Effects of the lowest Dirac modes on the spontaneous breaking of chiral symmetry and confinement

Mario Schröck
Advisor: Christian B. Lang

Jena, September 29, 2011

Outline

The Lattice Dirac operator

The lattice QCD fermionic action is given by

$$S_F[\psi, \overline{\psi}, U] = \sum_{n \in \Lambda} \overline{\psi}(n) \left(\sum_{\mu=1}^{4} \gamma_\mu \frac{U_\mu(n)\psi(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu})\psi(n-\hat{\mu})}{2} + m\psi(n) \right)$$

The Lattice Dirac operator

The lattice QCD fermionic action is given by

$$S_F[\psi, \overline{\psi}, U] = \sum_{n \in \Lambda} \overline{\psi}(n) \left(\sum_{\mu=1}^{4} \gamma_\mu \frac{U_\mu(n)\psi(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu})\psi(n-\hat{\mu})}{2} + m\psi(n) \right) = \sum_{n,m \in \Lambda} \overline{\psi}(n) D[U](n,m) \psi(m)$$

with the *naive* Dirac operator

$$D[U](n,m) = \frac{1}{2} \sum_{\mu=1}^{4} \left[\gamma_\mu U_\mu(n) \delta_{n,m-\hat{\mu}} - \gamma_\mu U_\mu^\dagger(n-\hat{\mu}) \delta_{n,m+\hat{\mu}} \right] + m\delta_{nm}$$

Why are the lowest Dirac eigenmodes interesting?

The Banks-Casher relation

$$\langle\overline{\psi}\psi\rangle = -\pi\rho(0)$$

directly relates the density of the Dirac modes near the origin $\rho(0)$ to the chiral condensate.

"Unbreaking" chiral symmetry

- Our goal is to construct meson correlators out of *reduced* quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., [DeGrand, Phys. Rev. D 69, 2004]).

"Unbreaking" chiral symmetry

- Our goal is to construct meson correlators out of *reduced* quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., [DeGrand, Phys. Rev. D 69, 2004]).
- Using these propagators we compute meson masses and we study the possible recovery of the degeneracies in the spectrum of (would be) chiral partners.

Correlators in Lattice QCD

Consider the correlator of an arbitrary operator, e.g., a meson $O(n) = \overline{\psi}_d(n)\Gamma\psi_u(n)$:

$$\left\langle O(n)\overline{O}(m)\right\rangle = \frac{1}{Z}\int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\; O(n)\overline{O}(m)e^{-S_G[U]-S_F[\psi,\overline{\psi},U]}$$

Correlators in Lattice QCD

Consider the correlator of an arbitrary operator, e.g., a meson $O(n) = \overline{\psi}_d(n)\Gamma\psi_u(n)$:

$$\left\langle O(n)\overline{O}(m)\right\rangle = \frac{1}{Z}\int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\; O(n)\overline{O}(m) e^{-S_G[U]-S_F[\psi,\overline{\psi},U]}$$

$$= \frac{1}{Z}\int \mathcal{D}U\; \mathrm{tr}\left[\Gamma D_u^{-1}(n,m)\Gamma D_d^{-1}(m,n)\right] e^{-S_G[U]-\ln\det D_u-\ln\det D_d}$$

Correlators in Lattice QCD

Consider the correlator of an arbitrary operator, e.g., a meson $O(n) = \overline{\psi}_d(n)\Gamma\psi_u(n)$:

$$\left\langle O(n)\overline{O}(m)\right\rangle = \frac{1}{Z}\int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\;O(n)\overline{O}(m)e^{-S_G[U]-S_F[\psi,\overline{\psi},U]}$$
$$= \frac{1}{Z}\int \mathcal{D}U\;\mathrm{tr}\left[\Gamma D_u^{-1}(n,m)\Gamma D_d^{-1}(m,n)\right]e^{-S_G[U]-\ln\det D_u-\ln\det D_d}$$

Fourier-transform and project to zero momentum

$$C(t) \equiv \left\langle \tilde{O}(t)\overline{O}(0)\right\rangle = A_0\,\mathrm{e}^{-tE_0} + \ldots$$

Correlators in Lattice QCD

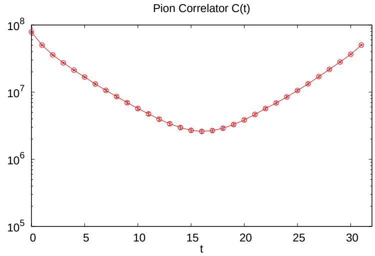
Consider the correlator of an arbitrary operator, e.g., a meson $O(n) = \overline{\psi}_d(n)\Gamma\psi_u(n)$:

$$\left\langle O(n)\overline{O}(m)\right\rangle = \frac{1}{Z}\int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\;O(n)\overline{O}(m)e^{-S_G[U]-S_F[\psi,\overline{\psi},U]}$$

$$= \frac{1}{Z}\int \mathcal{D}U\;\mathrm{tr}\left[\Gamma D_u^{-1}(n,m)\Gamma D_d^{-1}(m,n)\right]e^{-S_G[U]-\ln\det D_u-\ln\det D_d}$$

Fourier-transform and project to zero momentum

$$C(t) \equiv \left\langle \tilde{O}(t)\overline{O}(0)\right\rangle = A_0\,\mathrm{e}^{-tE_0} + \ldots$$



Correlators in Lattice QCD

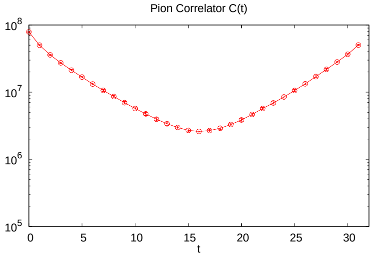
Consider the correlator of an arbitrary operator, e.g., a meson $O(n) = \overline{\psi}_d(n)\Gamma\psi_u(n)$:

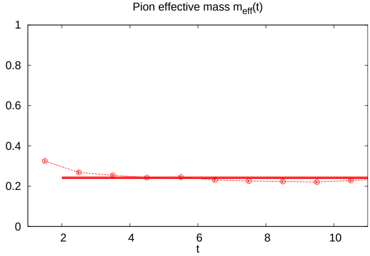
$$\left\langle O(n)\overline{O}(m)\right\rangle = \frac{1}{Z}\int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\; O(n)\overline{O}(m) e^{-S_G[U]-S_F[\psi,\overline{\psi},U]}$$

$$= \frac{1}{Z}\int \mathcal{D}U\;\mathrm{tr}\left[\Gamma D_u^{-1}(n,m)\Gamma D_d^{-1}(m,n)\right] e^{-S_G[U]-\ln\det D_u-\ln\det D_d}$$

Fourier-transform and project to zero momentum

$$C(t) \equiv \left\langle \tilde{O}(t)\overline{O}(0)\right\rangle = A_0\, \mathrm{e}^{-tE_0} + \ldots$$





Properties of $D_5 \equiv \gamma_5 D$

- D_5 is hermitian
- its eigenvalues are purely real

Properties of $D_5 \equiv \gamma_5 D$

- D_5 is hermitian
- its eigenvalues are purely real
- the spectral representation of the quark propagator using D_5 reads

$$S = \sum_{i=1}^{N} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5$$

Reducing quark propagators

- Split S into a low mode (lm5) part and a *reduced* (red5) part

$$\begin{aligned} S &= \sum_{i \leq k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 + \sum_{i > k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 \\ &= S_{\mathrm{lm5}(k)} + S_{\mathrm{red5}(k)} \end{aligned}$$

Reducing quark propagators

- Split S into a low mode (lm5) part and a *reduced* (red5) part

$$S = \sum_{i \leq k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5 + \sum_{i > k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5$$
$$= S_{\mathrm{lm5}(k)} + S_{\mathrm{red5}(k)}$$

