The effects of Dirac low-mode truncation on the hadron mass spectrum in lattice QCD

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in collaboration with
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Helsinki, May 28, 2013

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Outline

- Motivation and introduction
- Mesons under Dirac low-mode truncation
- Effects on the quark propagator
- Mesons: excited states
- Baryons: ground and excited states
- Summary

Motivation

- are dynamical chiral symmetry breaking (DχSB) and confinement ultimately interrelated?
- how is the mass of light-hadrons generated?
- how important is DχSB for the mass of light hadrons?
- would there be parity doubling in a chirally symmetric world?

Hadron spectroscopy

- on the lattice we study Euclidean correlation functions

$$\langle\, O(t)\, \overline{O}(0)\, \rangle = \sum_j \langle 0|\, \hat{O}\, |j\rangle\, \langle j|\, \hat{O}^\dagger\, |0\rangle\, \mathrm{e}^{-tE_j}$$

$$= A\, \mathrm{e}^{-tE_0}\, \left(1 + \mathcal{O}(\mathrm{e}^{-t\Delta E})\right)$$

- where O is an interpolating field with the quantum numbers of the state one is interested in, e.g., a pion:

$$O_\pi(n) = \overline{\psi}^{d}(n)\gamma_5\psi^u(n)$$

- projection to zero momentum allows the identification of the exponential with the effective mass $m_{\mathrm{eff}}(t)$

The variational analysis

- we collect different interpolators O_i describing the same state, and define the cross correlation matrix

$$C_{ij}(t) \equiv \langle\, O_i(t)\, \overline{O}_j(0)\, \rangle$$

- solving the generalized eigenvalue problem

$$C(t)\vec{v} = \lambda(t) C(t_0)\vec{v}$$

- gives an estimate for the energies

$$\lambda_k(t) \sim \mathrm{e}^{-tE_k}\left(1 + \mathcal{O}(\mathrm{e}^{-t\Delta E_k})\right)$$

- the eigenvectors indicate the overlap of different states

[Lüscher, Wolf, Nucl. Phys. B **339** (1990) 222]

Chiral symmetry

- the QCD Lagrangian with two massless quark flavors is invariant under

$$\mathrm{SU}(2)_A \times \mathrm{SU}(2)_V \times \mathrm{U}(1)_A \times \mathrm{U}(1)_V$$

SU(2)_A is not really a group

- $\mathrm{U}(1)_V$ conserves the baryon number
- $\mathrm{SU}(2)_V$ is the isospin symmetry $(m_N \approx m_P)$
- $\mathrm{SU}(2)_A$ is broken by the dynamics of QCD
- $\mathrm{U}(1)_A$ is broken dynamically and explicitly by the quantization of QCD (axial anomaly)

Vafa-Witten theorem forbids spontaneous breaking of vector symmetry

Chiral symmetry on the lattice

- a chirally symmetric Dirac operator must obey

$$\{D,\, \gamma_5\} = 0$$

- No-go theorem: it is impossible to have a (naively) chirally invariant, doubler-free, local and translational invariant discretization of fermions on the lattice

[Nielsen and M. Ninomiya Phys. Lett. B **105** (1981) 219]

- way out: replace continuum condition with lattice version to obtain an exact formulation of chiral symmetry on the lattice (GW equation):

$$\{D,\, \gamma_5\} = aD\gamma_5 D$$

[Ginsparg, Wilson, Phys. Rev. D **25** (1982) 2649]

The CI Dirac operator

- the chirally improved (CI) Dirac operator is an approximate solution to the GW equation
- it is obtained by expanding the most general Dirac operator in a basis of simple operators

$$D(x,y) = \sum_{i=1}^{16} c_{xy}^{(i)}(U)\Gamma_i + m_0$$

paths of length 4 (3?)

- inserting this into the GW eq. then turns into a system of coupled quadratic equations for the expansion coefficients $c_{xy}^{(i)}(U)$
- this expansion provides for a natural cutoff that turns the quadratic equations into a simple finite system.

