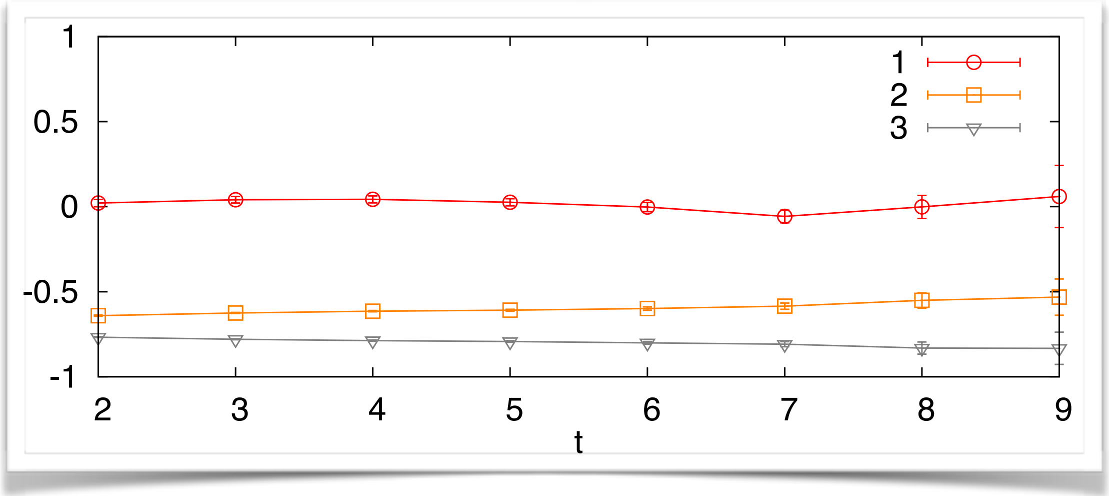
Correlators (eigenvalues) of all states



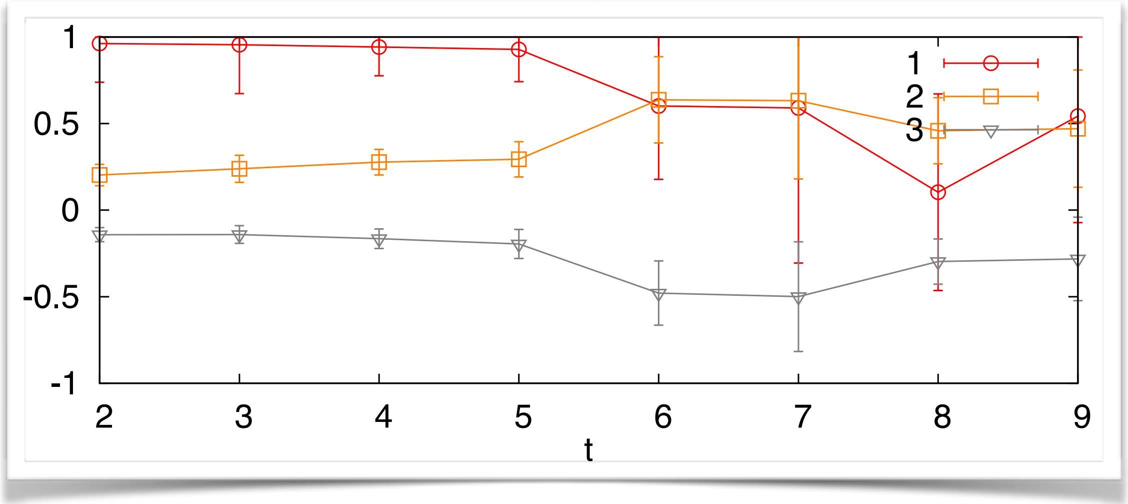
Effective masses of lowest two states



Eigenvectors of ground state