- In this work we investigate the *reduced* (red5) part of the propagator

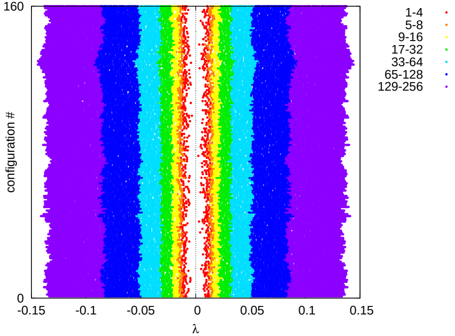
$$S_{\mathrm{red5}(k)} = S - S_{\mathrm{lm5}(k)}$$

The Setup

- 161 configurations [Gattringer, Hagen, Lang, Limmer, Mohler, Schäfer, Phys. Rev. D 79, 2009]
- size $16^3 \times 32$
- two degenerate flavors of light fermions, $m_\pi = 322(5)\,\mathrm{MeV}$
- lattice spacing $a = 0.1440(12)\,\mathrm{fm}$
- Chirally Improved Dirac operator [Gattringer, Phys. Rev. D 63, 2001] (approximate solution of the Ginsparg-Wilson equation)
- Jacobi smeared quark sources

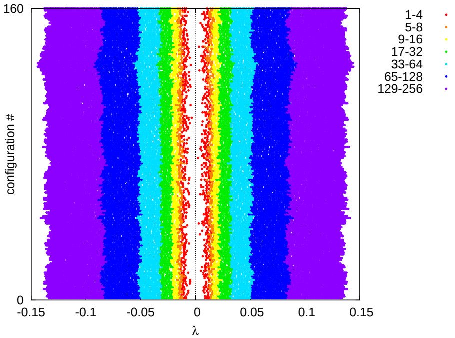
Eigenvalues

D_5

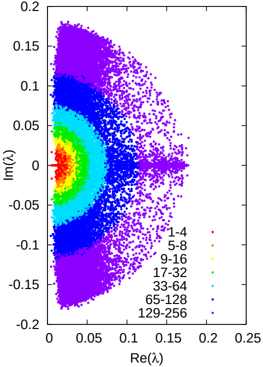


Eigenvalues

D_5



D



Chiral symmetry and light mesons

Two dynamical flavors of quarks (neglect mass), underlying symmetry group is

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

whereas the chiral symmetry $SU(2)_L \times SU(2)_R$ is broken spontaneously in the vacuum and the $U(1)$ axial symmetry is broken explicitly in the quantized theory (axial anomaly).

Chiral symmetry and light mesons

Two dynamical flavors of quarks (neglect mass), underlying symmetry group is

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

whereas the chiral symmetry $SU(2)_L \times SU(2)_R$ is broken spontaneously in the vacuum and the $U(1)$ axial symmetry is broken explicitly in the quantized theory (axial anomaly).
We will explore following particles which would be connected via the above symmetries [L.Ya. Glozman, Physics Reports, Volume 444, 2007]

$U(1)_A$	$SU(2)_L \times SU(2)_R$ (axial)
$\pi \longleftrightarrow a_0$	

Chiral symmetry and light mesons

Two dynamical flavors of quarks (neglect mass), underlying symmetry group is

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

whereas the chiral symmetry $SU(2)_L \times SU(2)_R$ is broken spontaneously in the vacuum and the $U(1)$ axial symmetry is broken explicitly in the quantized theory (axial anomaly).
We will explore following particles which would be connected via the above symmetries [L.Ya. Glozman, Physics Reports, Volume 444, 2007]

$U(1)_A$	$SU(2)_L \times SU(2)_R$ (axial)
$\pi \longleftrightarrow a_0$	$\rho \longleftrightarrow a_1$

Chiral symmetry and light mesons

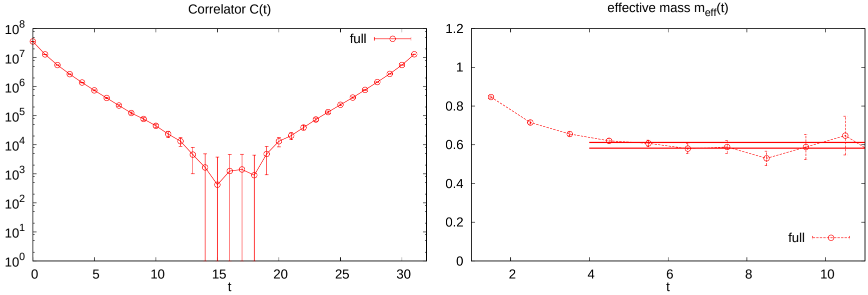
Two dynamical flavors of quarks (neglect mass), underlying symmetry group is

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

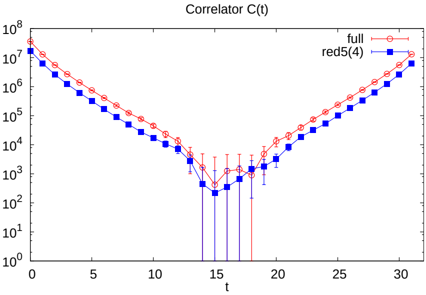
whereas the chiral symmetry $SU(2)_L \times SU(2)_R$ is broken spontaneously in the vacuum and the $U(1)$ axial symmetry is broken explicitly in the quantized theory (axial anomaly).
We will explore following particles which would be connected via the above symmetries [L.Ya. Glozman, Physics Reports, Volume 444, 2007]

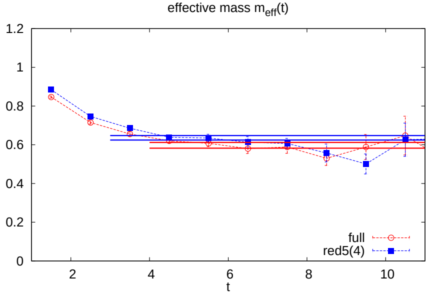
$U(1)_A$	$SU(2)_L \times SU(2)_R$ (axial)
$\pi \longleftrightarrow a_0$	$\rho \longleftrightarrow a_1$
	$\pi \longleftrightarrow f_0$
	$a_0 \longleftrightarrow \eta$

ρ, $J^{PC} = 1^{--}$, $\bar{u}\gamma_i d$

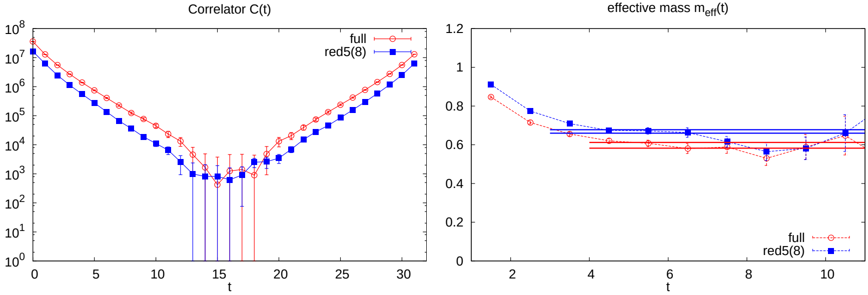


ρ, $J^{PC} = 1^{--}$, $\bar{u}\gamma_i d$

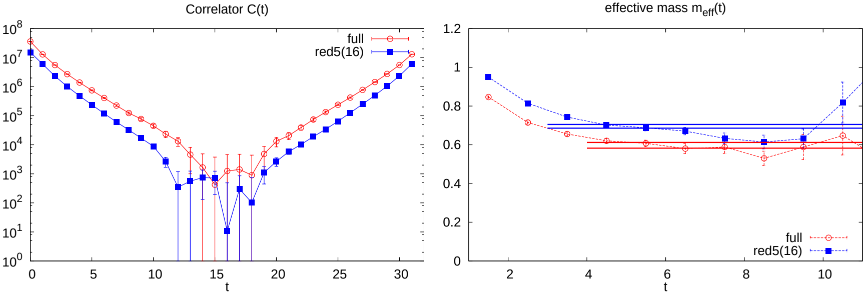




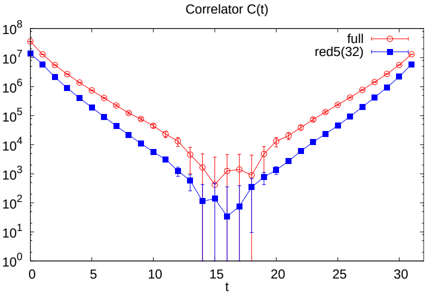
$\rho,\ J^{PC} = 1^{--},\ \overline{u}\gamma_i d$