[Gattringer, Phys. Rev. D **63** (2001) 114501]

Eigenvalues of the Dirac operator

- the difference of left- and right-handed zero modes of the Dirac operator accounts for the *topological charge* which is responsible for the axial anomaly

[Atiyah, Singer, Ann. Math. **93** (1971) 139]

- the spectrum of non-GW fermions exhibits purely real modes which 'would-be' the zero modes

- the density of the smallest eigenvalues is related to the chiral condensate

$$\langle \overline{\psi}\psi \rangle = -\pi \rho(0)$$

order of limits important:
1.) infinite volume
2.) m->0

[Banks, Casher, Nucl. Phys. B **169** (1980) 103]

“Unbreaking” chiral symmetry

- we subtract the Dirac low-mode contribution from the valence quark propagators

$$S_{\mathrm{red}(k)} = S - \sum_{i=1}^{k} \mu_i^{-1} \left| w_i \right\rangle \left\langle w_i \right| \gamma_5$$

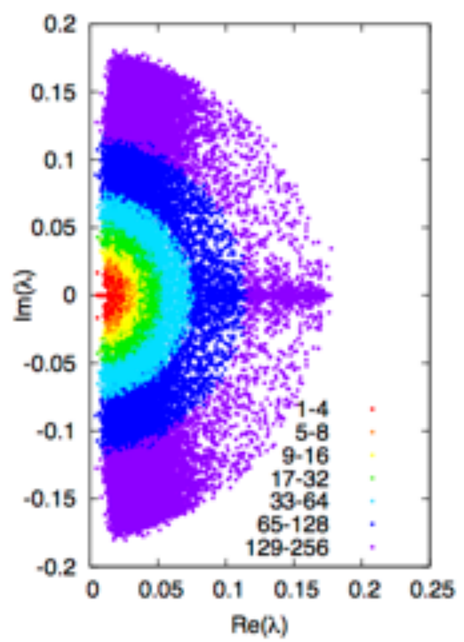
- $\mu_i, \ |w_i\rangle$ are the eigenvalues and vectors of the Hermitian Dirac operator $D_5 = \gamma_5 D$ and k denotes the truncation level

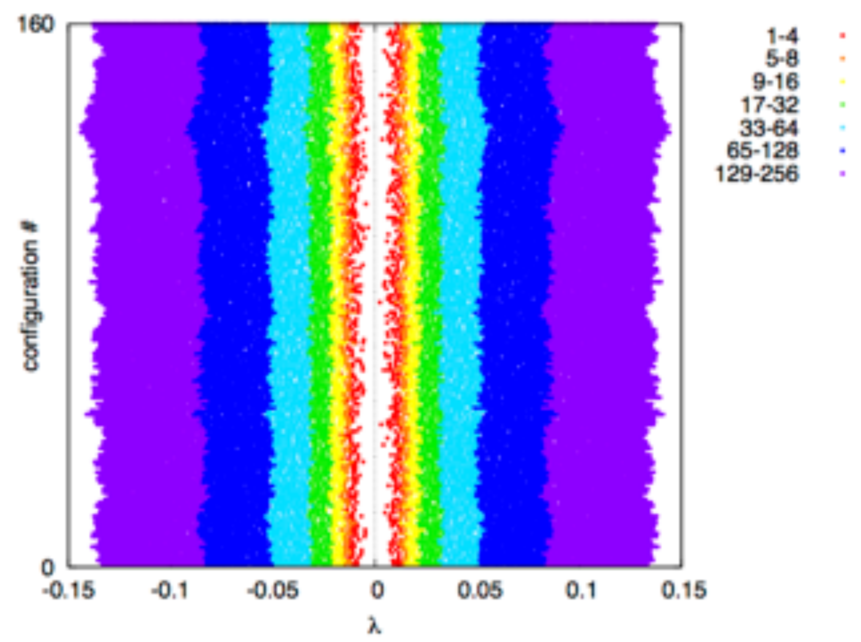
- this truncation corresponds to removing the chiral condensate of the valence quark sector by hand

- in the following we are going to perform a hadron spectroscopy with the truncated quark propagators

The setup

- we adopt 161 gauge field configurations with two flavors of degenerate CI fermions [Gattringer et al., PRD **79** (2009) 054501]
- pion mass $m_\pi = 322(5)\,\mathrm{MeV}$
- lattice size $16^3 \times 32$
- lattice spacing $a = 0.144(1)\,\mathrm{fm}$
- $L \cdot m_\pi \approx 3.75$