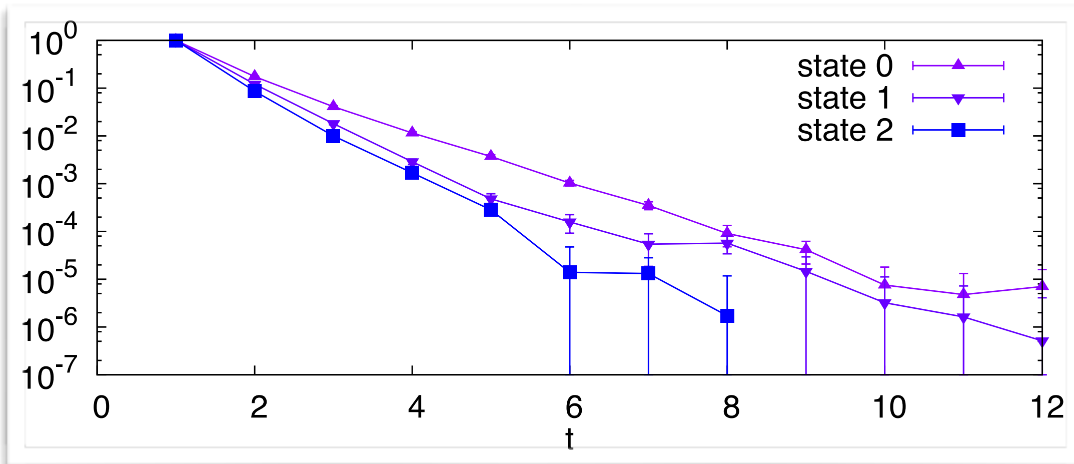
Eigenvectors of first excited state



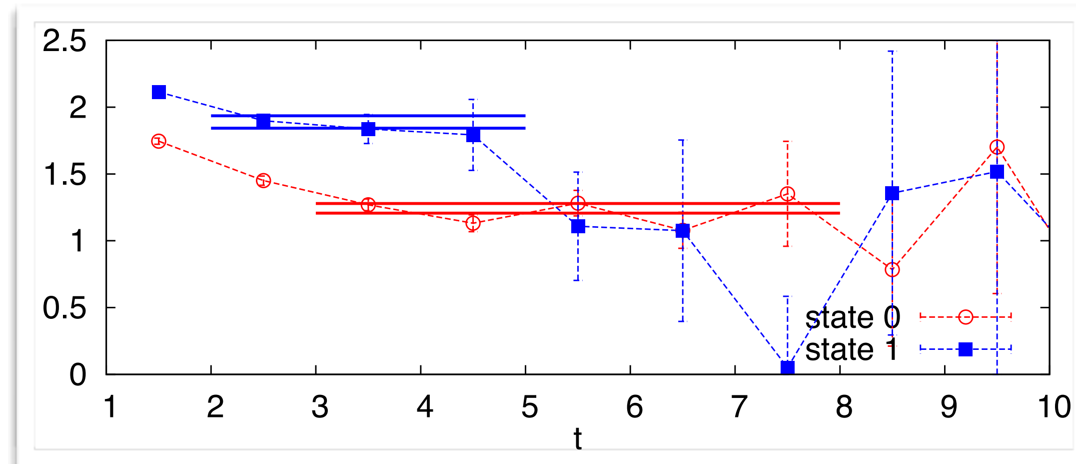
[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Truncation $k = 16$ of $\Delta(1/2^-)$