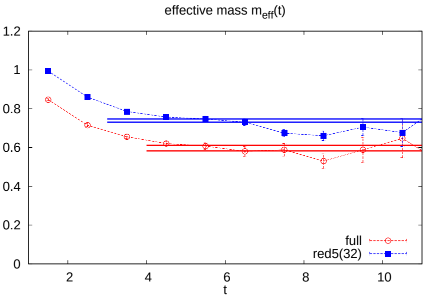


ρ, $J^{PC} = 1^{--}$, $\bar{u}\gamma_i d$

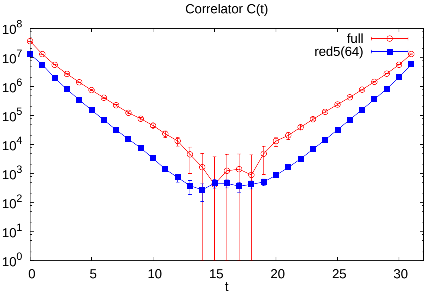


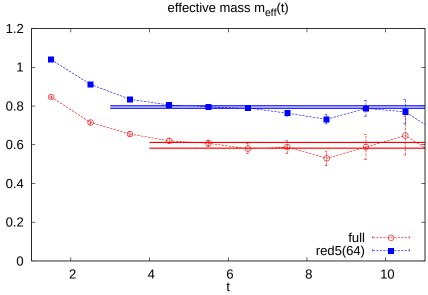
$\rho,\ J^{PC} = 1^{--},\ \overline{u}\gamma_i d$



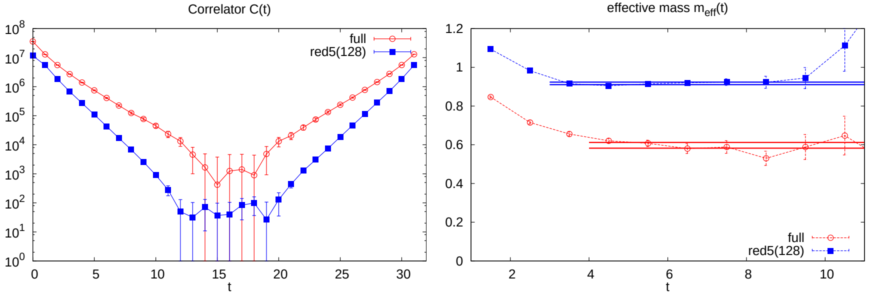


ρ, $J^{PC} = 1^{--}$, $\overline{u}\gamma_i d$

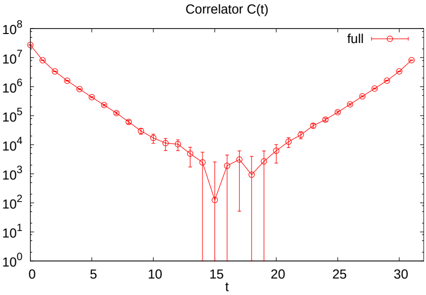


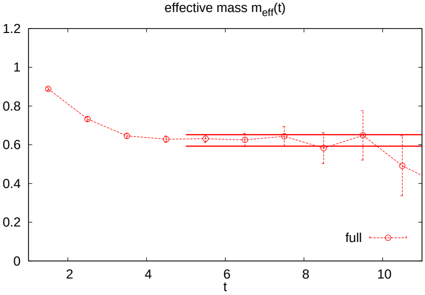


ρ, $J^{PC} = 1^{--}$, $\overline{u}\gamma_i d$

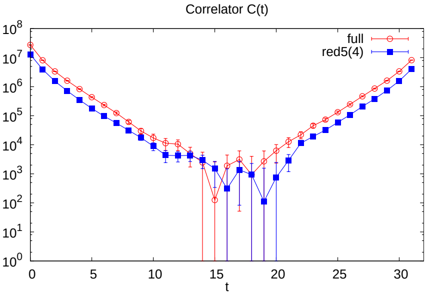


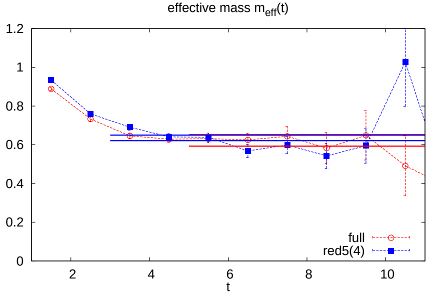
ρ, $J^{PC} = 1^{--}$, $\overline{u}\gamma_4\gamma_i d$



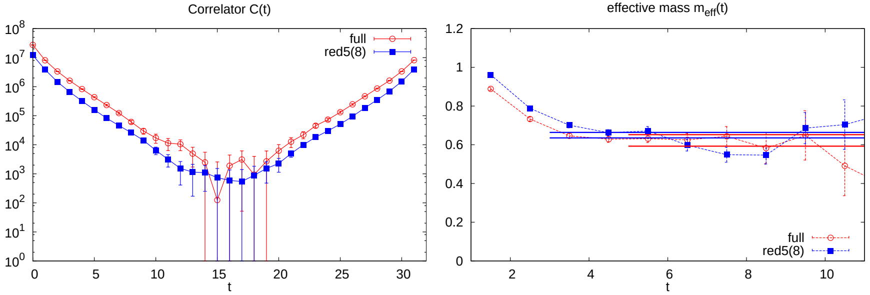


ρ, $J^{PC} = 1^{--}$, $\overline{u}\gamma_4\gamma_i d$

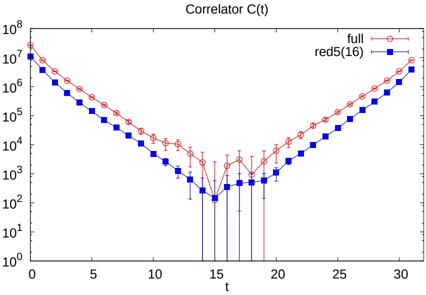


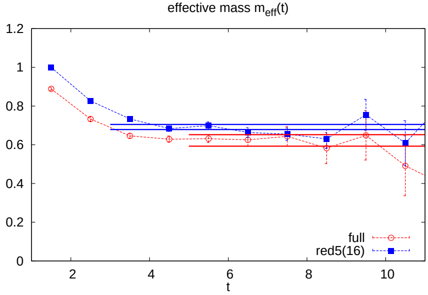


ρ, $J^{PC} = 1^{--}$, $\overline{u}\gamma_4\gamma_i d$

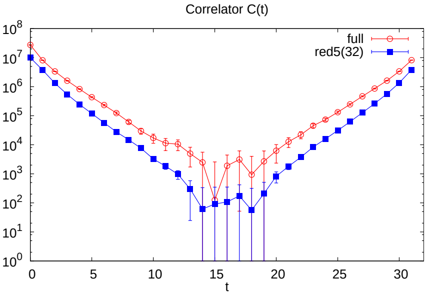


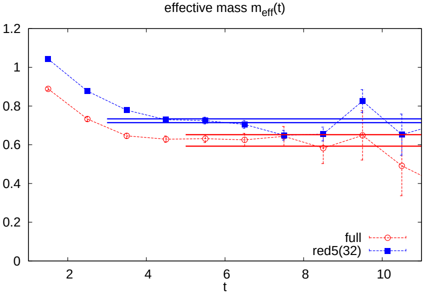
ρ, $J^{PC} = 1^{--}$, $\bar{u}\gamma_4\gamma_i d$



