Effects of the lowest Dirac modes on the spectrum of ground state mesons

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Outline

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Motivation

Method

Results

Summary

Chiral symmetry and the low-lying Dirac eigenmodes

- Neglecting the two lightest quark masses the QCD Lagrangian is invariant under the symmetry group

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_A.$$

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- The single flavor axial symmetry $\mathrm{U}(1)_A$ mixes the currents of the same isospin but opposite parity. It is not only broken spontaneously but also explicitly by the anomaly.
- The Banks-Casher relation relates the density $\rho(0)$ of the Dirac modes near the origin to the chiral condensate.

"Unbreaking" chiral symmetry

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"Unbreaking" chiral symmetry

- We construct meson correlators out of *reduced* quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., **[DeGrand, Phys. Rev. D 69, 2004]**).
- Using these propagators we compute meson masses and we study the possible recovery of the degeneracies in the spectrum of (would be) chiral partners.

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- The dynamics of QCD seem to be more sensitive to the low-modes of D_5 rather than of D.
- The spectral representation of the quark propagator using D_5 reads

$$S = \sum_{i=1}^{N} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5$$

Reducing quark propagators

- Split S into a low mode (LM) part and a *reduced* (RD) part

$$
\begin{aligned}
S &= \sum_{i \le k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 + \sum_{i > k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 \\
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$$= S_{\mathrm{LM}(k)} + S_{\mathrm{RD}(k)}$$

- In this work we investigate the *reduced* (RD) part of the propagator

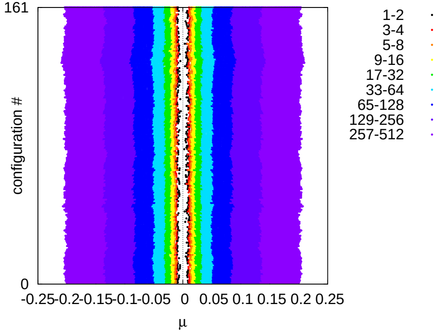
$$S_{\mathrm{RD}(k)} = S - S_{\mathrm{LM}(k)}$$

The Setup

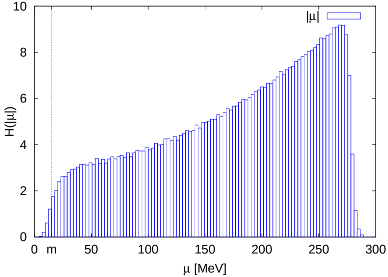
- 161 configurations **[Gattringer et al., Phys. Rev. D 79, 2009]**
- size $16^3 \times 32$
- two degenerate flavors of light fermions, $m_\pi = 322(5)\,\mathrm{MeV}$
- lattice spacing $a = 0.1440(12)\,\mathrm{fm}$
- Chirally Improved Dirac operator **[Gattringer, Phys. Rev. D 63, 2001]** (approximate solution of the Ginsparg-Wilson equation)
- Jacobi smeared quark sources

Eigenvalues of D_5

The lowest 512 eigenvalues

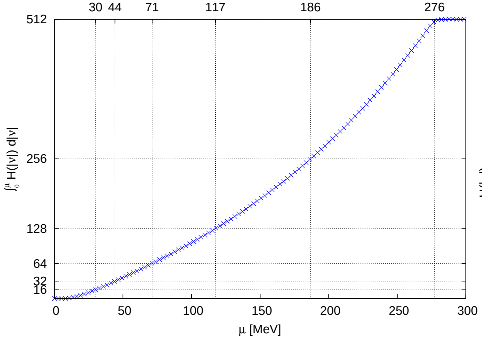


Histogram $H(|\mu|)$



Eigenvalues of D_5

$\int_0^{\mu} H(|\nu|)\, d|\nu|$



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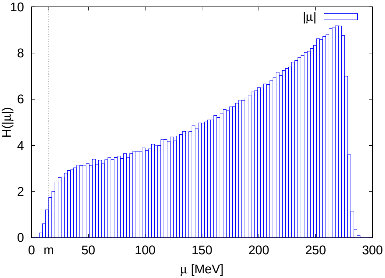

Chiral partners under investigation

We restrict ourselves to the study of isovectors:

- the vector meson $\rho(J^{PC} = 1^{--})$ with interpolating fields

$$\overline{u}\gamma_i d, \quad \overline{u}\gamma_t\gamma_i d$$

and the axial vector $a_1(J^{PC} = 1^{++})$ with

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which (would) get mixed via the isospin axial transformations.

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  which (would) get mixed via the isospin axial transformations.
- The pseudo-scalar $\pi(J^{PC} = 0^{-+})$ with interpolating fields

  $$\overline{u}\gamma_5 d, \quad \overline{u}\gamma_t\gamma_5 d$$

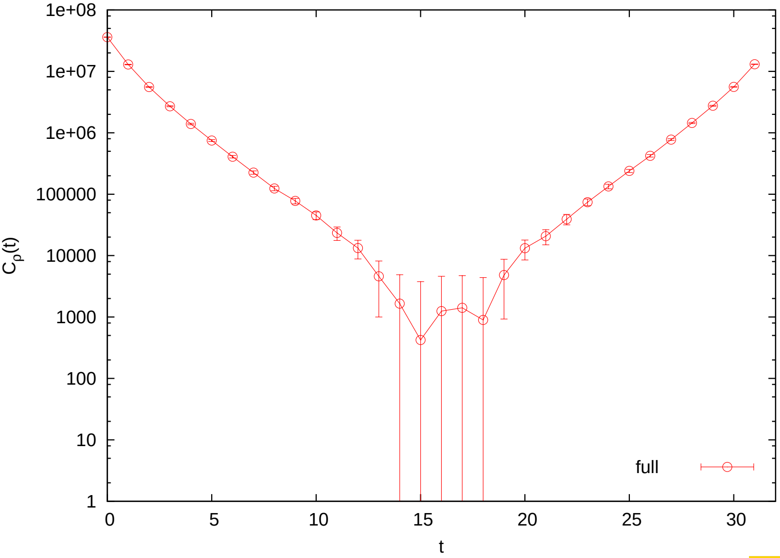
  and the scalar $a_0(J^{PC} = 0^{++})$,

  $$\overline{u}d$$

  which (would) get mixed via the $\mathrm{U}(1)_A$ transformation.

Outline

ρ, $J^{PC} = 1^{--}$, $\overline{u}\gamma_i d$



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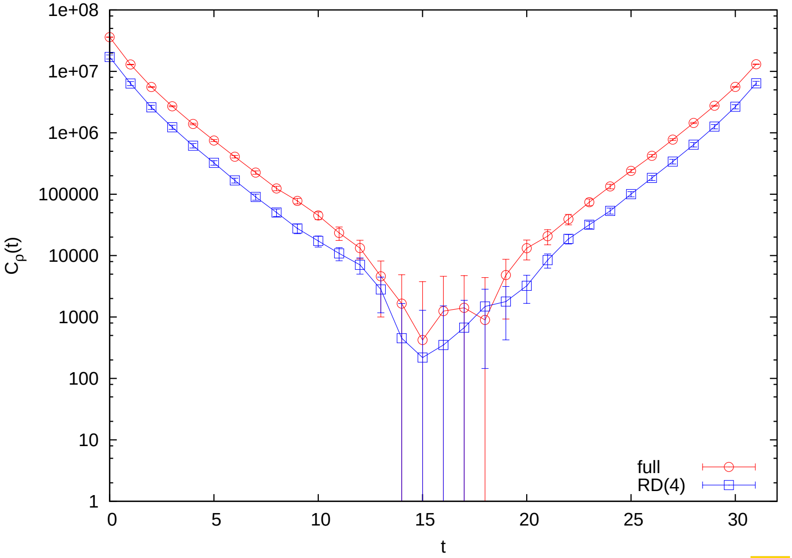

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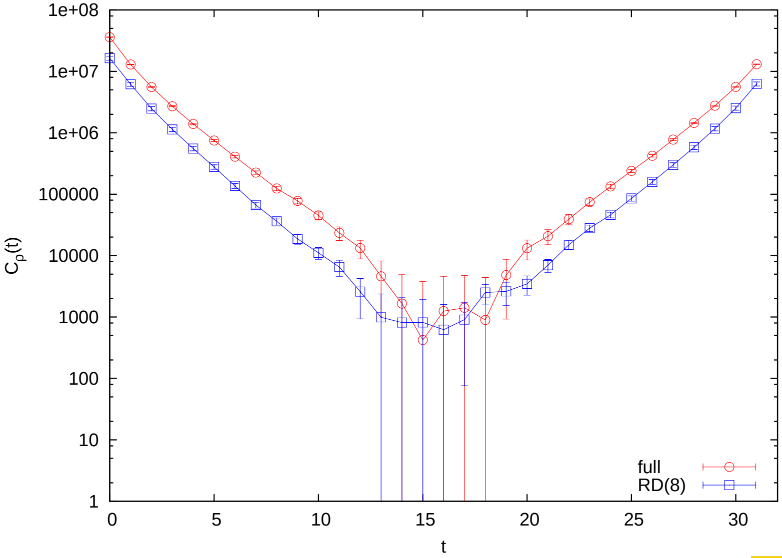