Mesons under low-mode truncation

- we restrict ourselves to the study of isovector mesons (no need for disconnected diagrams)
- the following Dirac low-mode truncated meson correlators will be investigated:

$\rho\,(1^{--})$ $\qquad$ $a_1\,(1^{++})$

$\pi\,(0^{-+})$ $\qquad$ $a_0\,(0^{++})$

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Pion low-modes only

[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode contribution to the correlators for the $J^{PC} = 0^{-+}$ sector in comparison to the correlators from full propagators
- interpolators: (a) $\bar{u}\gamma_5 d$ (b) $\bar{u}\gamma_4\gamma_5 d$

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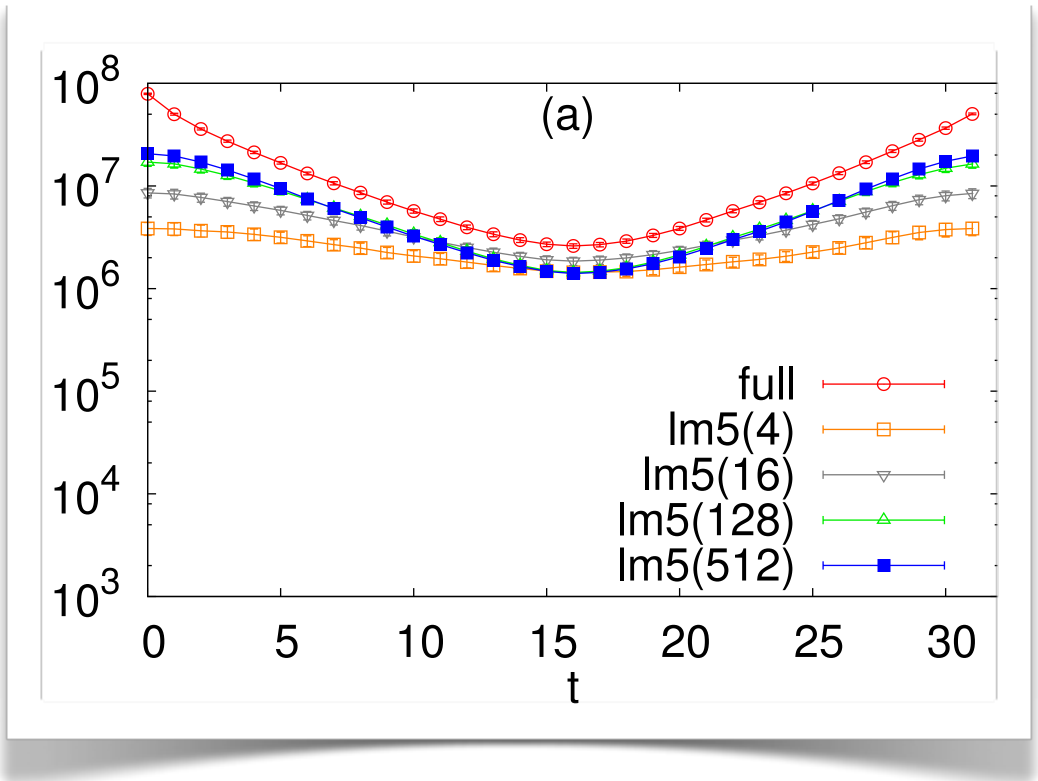