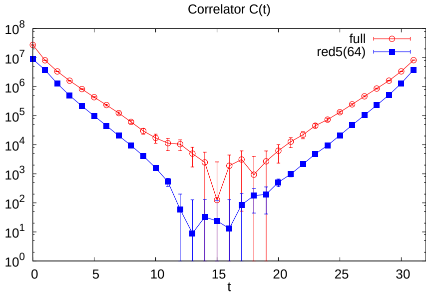


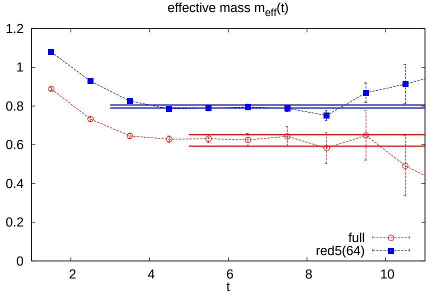
ρ, $J^{PC} = 1^{--}$, $\overline{u}\gamma_4\gamma_i d$



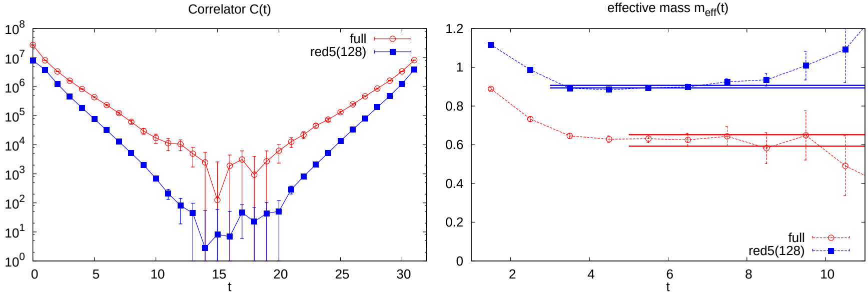


$\rho,\ J^{PC} = 1^{--},\ \bar{u}\gamma_4\gamma_i d$

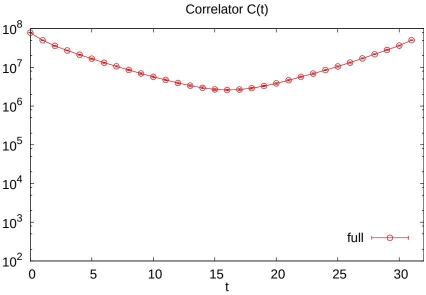


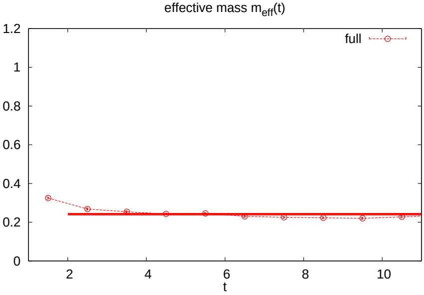


$\rho,\ J^{PC} = 1^{--},\ \bar{u}\gamma_4\gamma_i d$

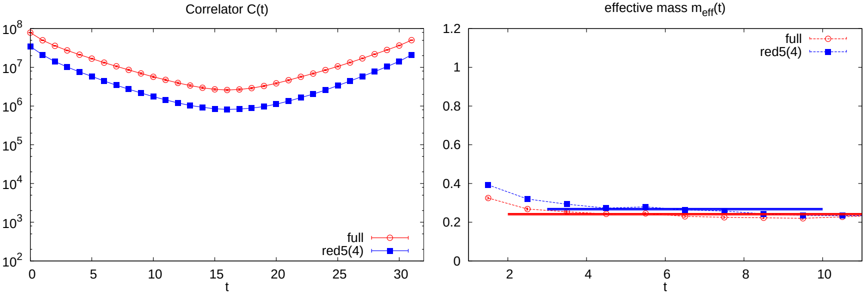


π, $J^{PC} = 0^{-+}$, $\overline{u}\gamma_5 d$

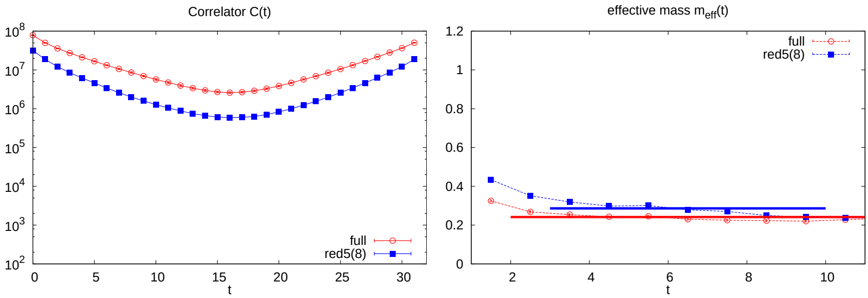




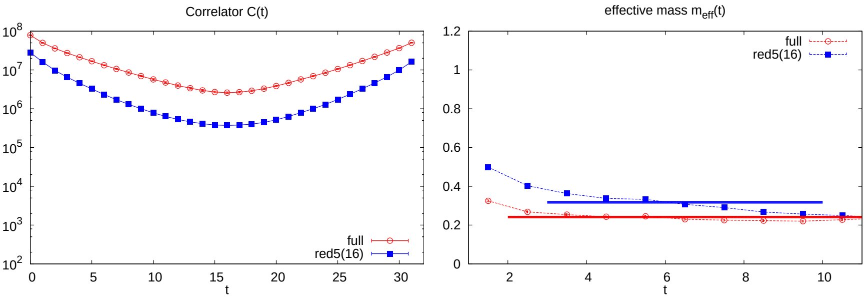
$\pi,\ J^{PC} = 0^{-+},\ \bar{u}\gamma_5 d$



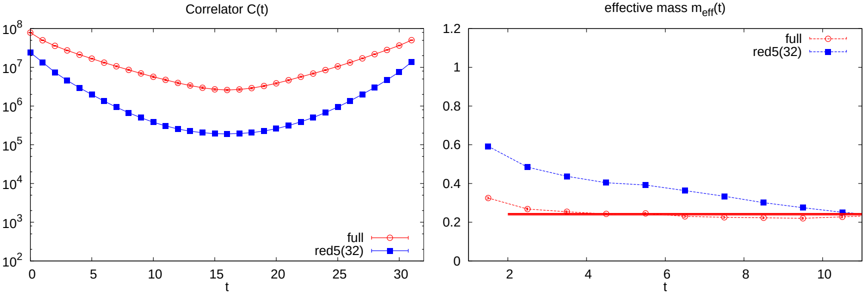
π, $J^{PC} = 0^{-+}$, $\bar{u}\gamma_5 d$



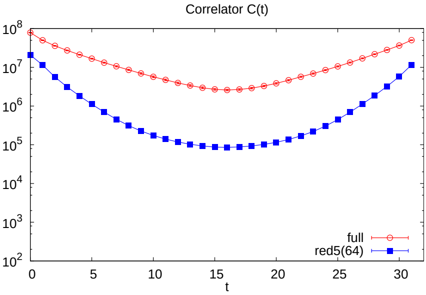
$\pi,\ J^{PC} = 0^{-+},\ \overline{u}\gamma_5 d$