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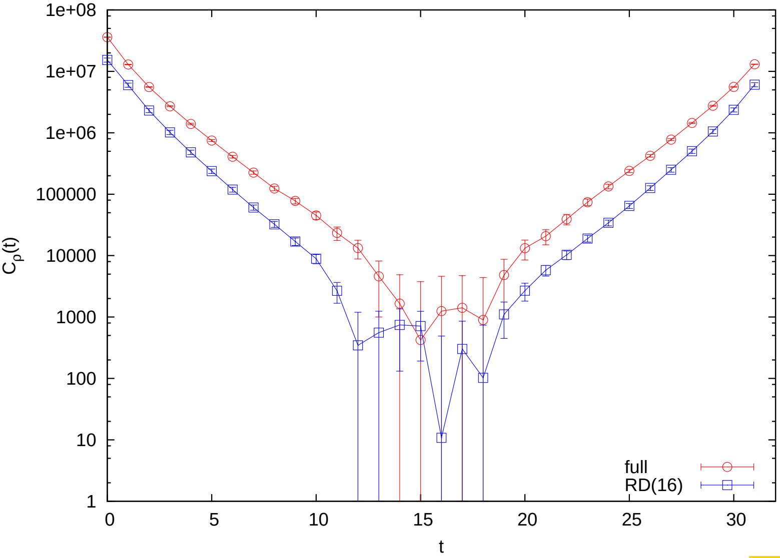

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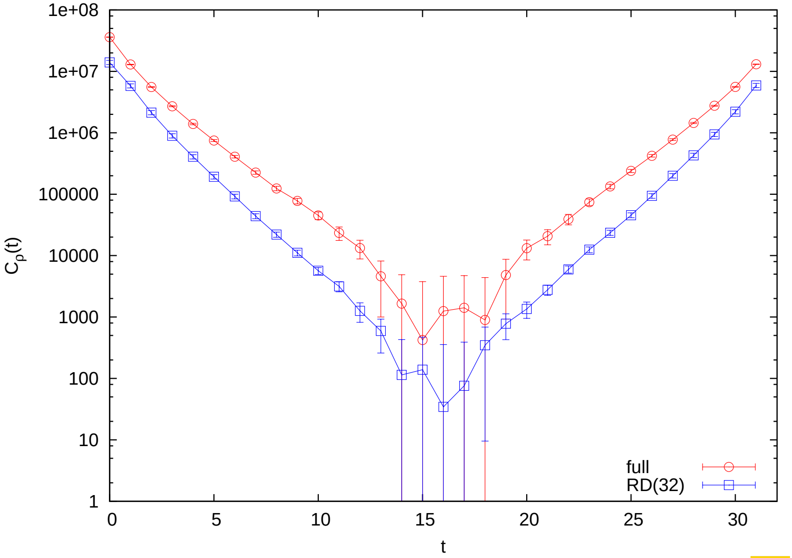

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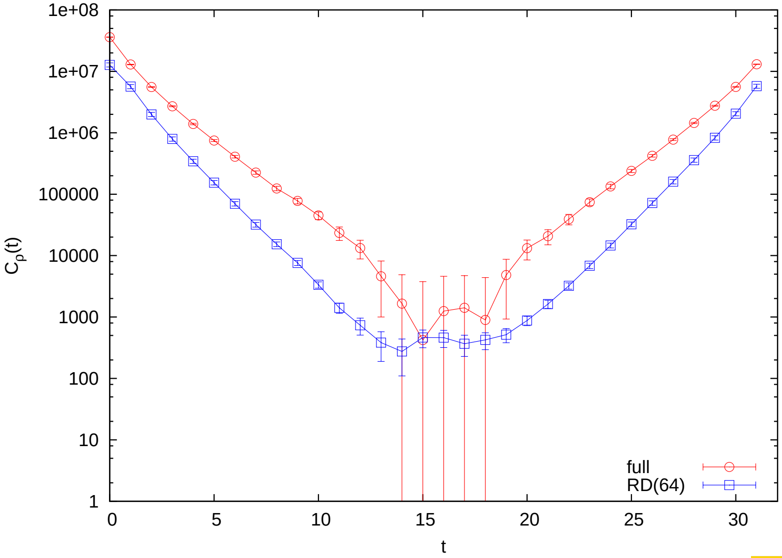

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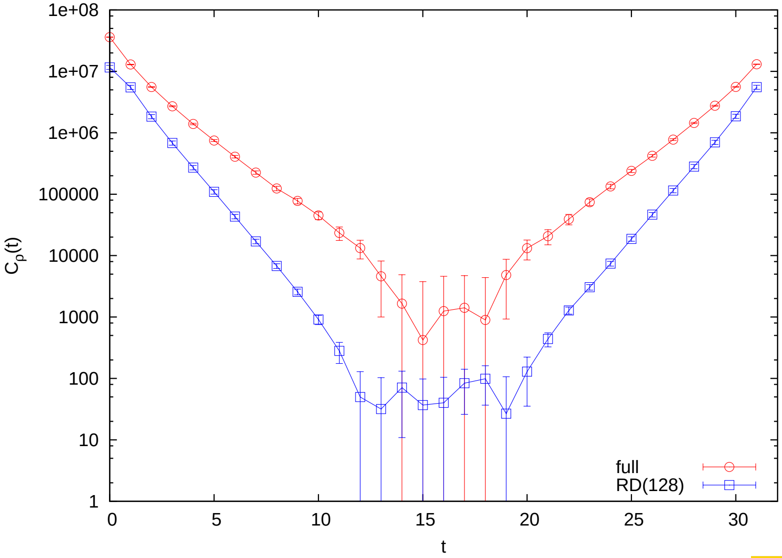

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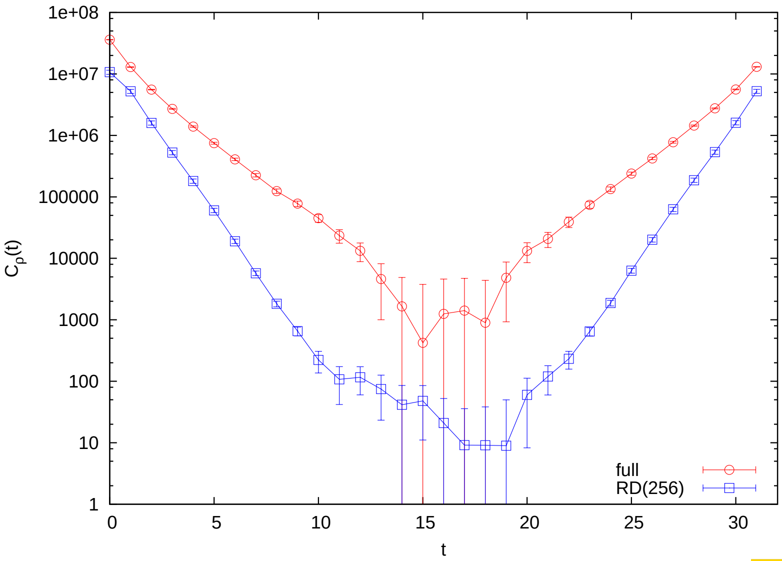

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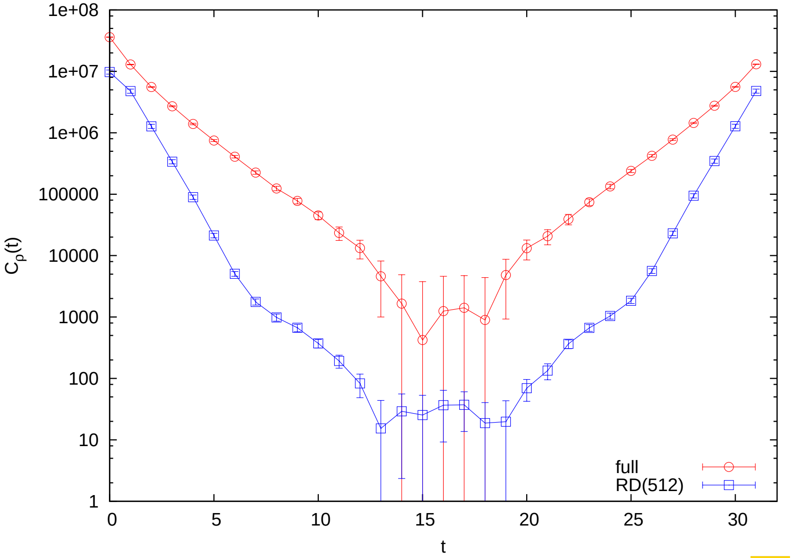

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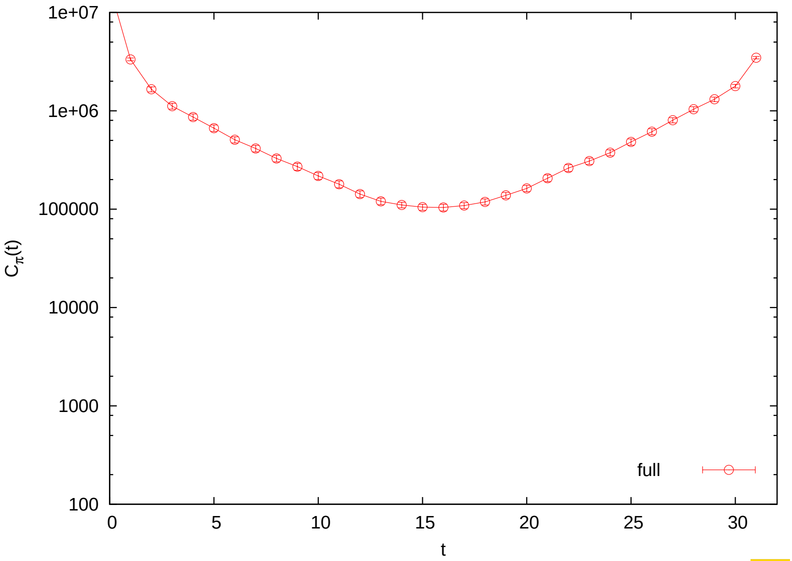