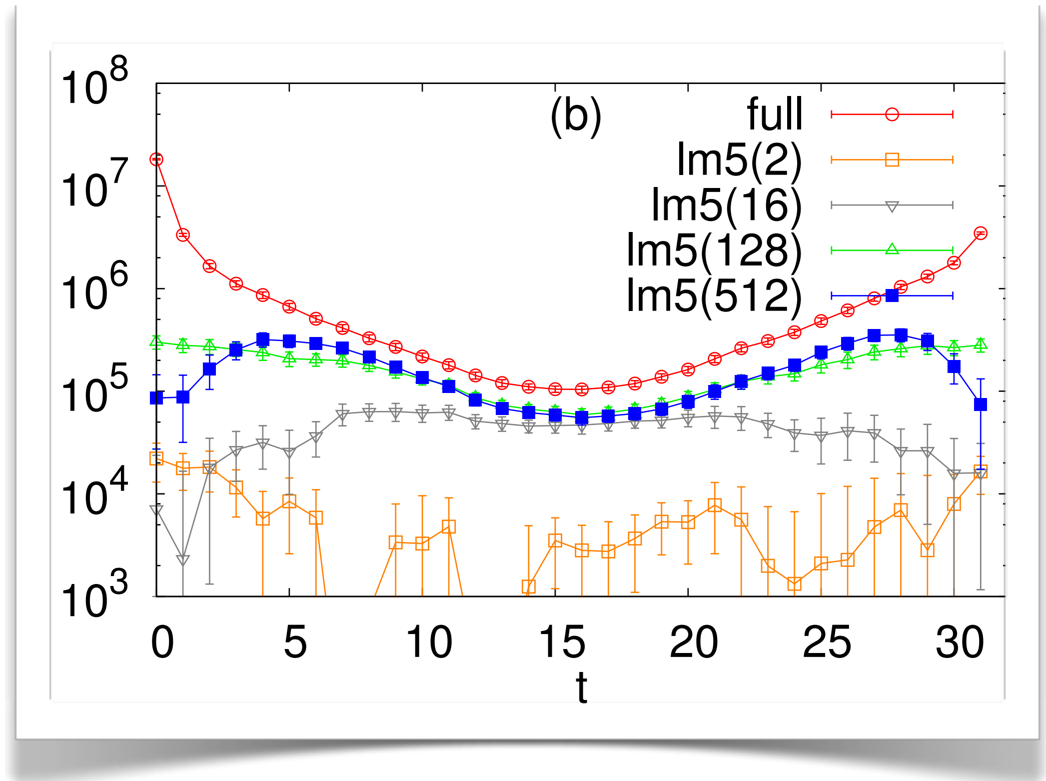

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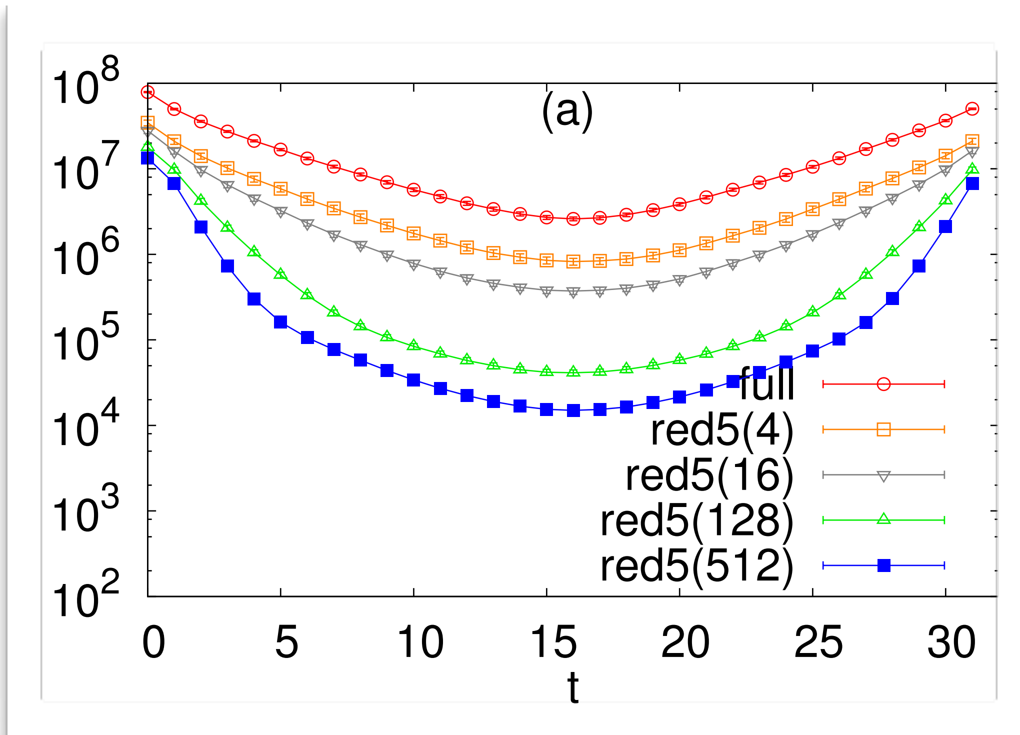