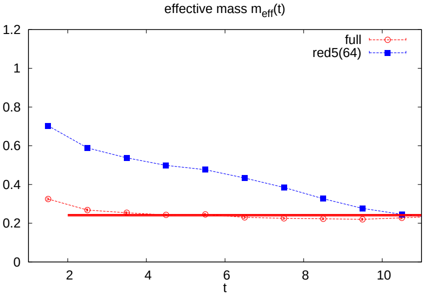


$\pi,\ J^{PC} = 0^{-+},\ \bar{u}\gamma_5 d$

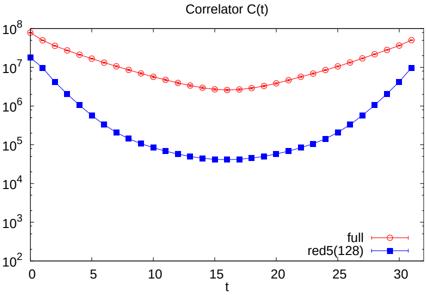


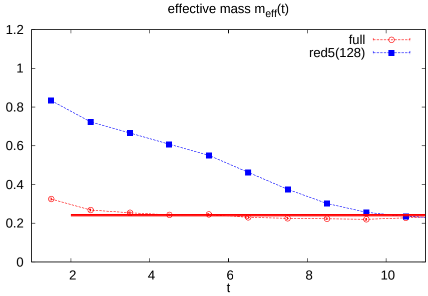
$\pi,\ J^{PC} = 0^{-+},\ \overline{u}\gamma_5 d$



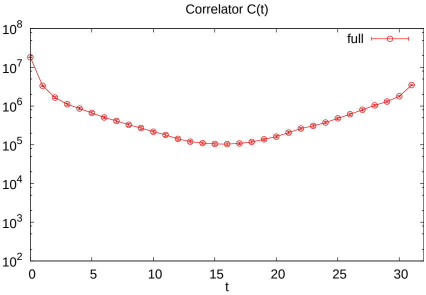


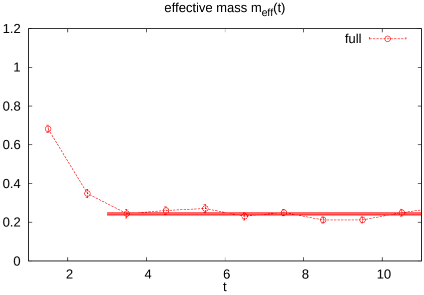
$\pi, \; J^{PC} = 0^{-+}, \; \bar{u}\gamma_5 d$



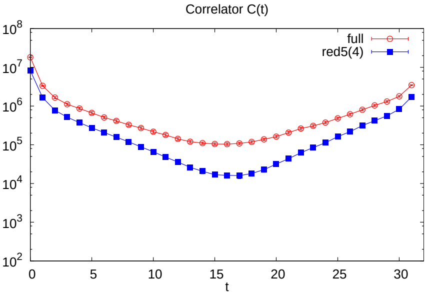


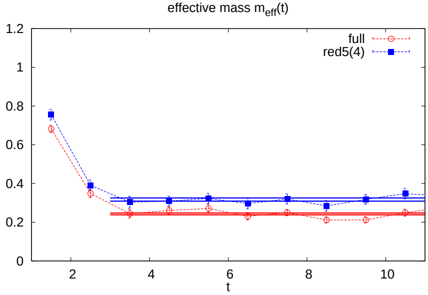
π, $J^{PC} = 0^{-+}$, $\overline{u}\gamma_4\gamma_5 d$



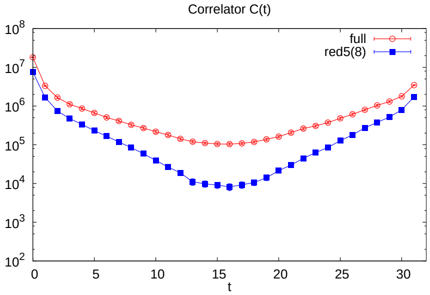


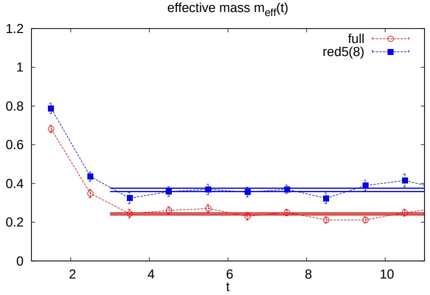
π, $J^{PC} = 0^{-+}$, $\overline{u}\gamma_4\gamma_5 d$



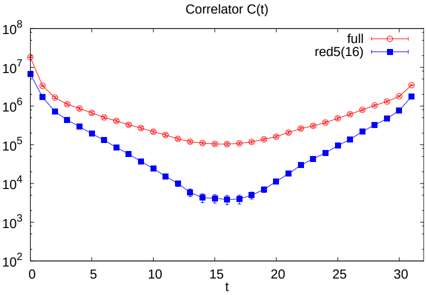


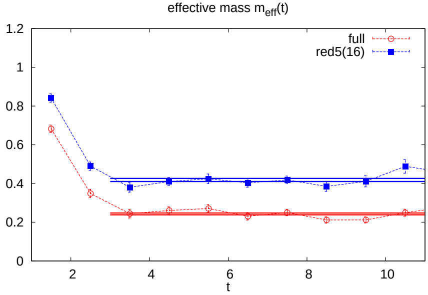
π, $J^{PC} = 0^{-+}$, $\bar{u}\gamma_4\gamma_5 d$



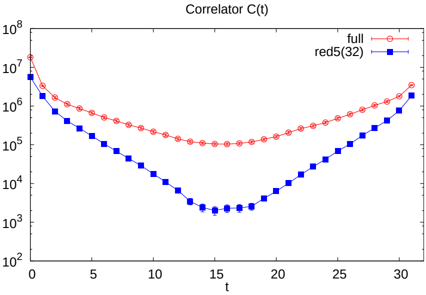


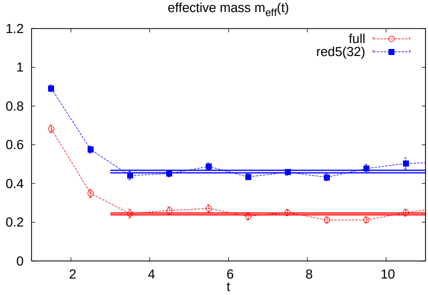
$\pi,\ J^{PC} = 0^{-+},\ \overline{u}\gamma_4\gamma_5 d$



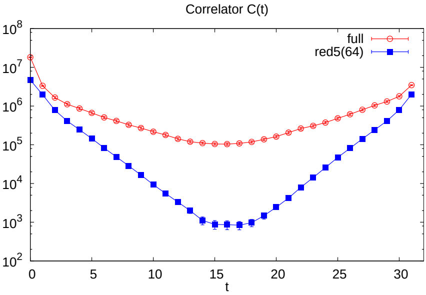


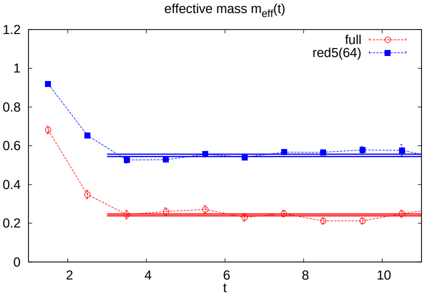
$\pi,\ J^{PC} = 0^{-+},\ \bar{u}\gamma_4\gamma_5 d$