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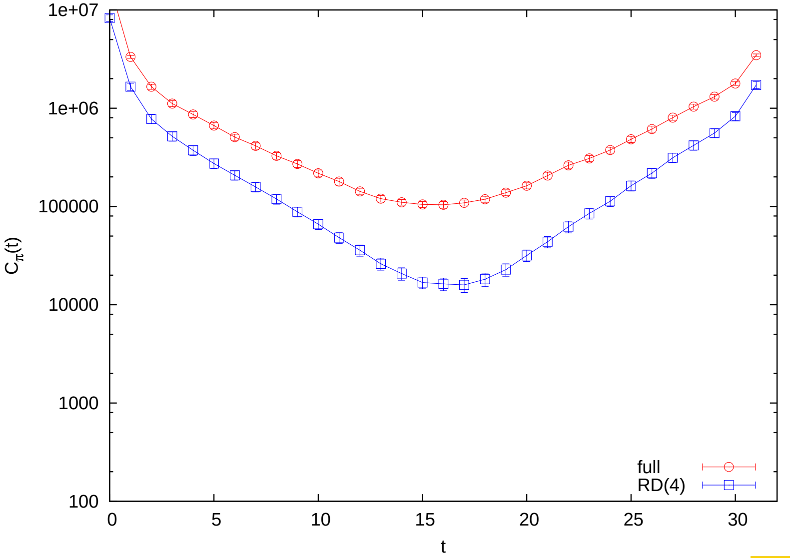

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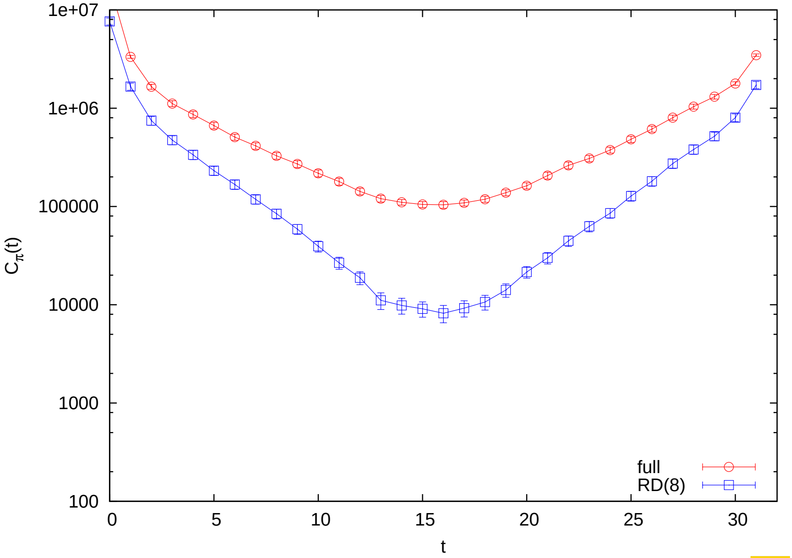

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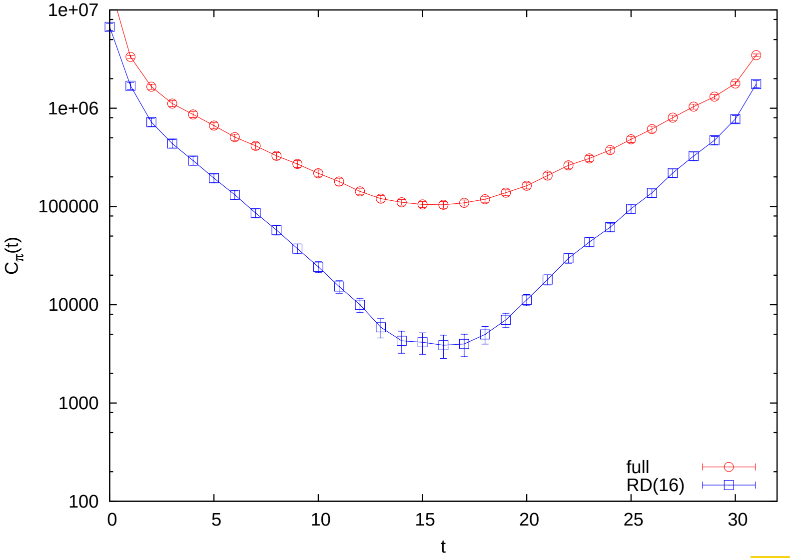

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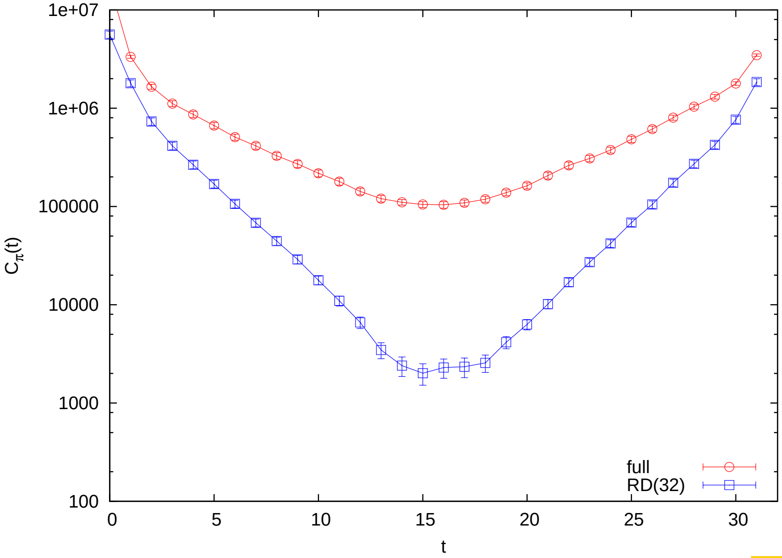

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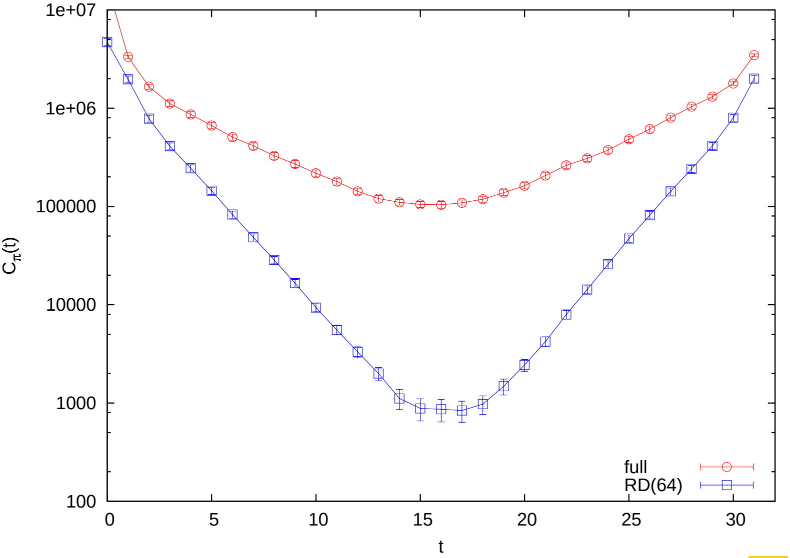

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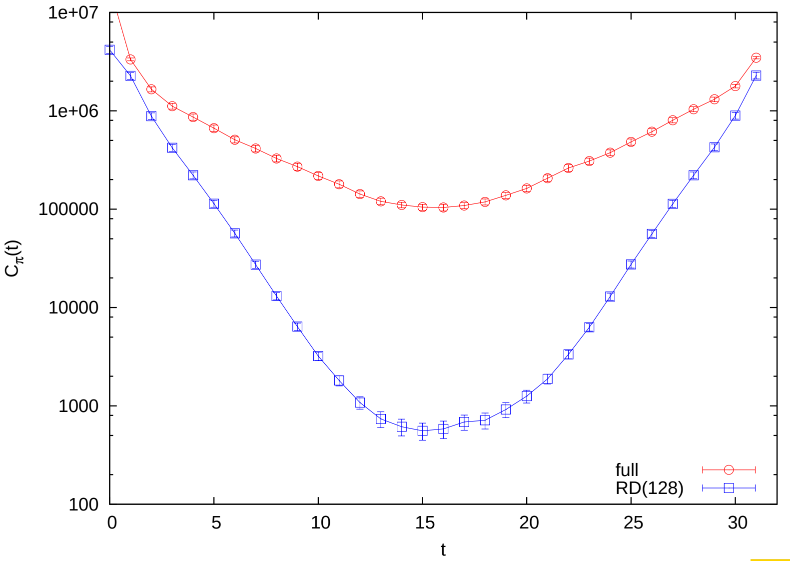

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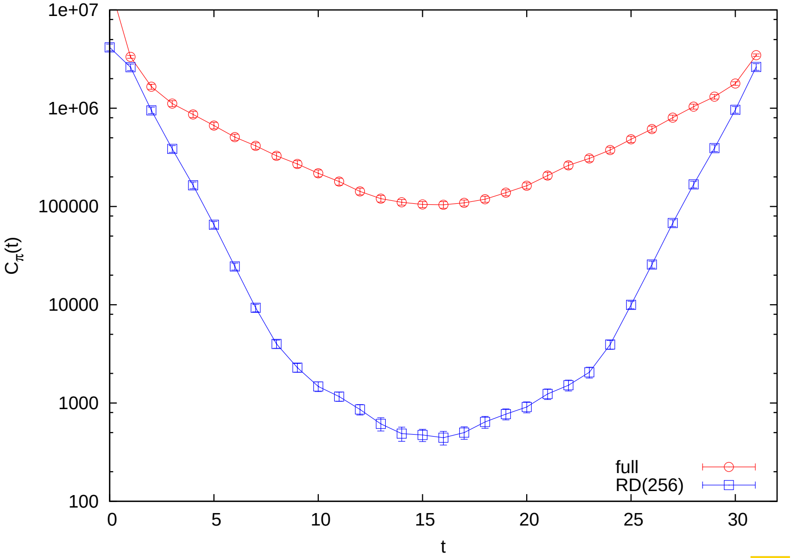