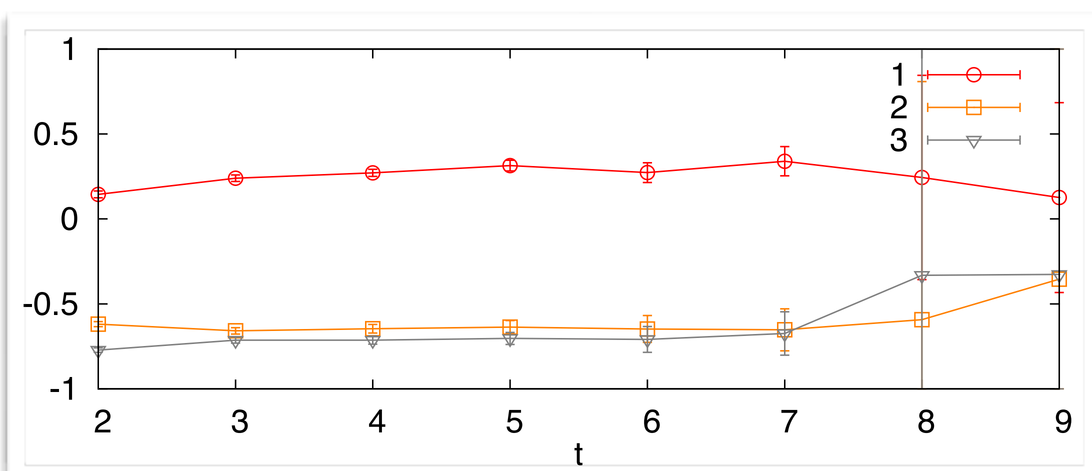
Correlators (eigenvalues) of all states



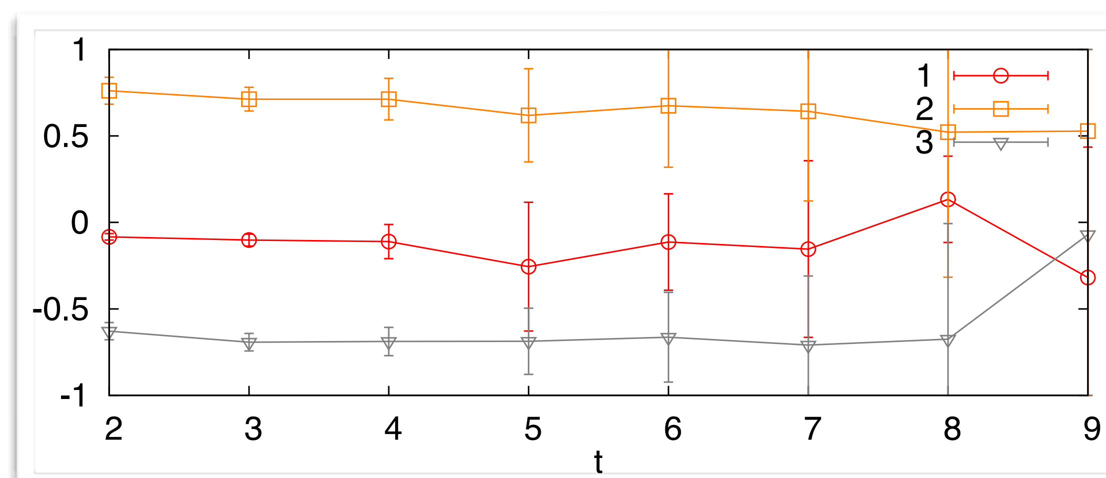
Effective masses of lowest two states



Eigenvectors of ground state

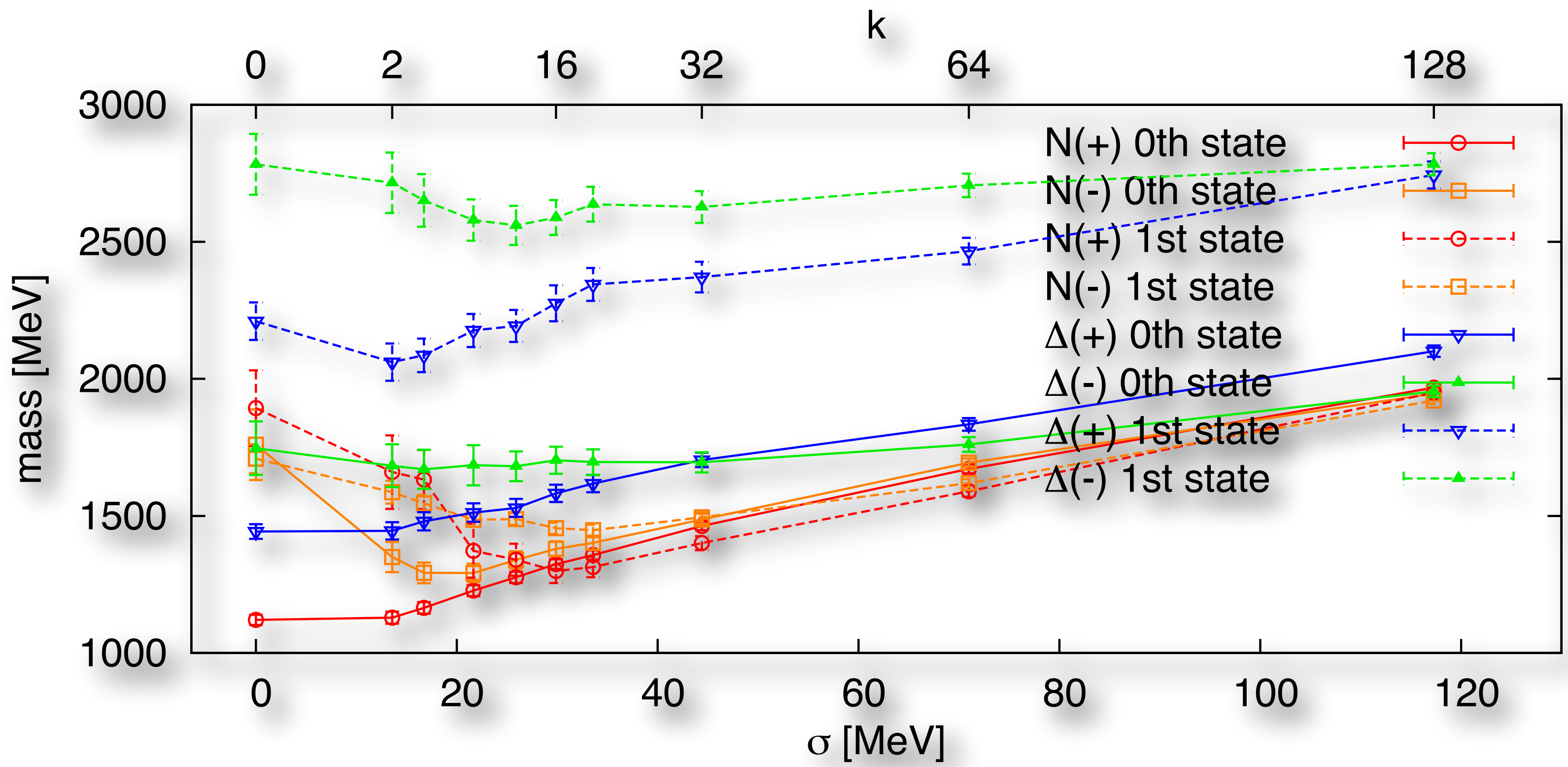


Eigenvectors of first excited state



[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Baryon mass evolution