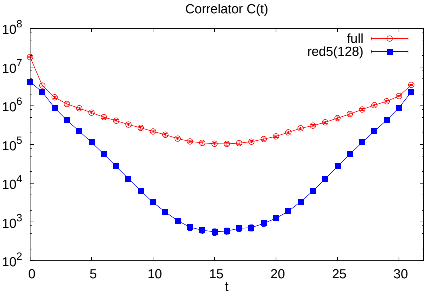


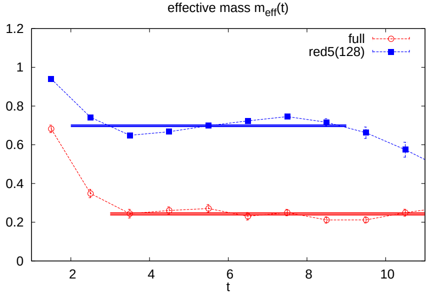
$\pi,\ J^{PC} = 0^{-+},\ \overline{u}\gamma_4\gamma_5 d$



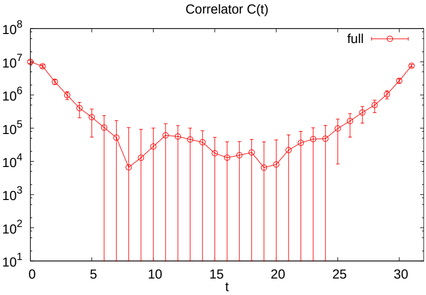


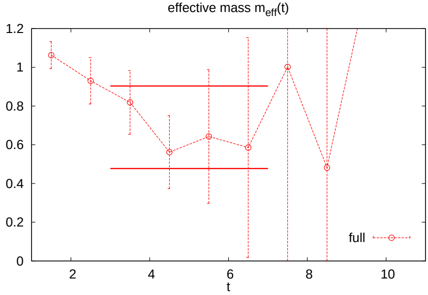
π, $J^{PC} = 0^{-+}$, $\overline{u}\gamma_4\gamma_5 d$