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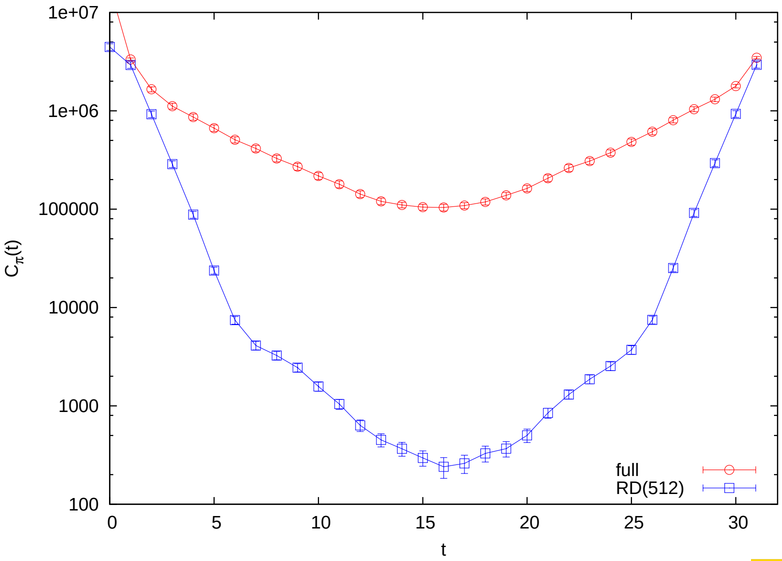

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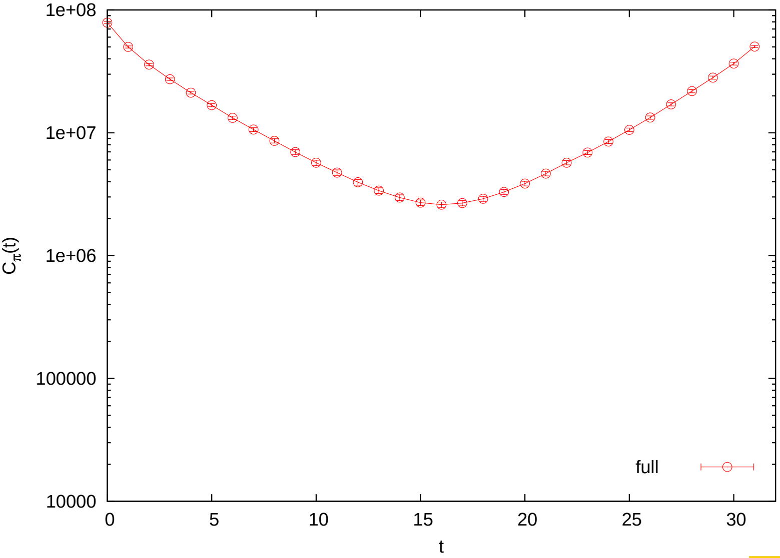

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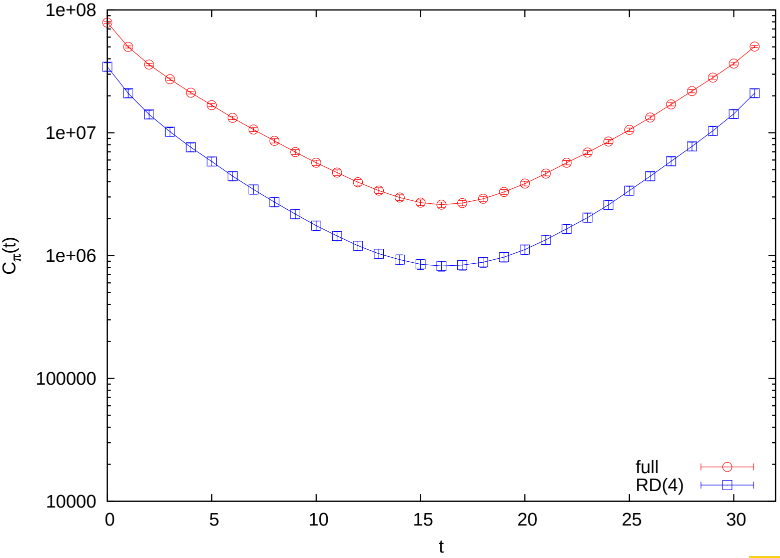

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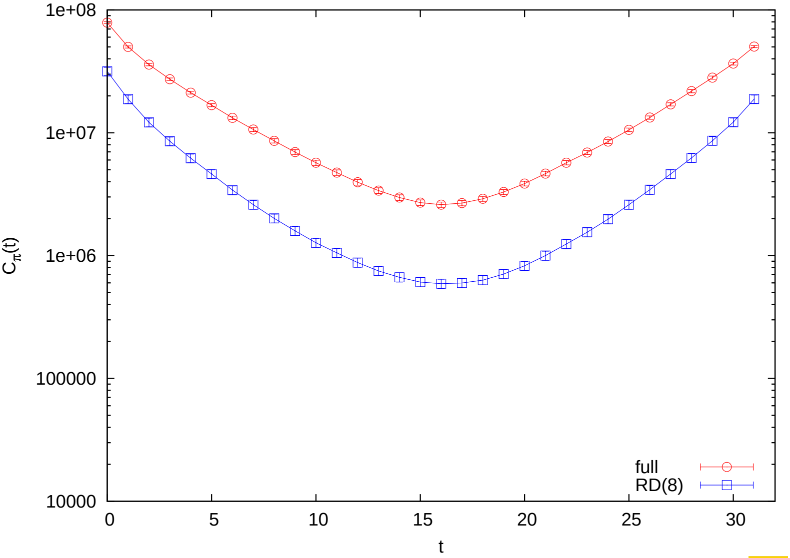

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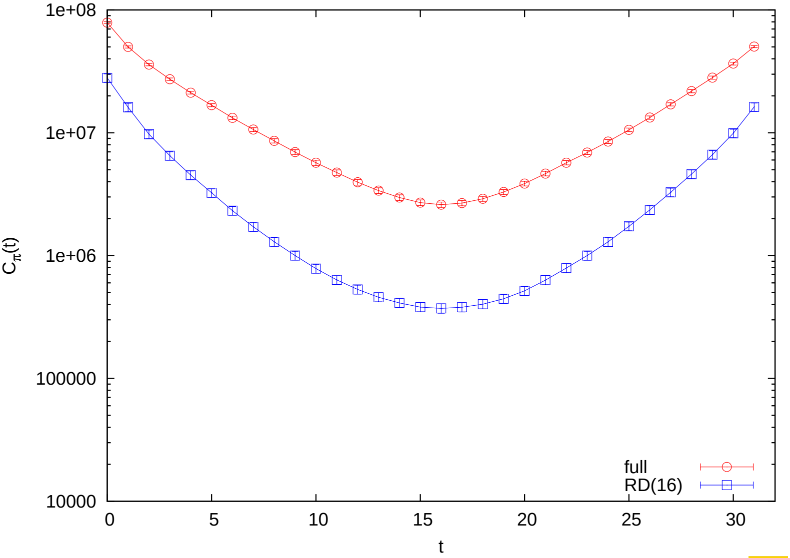

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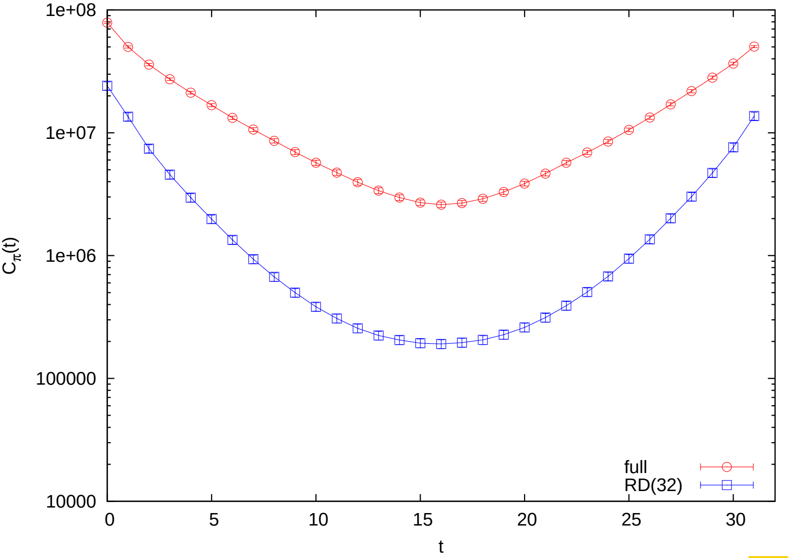

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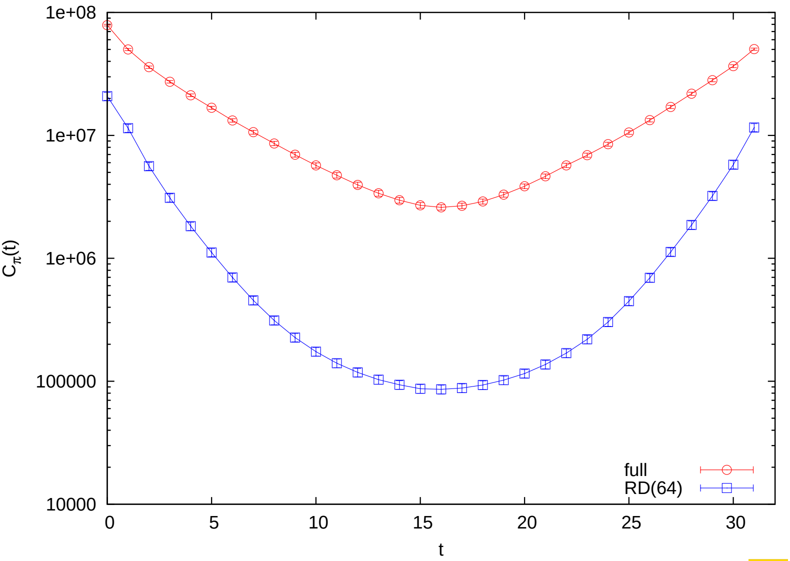