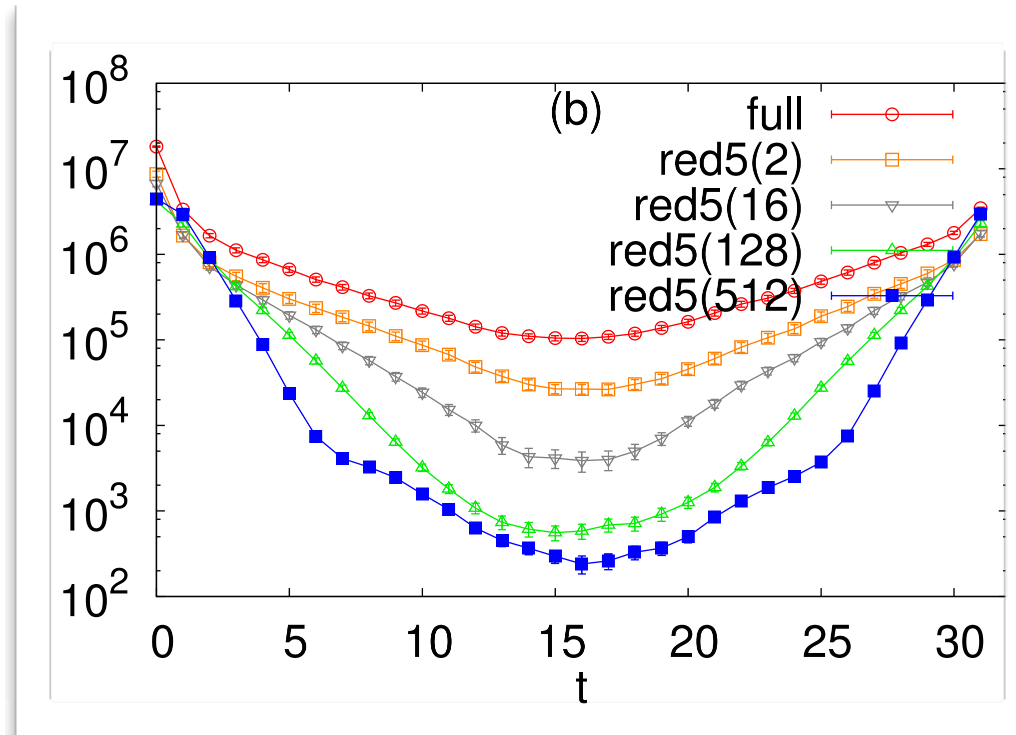
Pion without low-modes





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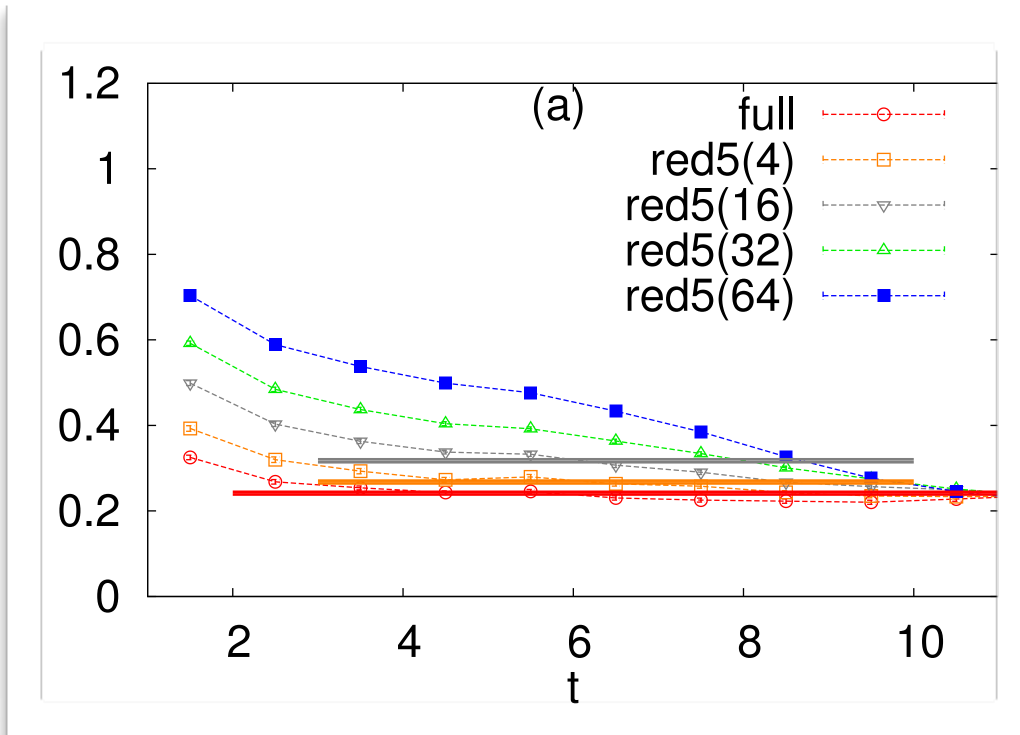