[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

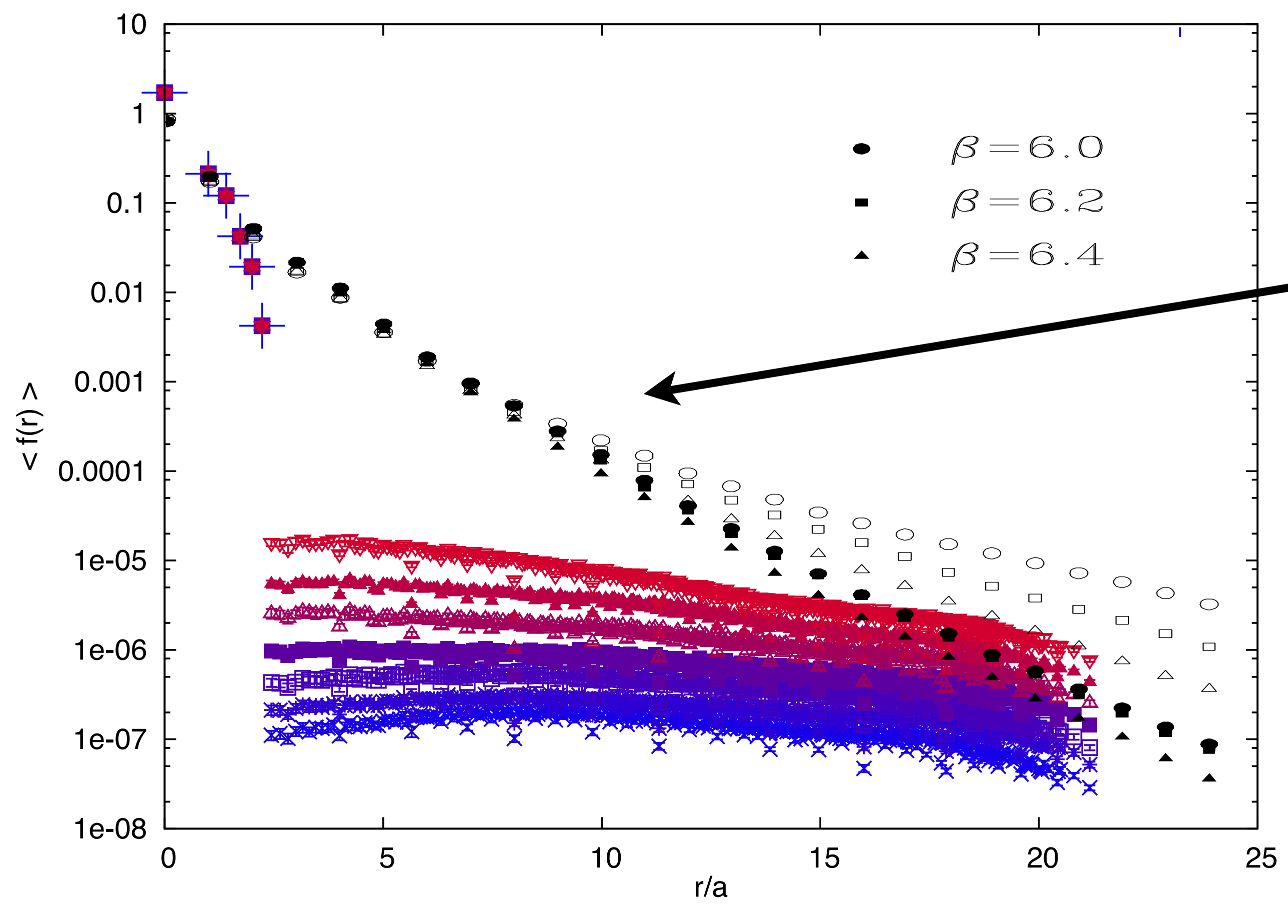
- parity doubling in the $J = 1/2$ and $J = 3/2$ channels
- degeneracy of nucleon ground and exited states
- splitting of Δ ground vs. excited state remains: persistence of confinement

Locality properties

- to what extent is the locality of the low-mode truncated Dirac operator violated?

$$\psi(x)^{[x_0,\alpha_0,a_0]} = \sum_y D_5(x,y)\,\eta(y)^{[x_0,\alpha_0,a_0]}$$

$$f(r) = \max_{x,\alpha_0,a_0} \{\|\psi(x)\| \mid \| x \| = r\}$$



(non)locality of the overlap operator

[Hernandez et al., Nucl. Phys. B **552** (1999) 363–378]

Summary

We removed the lowest lying Dirac eigenmodes of the valence quark sector and found the following effects thereupon

✦ on the quarks:

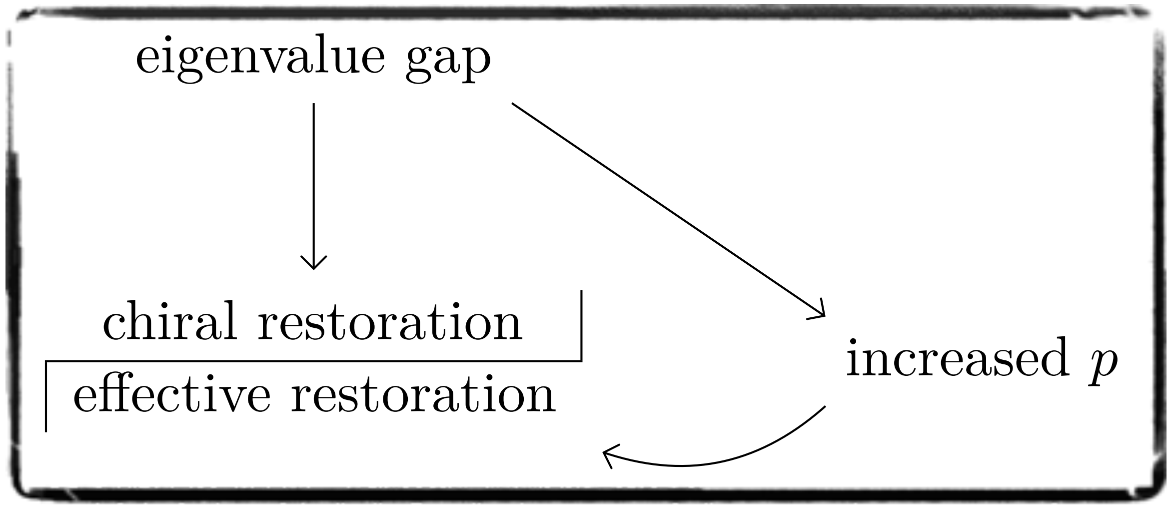
- vanishing of the dynamically generated mass
- no effect on the bare quark mass
- increasing of the quark momenta

✦ on the hadron masses:

- hadron mass increases with the truncation level, due to the increased quark momenta

✦ on the hadron spectrum:

- persistence of confinement
- restoration of chiral symmetry
- no restoration of $\mathrm{U}(1)_A$



Appendix

Baryon interpolators

$$N^{(i)} = \epsilon_{abc}\,\Gamma_1^{(i)}\,u_a\left(u_b^T\,\Gamma_2^{(i)}\,d_c - d_b^T\,\Gamma_2^{(i)}\,u_c\right)\,,$$

$$\Delta_k = \epsilon_{abc}\,u_a\left(u_b^T\,C\,\gamma_k\,u_c\right)$$

$\chi^{(i)}$	$\Gamma_1^{(i)}$	$\Gamma_2^{(i)}$	smearing	$\#_N$
$\chi^{(1)}$	1	$C\,\gamma_5$	$(nn)n$	1
			$(nn)w$	2
			$(nw)n$	3
			$(nw)w$	4
			$(ww)n$	5
			$(ww)w$	6
$\chi^{(2)}$	γ_5	C	$(nn)n$	7
			$(nn)w$	8
			$(nw)n$	9
			$(nw)w$	10
			$(ww)n$	11
			$(ww)w$	12
$\chi^{(3)}$	$i\,1$	$C\,\gamma_t\,\gamma_5$	$(nn)n$	13
			$(nn)w$	14
			$(nw)n$	15
			$(nw)w$	16
			$(ww)n$	17
			$(ww)w$	18

smearing	$\#_\Delta$
$(nn)n$	1
$(nn)w$	2
$(nw)n$	3
$(nw)w$	4
$(ww)n$	5
$(ww)w$	6

Meson interpolators

$\#_\rho$	interpolator(s)
1	$\overline{a}_n \gamma_k b_n$
8	$\overline{a}_w \gamma_k \gamma_t b_w$
12	$\overline{a}_{\partial_k} b_w - \overline{a}_w b_{\partial_k}$
17	$\overline{a}_{\partial_i} \gamma_k b_{\partial_i}$
22	$\overline{a}_{\partial_k} \varepsilon_{ijk} \gamma_j \gamma_5 b_w - \overline{a}_w \varepsilon_{ijk} \gamma_j \gamma_5 b_{\partial_k}$
$\#_{a_1}$	interpolator(s)
1	$\overline{a}_n \gamma_k \gamma_5 b_n$
2	$\overline{a}_n \gamma_k \gamma_5 b_w + \overline{a}_w \gamma_k \gamma_5 b_n$
4	$\overline{a}_w \gamma_k \gamma_5 b_w$
$\#_{b_1}$	interpolator(s)
6	$\overline{a}_{\partial_k} \gamma_5 b_n - \overline{a}_n \gamma_5 b_{\partial_k}$
8	$\overline{a}_{\partial_k} \gamma_5 b_w - \overline{a}_w \gamma_5 b_{\partial_k}$