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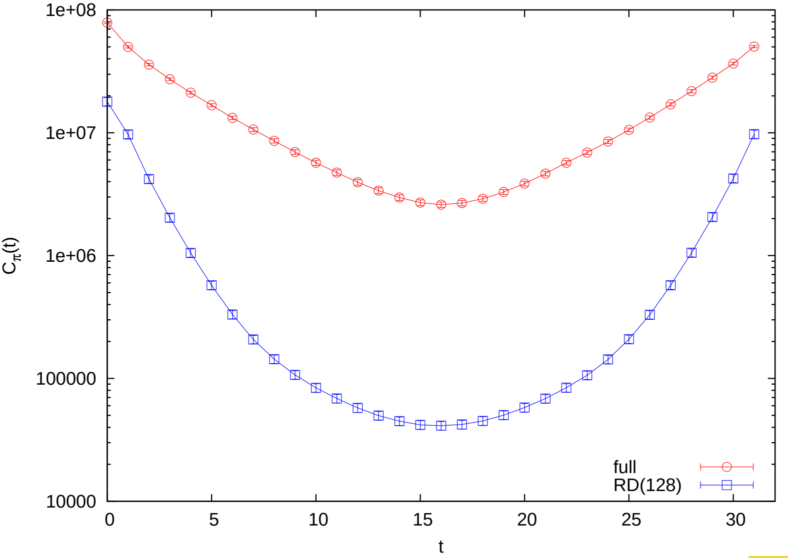

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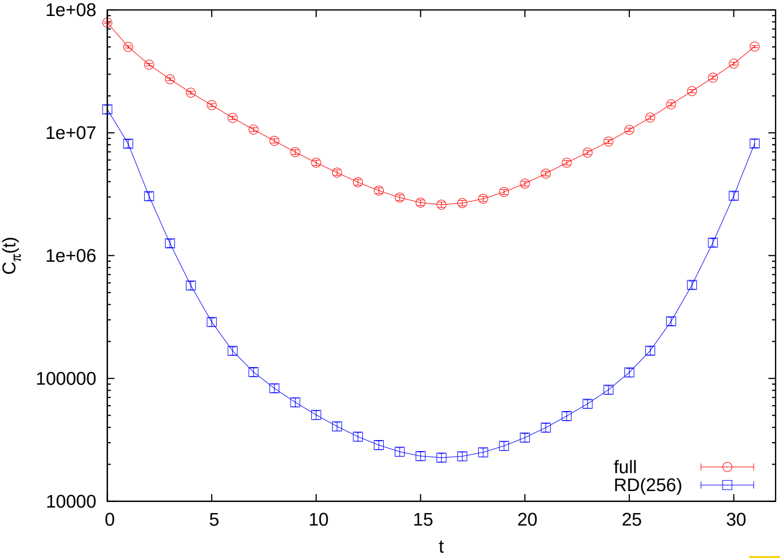

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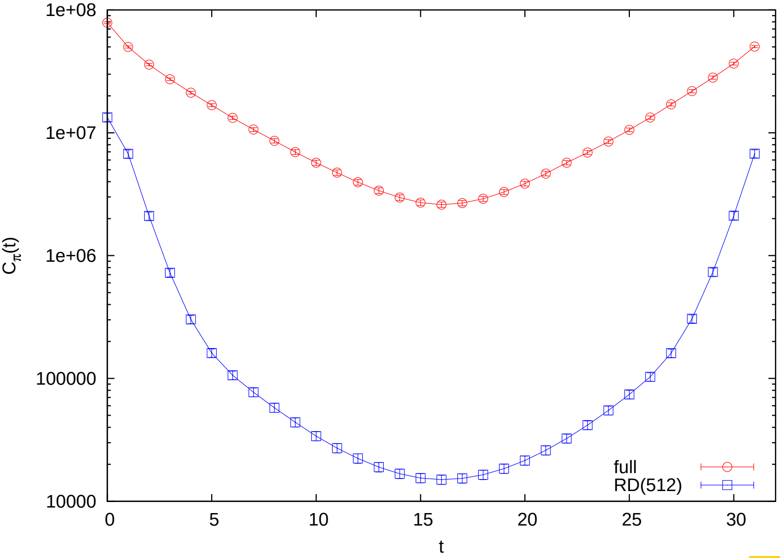

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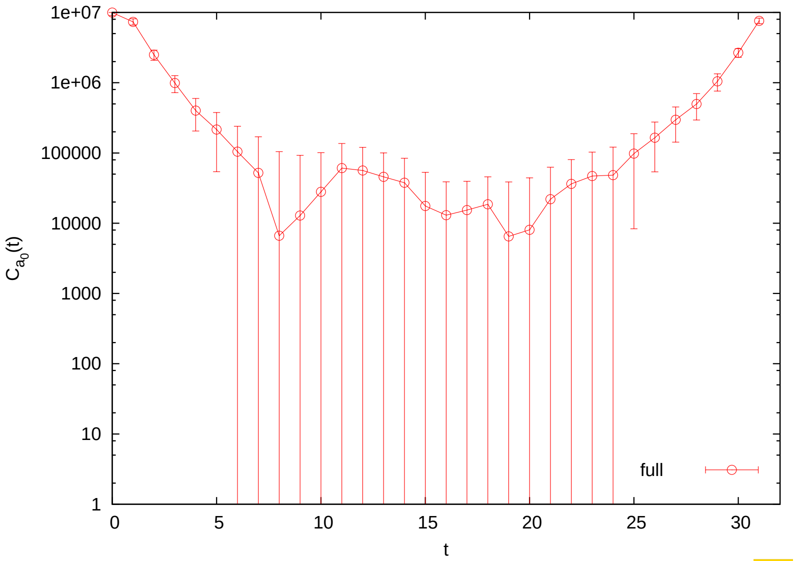

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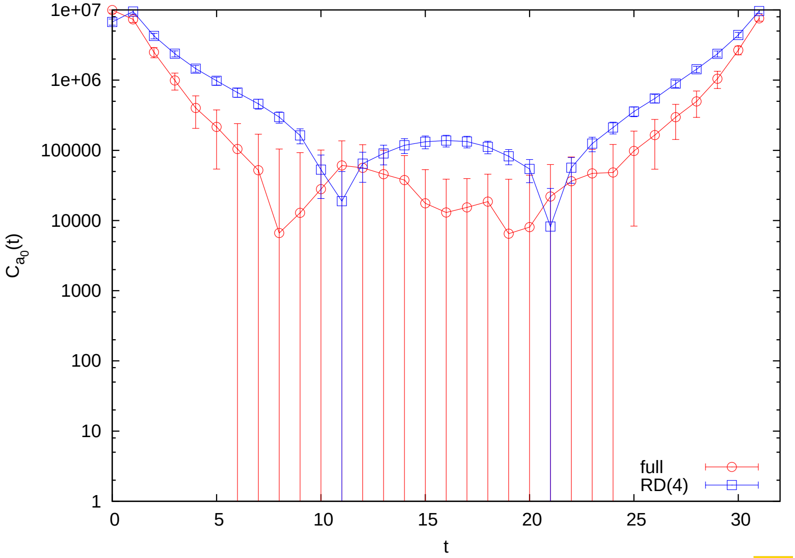

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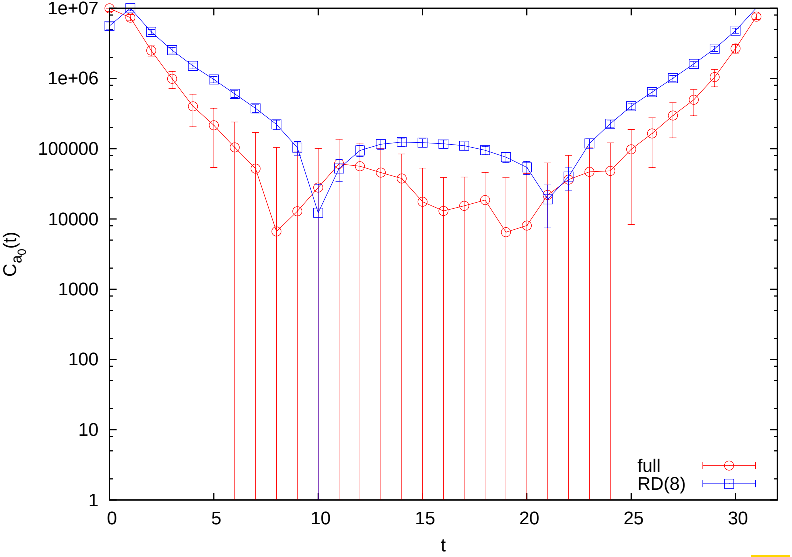

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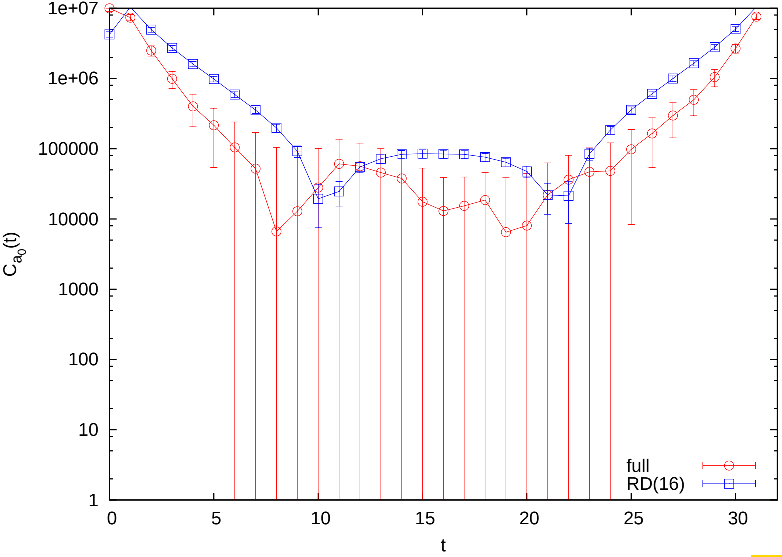