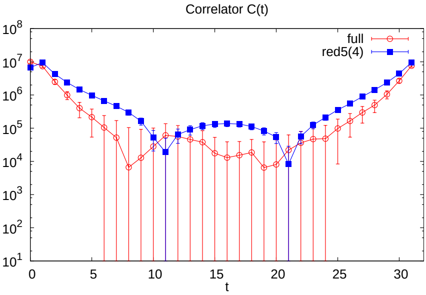


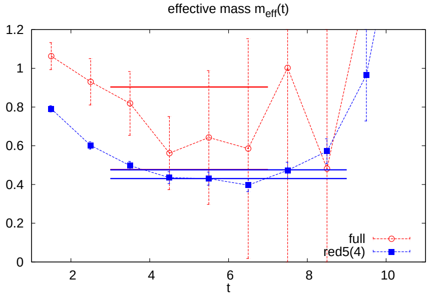
$a_0,\ J^{PC} = 0^{++},\ \overline{u}d$



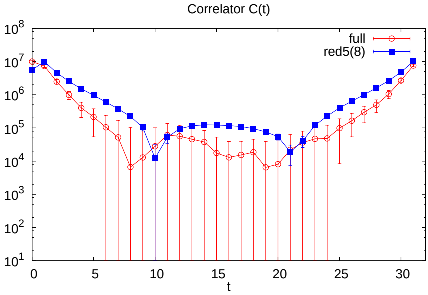


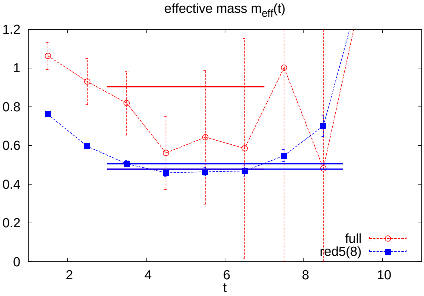
a_0, $J^{PC} = 0^{++}$, $\overline{u}d$



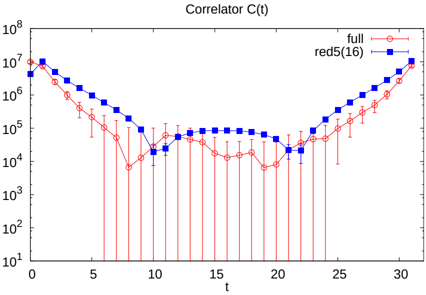


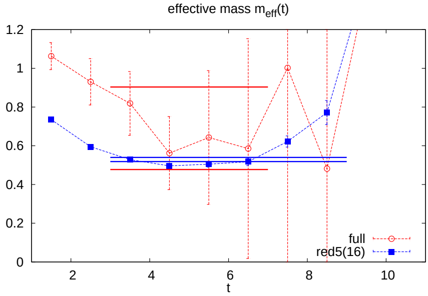
$a_0,\ J^{PC} = 0^{++},\ \overline{u}d$



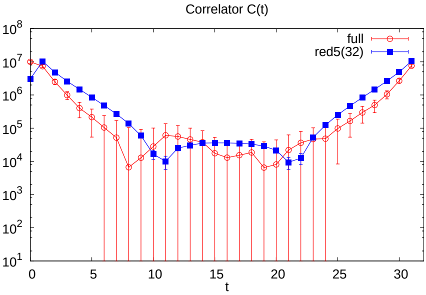


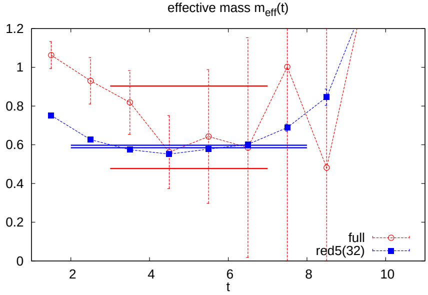
$a_0,\ J^{PC} = 0^{++},\ \bar{u}d$



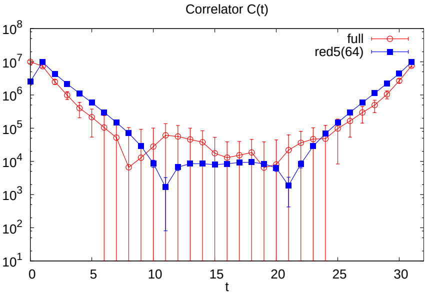


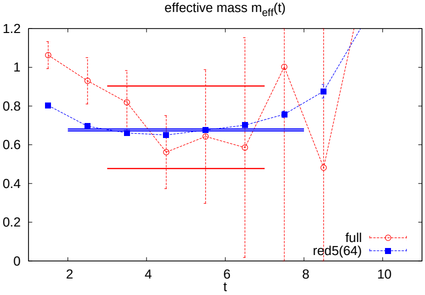
$a_0,\ J^{PC} = 0^{++},\ \overline{u}d$



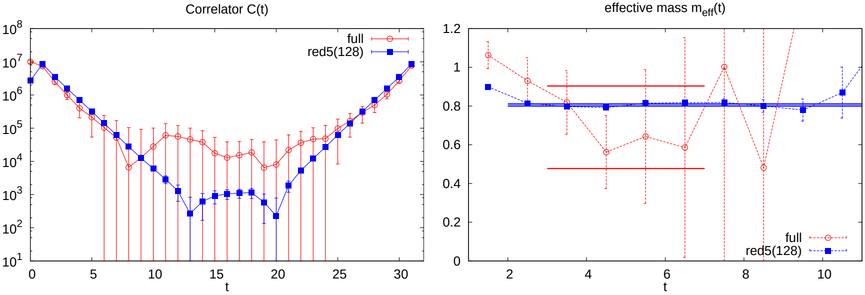


$a_0,\ J^{PC} = 0^{++},\ \overline{u}d$

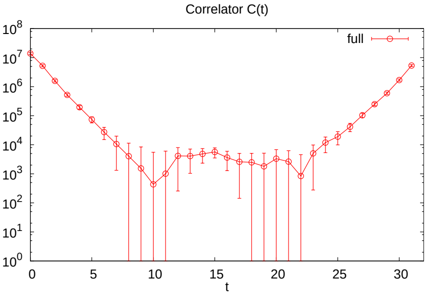


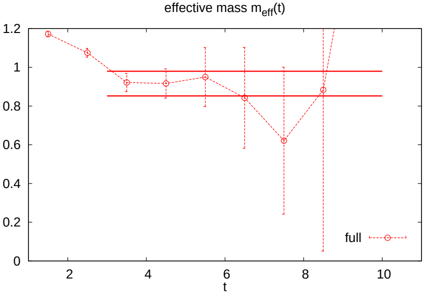


$a_0,\ J^{PC} = 0^{++},\ \overline{u}d$

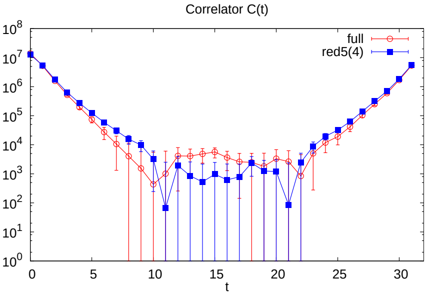


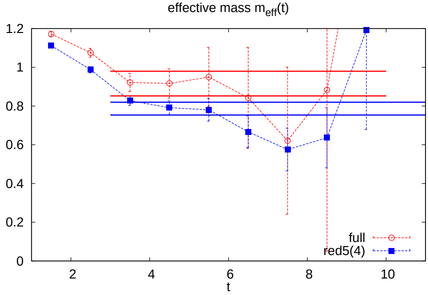
a_1, $J^{PC} = 1^{++}$, $\overline{u}\gamma_i\gamma_5 d$



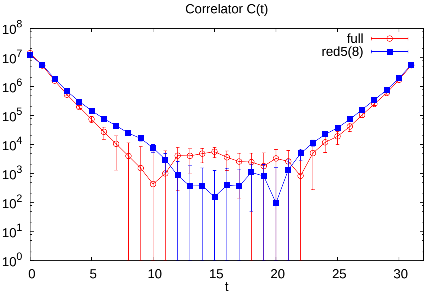


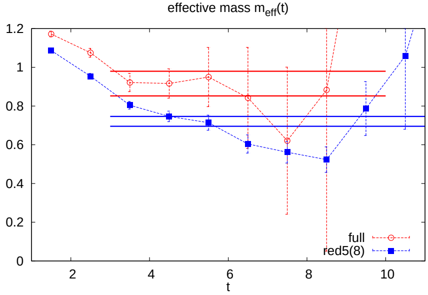
$a_1,\ J^{PC} = 1^{++},\ \overline{u}\gamma_i\gamma_5 d$



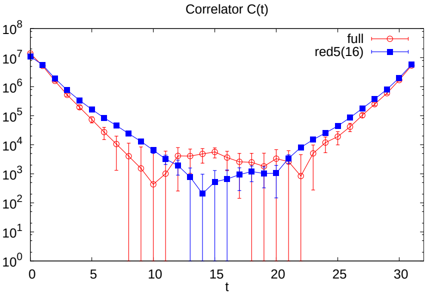


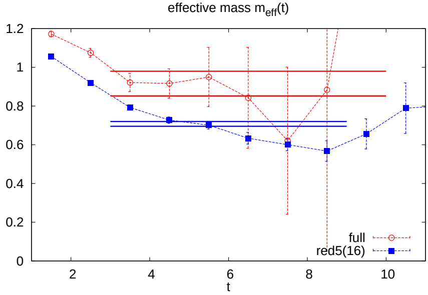
a_1, $J^{PC} = 1^{++}$, $\overline{u}\gamma_i\gamma_5 d$



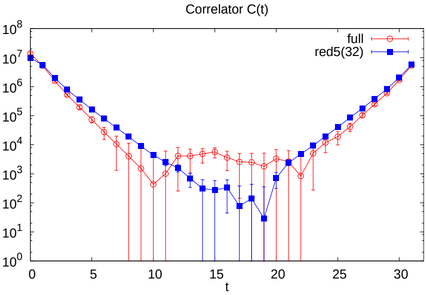


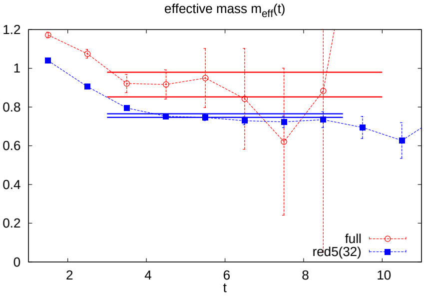
a_1, $J^{PC} = 1^{++}$, $\overline{u}\gamma_i\gamma_5 d$



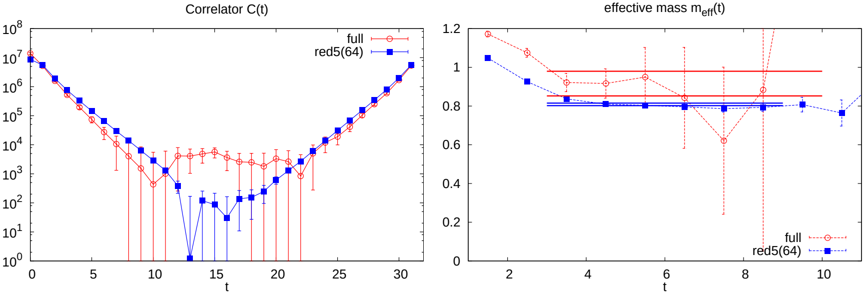


$a_1,\ J^{PC} = 1^{++},\ \bar{u}\gamma_i\gamma_5 d$

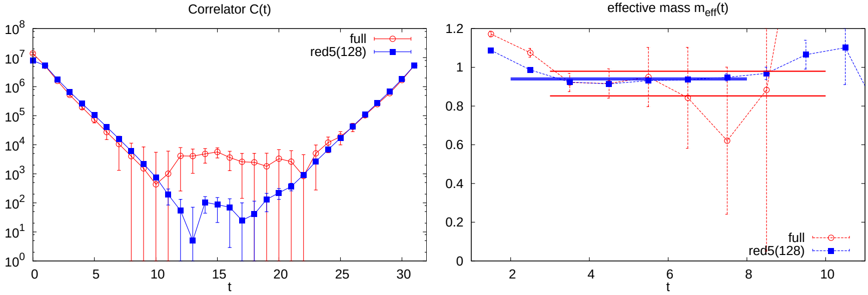




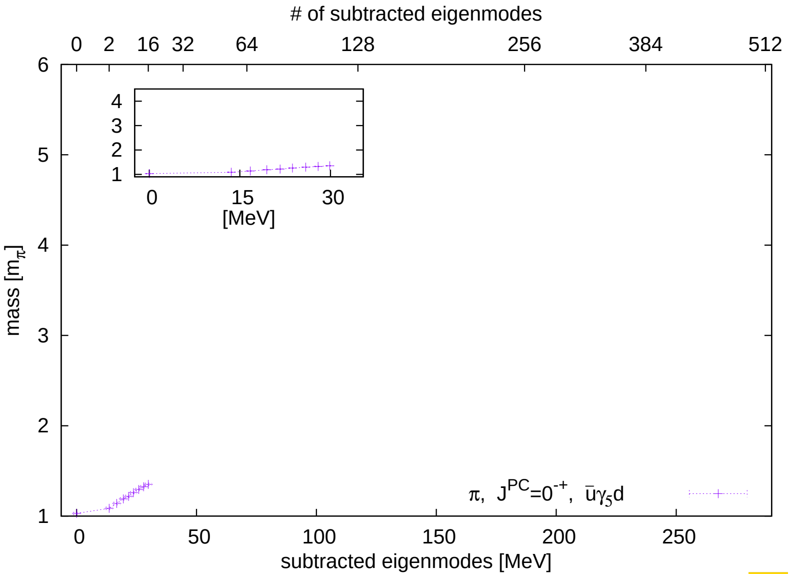
$a_1,\ J^{PC} = 1^{++},\ \overline{u}\gamma_i\gamma_5 d$