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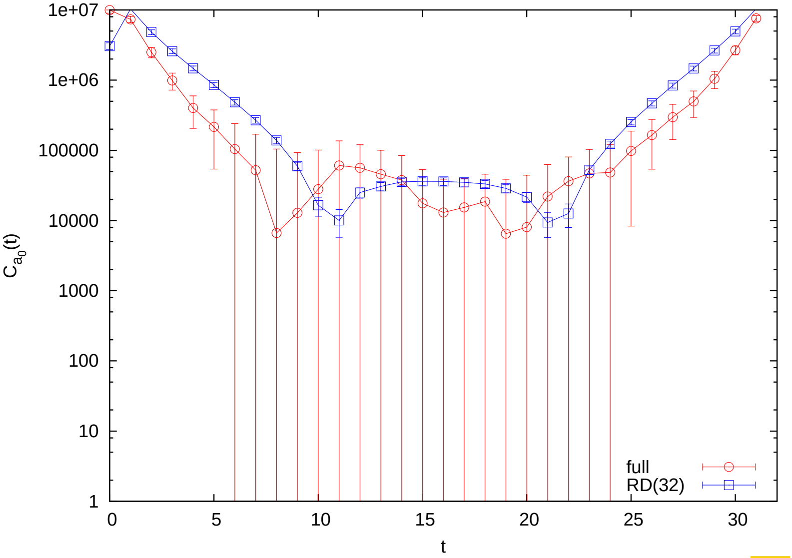

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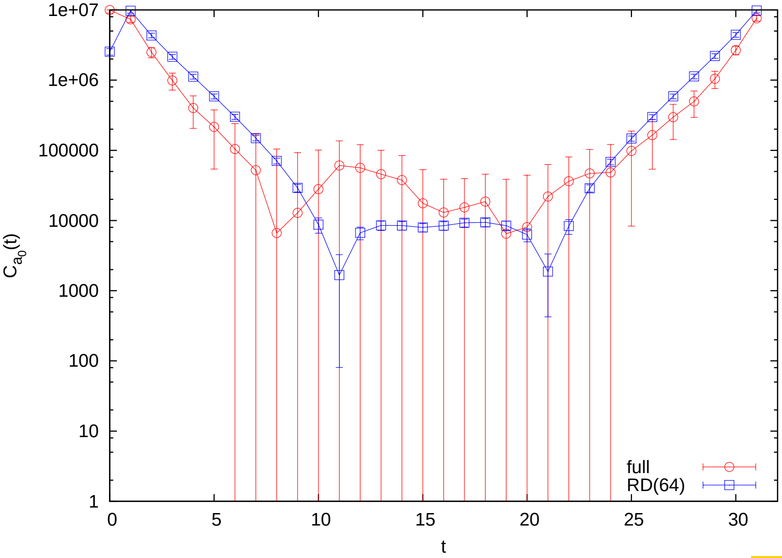

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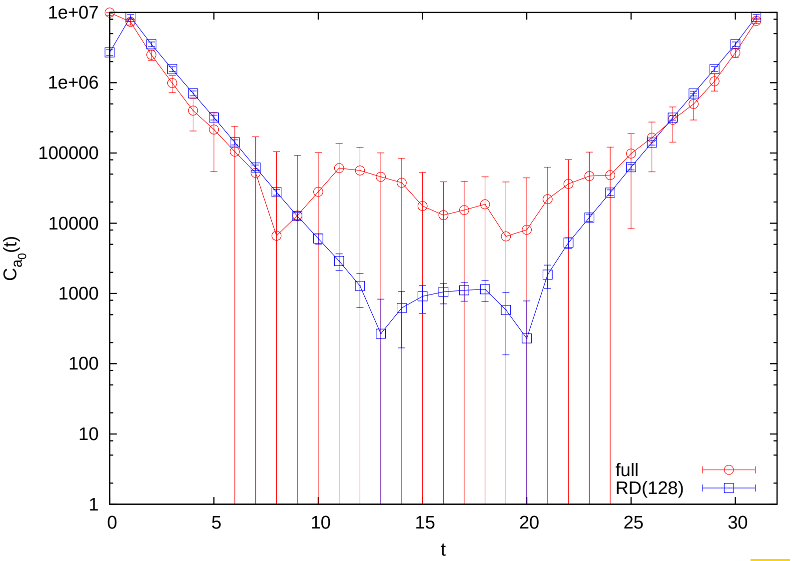

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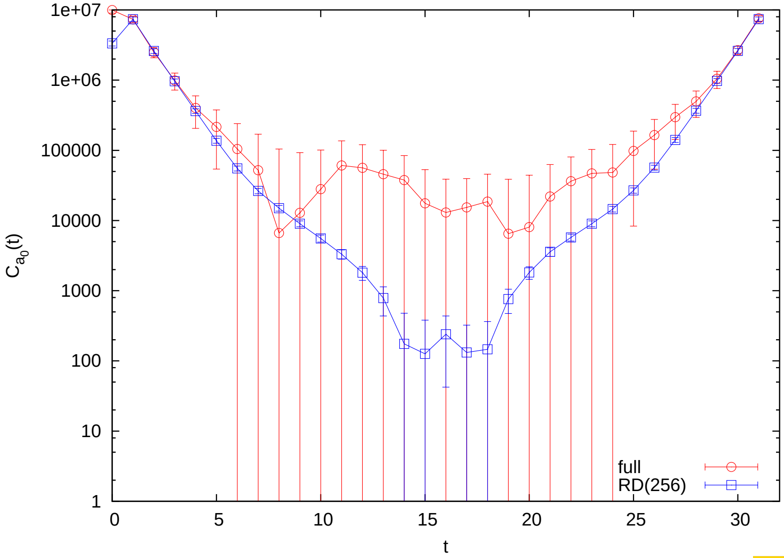

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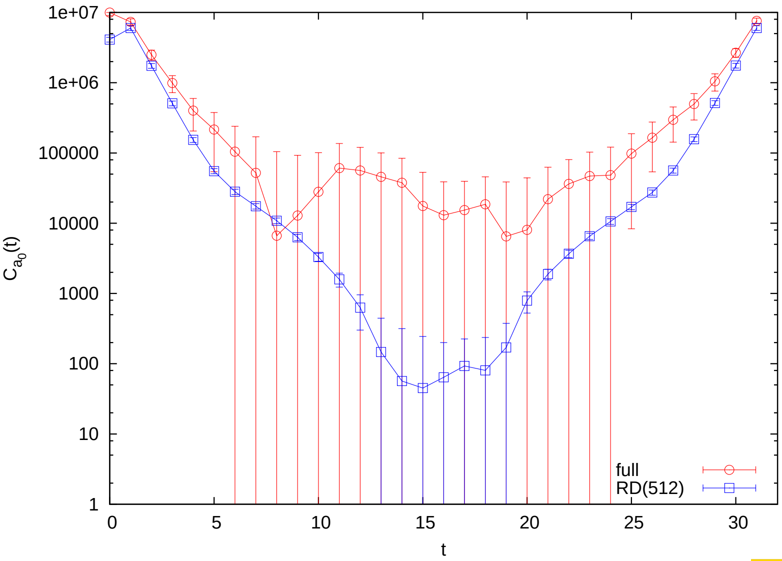

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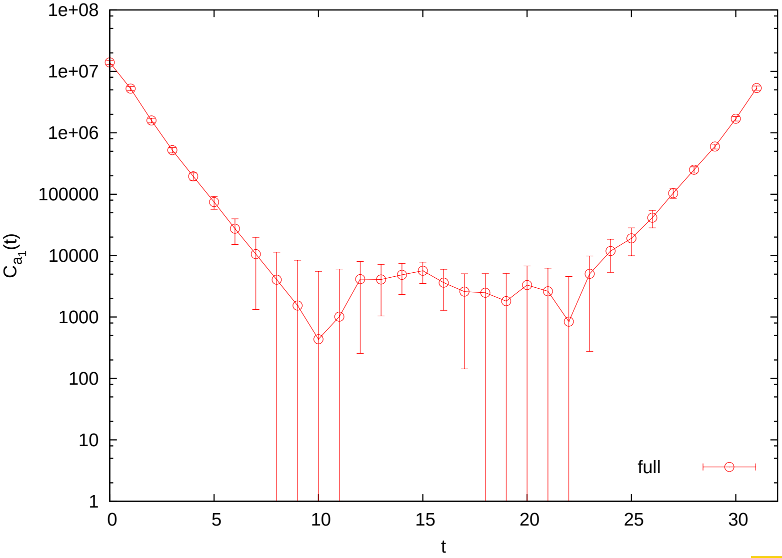

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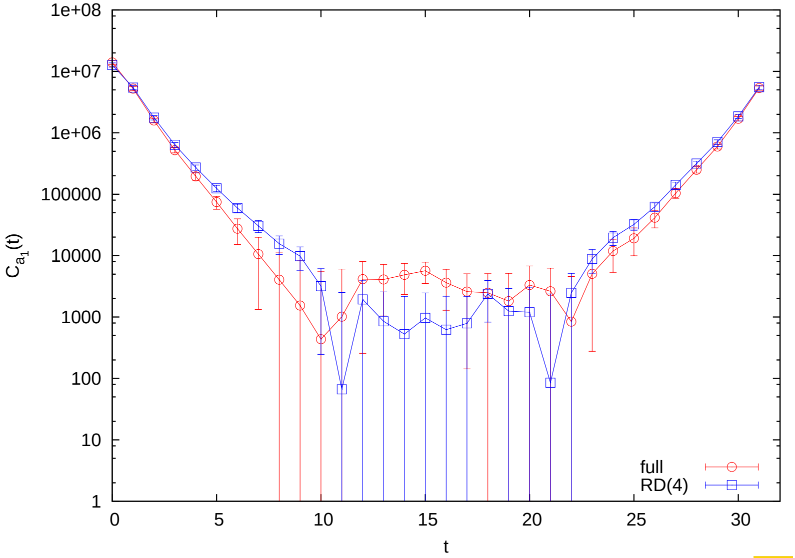

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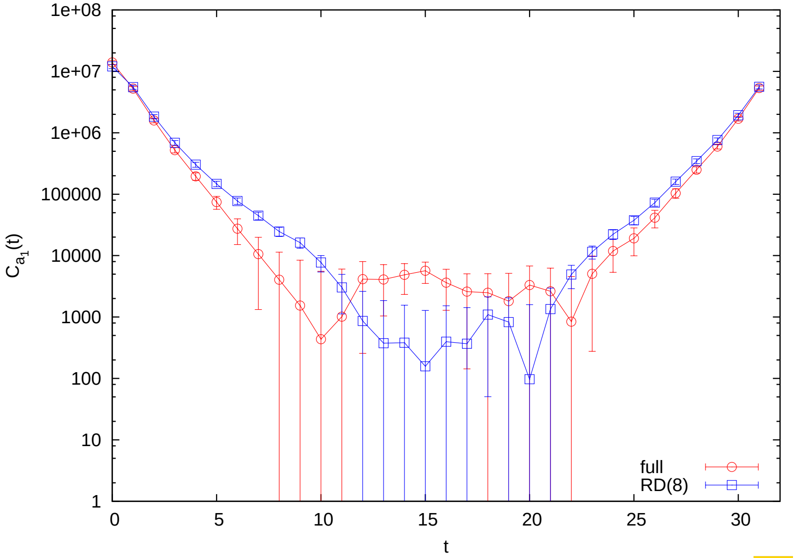

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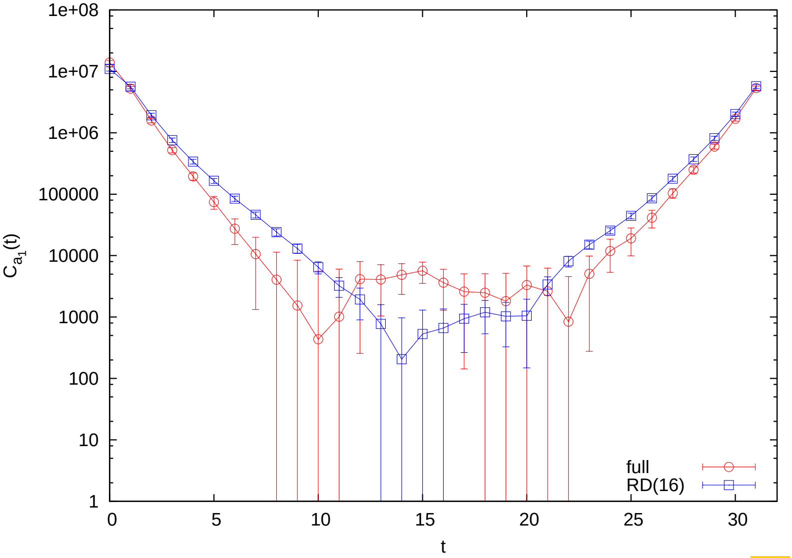

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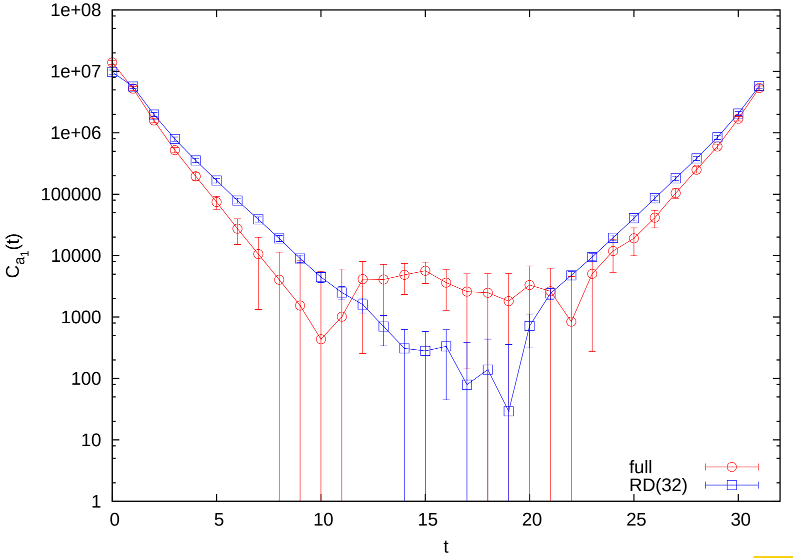

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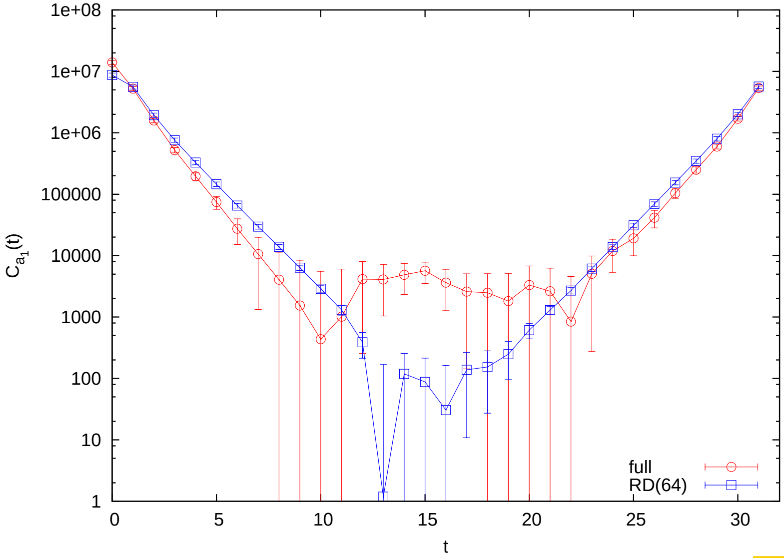

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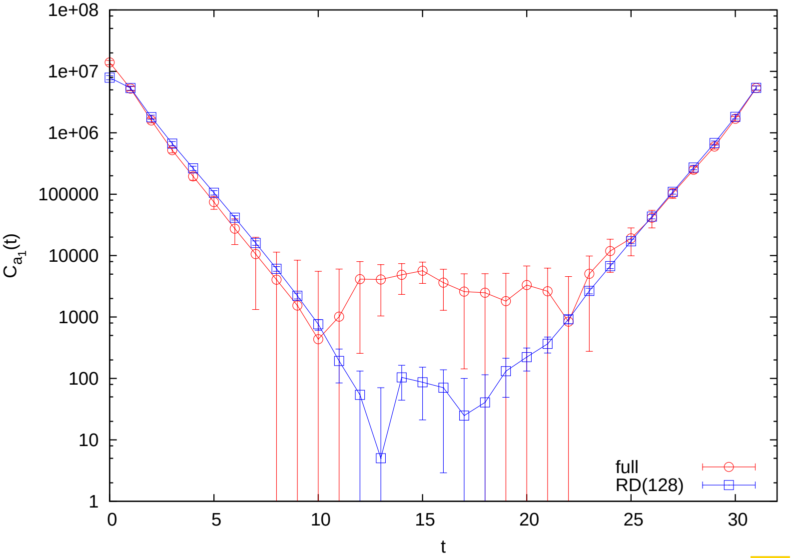

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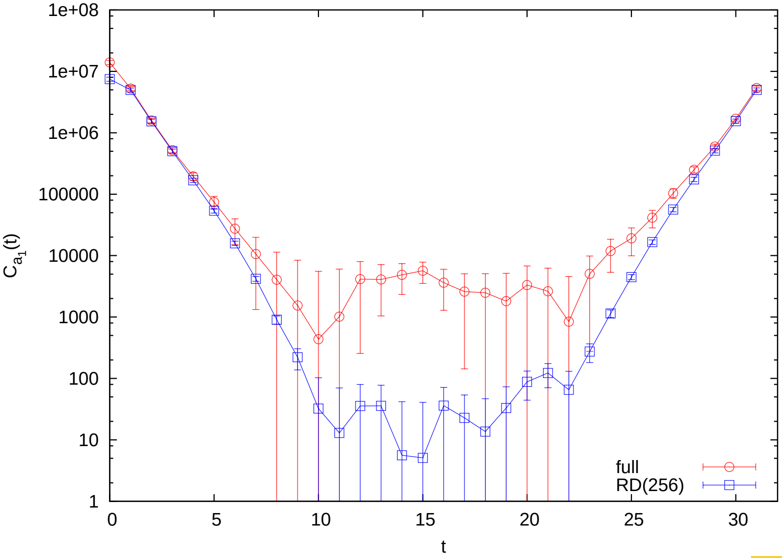