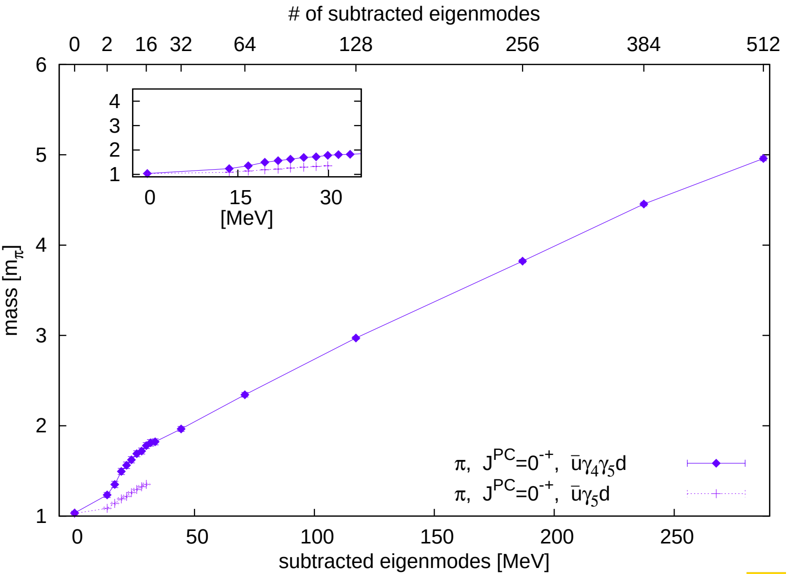
$a_1,\ J^{PC} = 1^{++},\ \overline{u}\gamma_i\gamma_5 d$



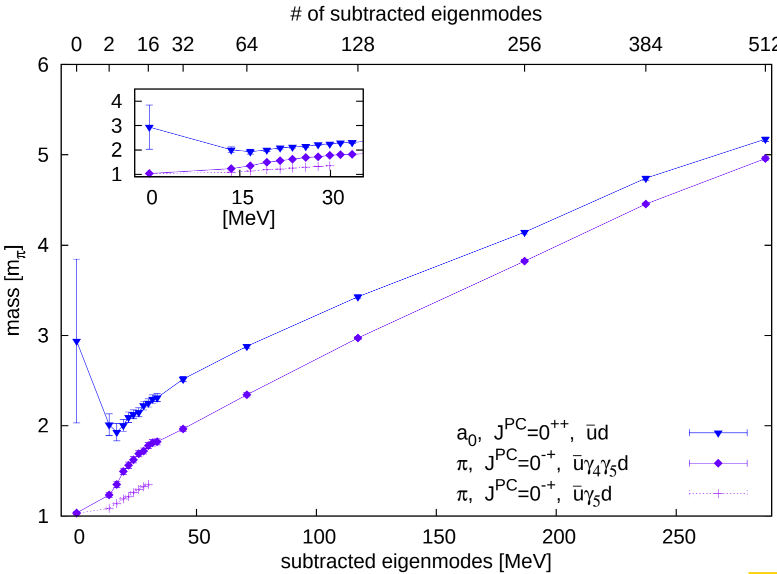
Meson masses vs. eigenmode reduction level



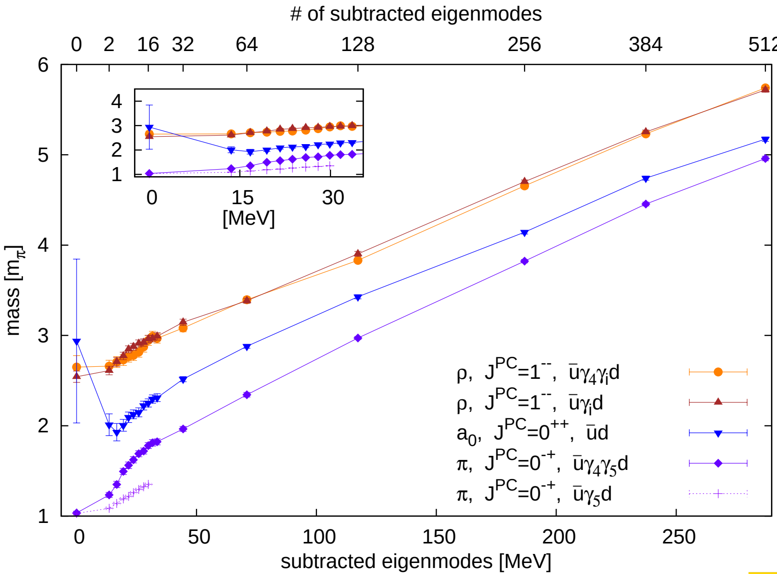
Meson masses vs. eigenmode reduction level



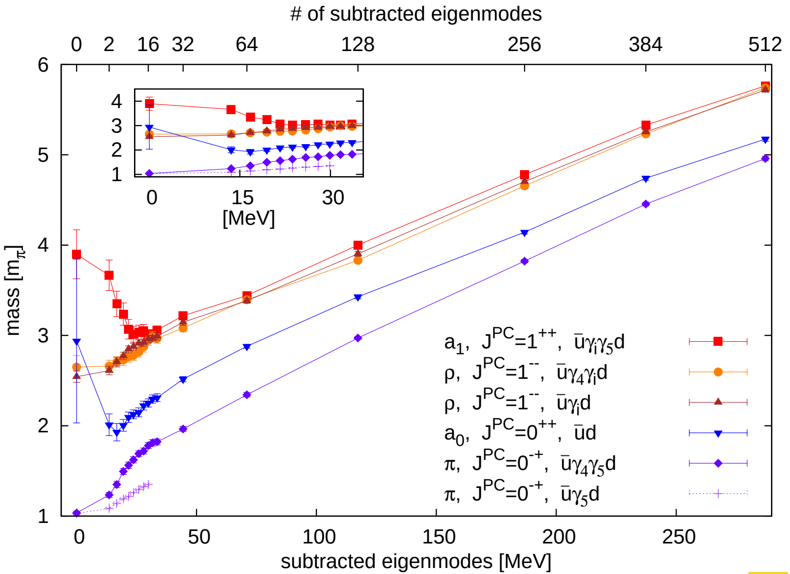
Meson masses vs. eigenmode reduction level



Meson masses vs. eigenmode reduction level



Meson masses vs. eigenmode reduction level



Conclusions

- low lying eigenvalues of the Dirac operator are associated with chiral symmetry breaking
- we have computed hadron propagators while removing increasingly more of the low lying eigenmodes of the Dirac operator
- this amounts to partially restoring chiral symmetry (in the valence quarks)
- We find drastic behavior for some meson interpolators when starting to remove low eigenmodes. At truncation level 16 the behavior saturates and then the mass values rise uniformly with roughly parallel slopes.
- the confinement properties remain intact, i.e., we still observe clear bound states for most of the studied isovector (scalar, axial vector and vector) mesons
- an exception is the pion, where no clear exponential decay of the correlation function is seen in the $\overline{u}\gamma_5 d$ interpolator
- the mass values of the vector meson chiral partners a_1 and ρ approach each other when 8 or more low modes are removed