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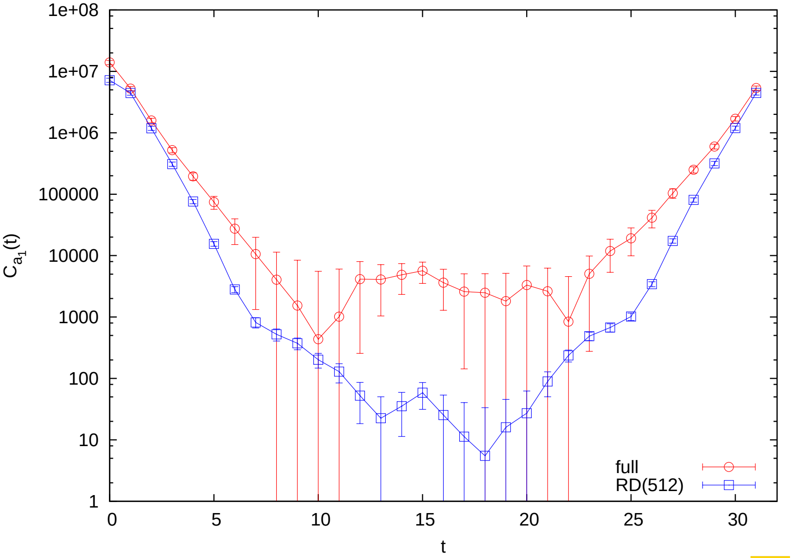
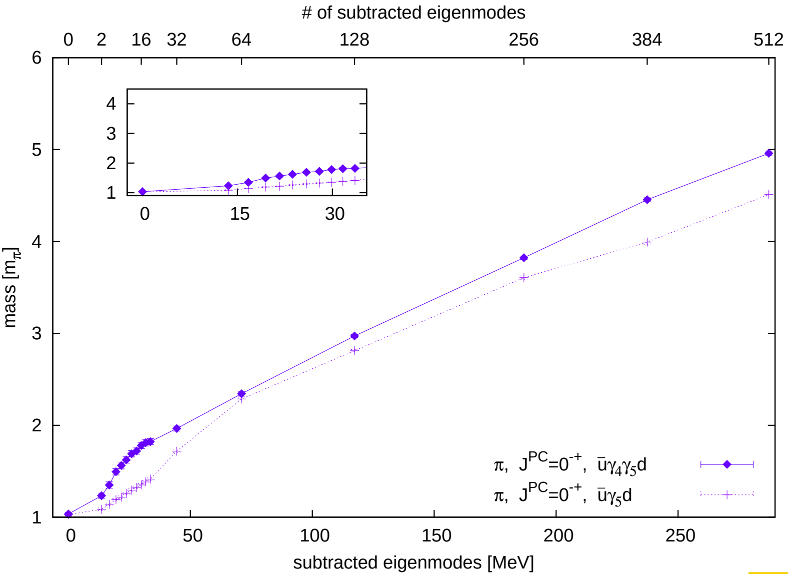
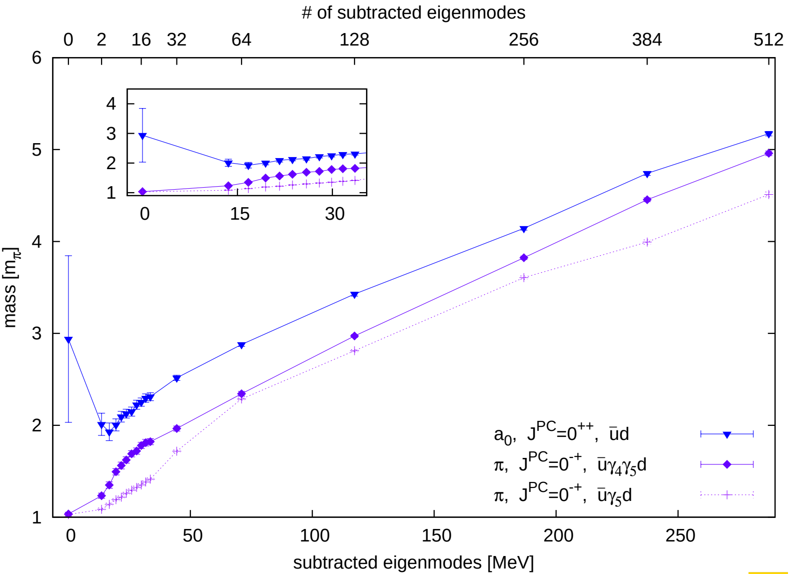


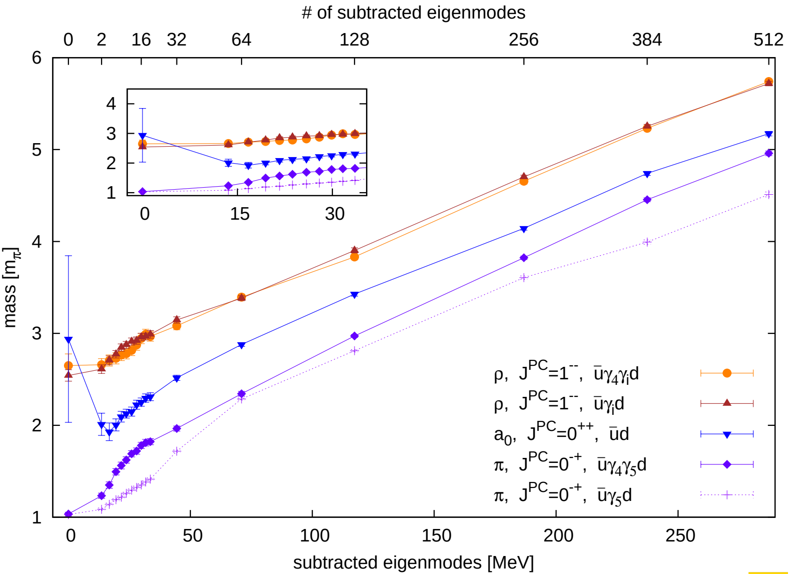
Meson masses under removal of the lowest Dirac modes



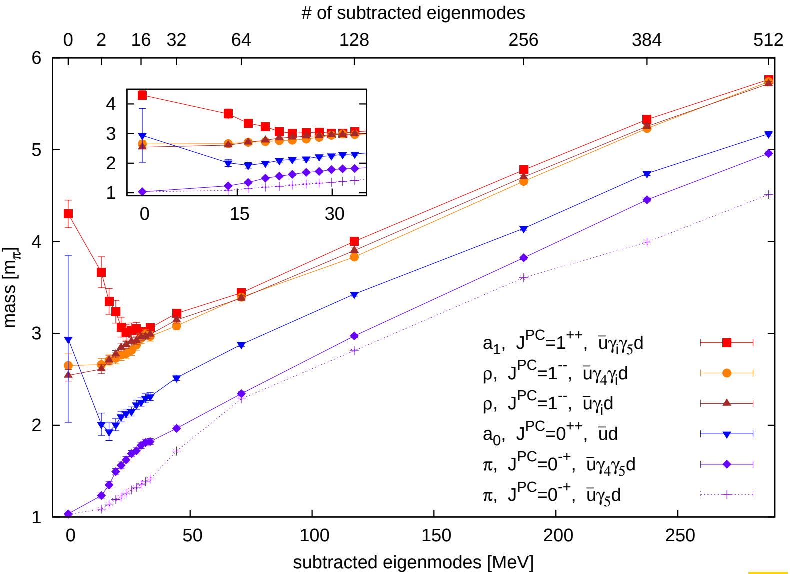
Meson masses under removal of the lowest Dirac modes



Meson masses under removal of the lowest Dirac modes



Meson masses under removal of the lowest Dirac modes



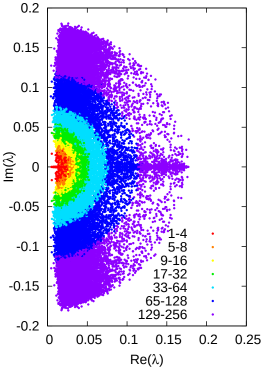
Outline

Summary

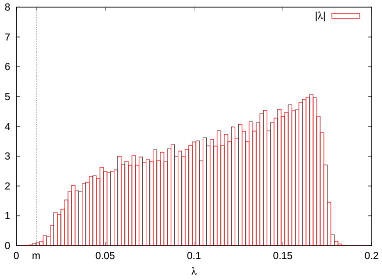
- The low-modes of the Dirac operator are related to $D\chi SB$.
- We constructed reduced quark propagators which exclude a variable number of Dirac eigenmodes.
- Therewith we built meson correlators and computed meson masses.
- By subtracting the lowest 16 eigenmodes of D_5, corresponding to the spectrum of up to 30 MeV, we find:
    - negative parity mesons become heavier,
    - positive parity mesons become lighter,
    - the degeneracy in the ground state spectrum of the ρ and the a_1 got restored.
- Subtracting more than 16 eigenmodes results in a linear growing of the masses of all considered mesons whereby the artificially restored chiral symmetry stays intact.

Eigenvalues of D

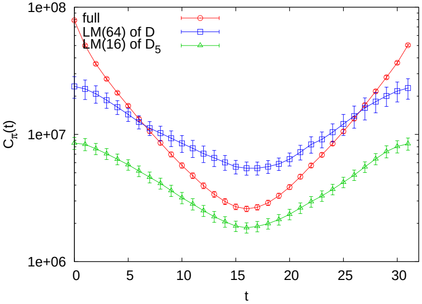
The lowest 256 eigenvalues

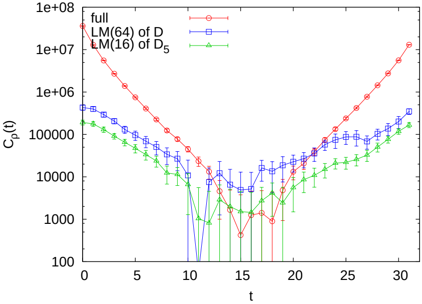


Histogram



Low-mode contribution of D and D_5 to the π and ρ correlators





Low-mode contribution of D and D_5 to the π and ρ correlators