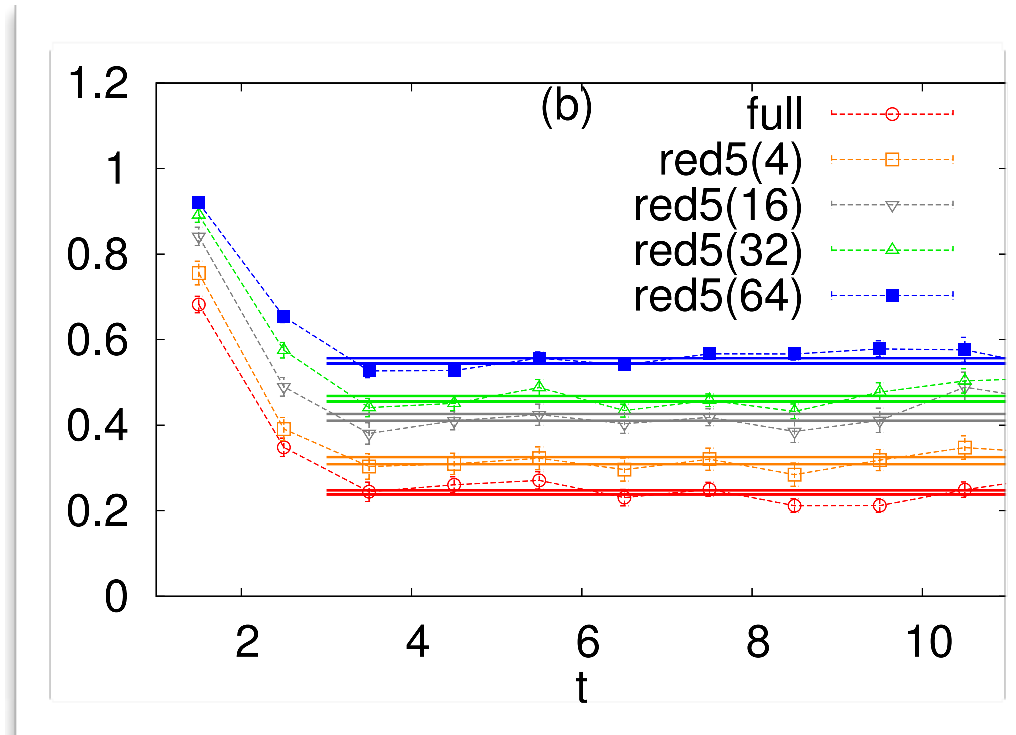

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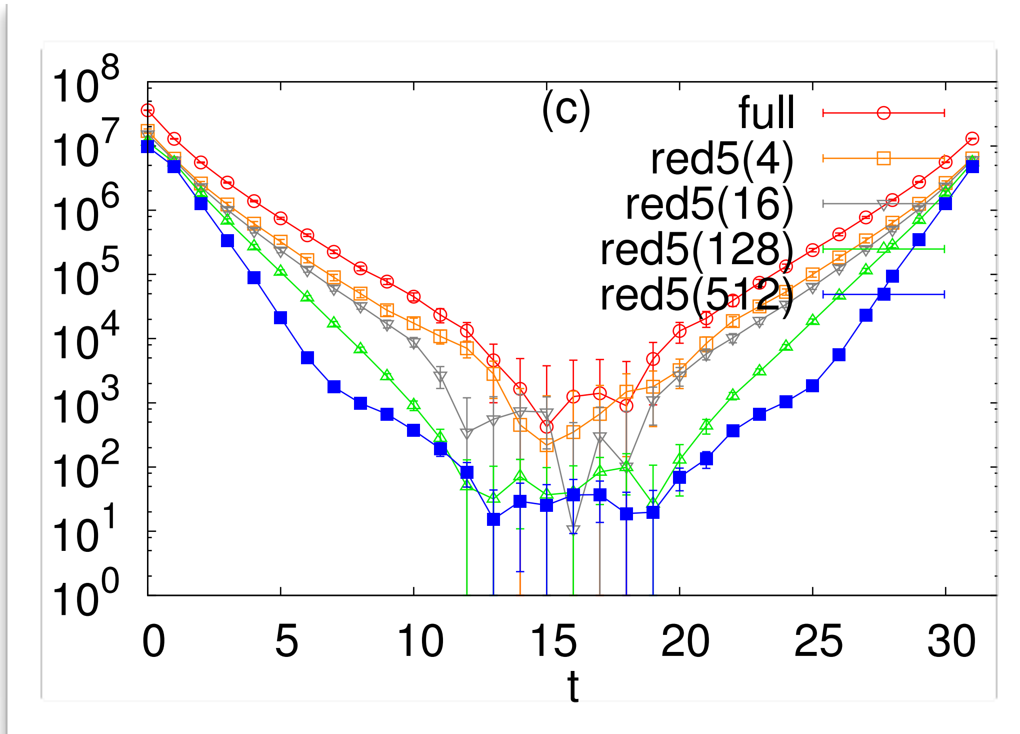
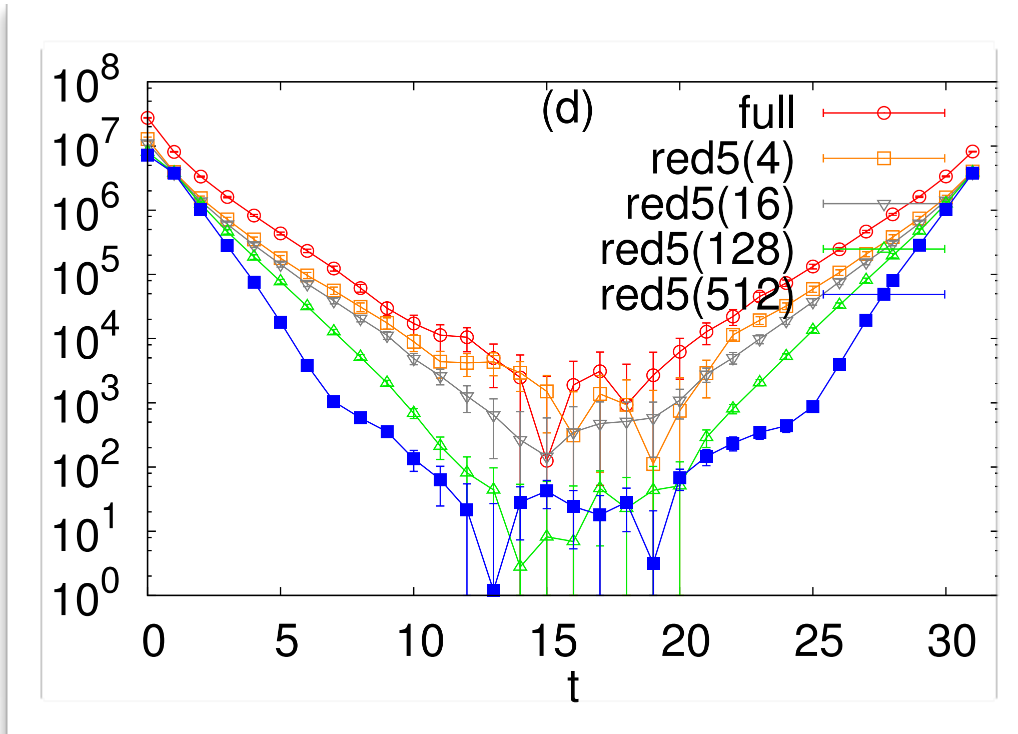
Rho without low-modes





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- Low-mode truncated correlators of the $J^{PC} = 1^{--}$ sector in comparison to the correlators from full propagators

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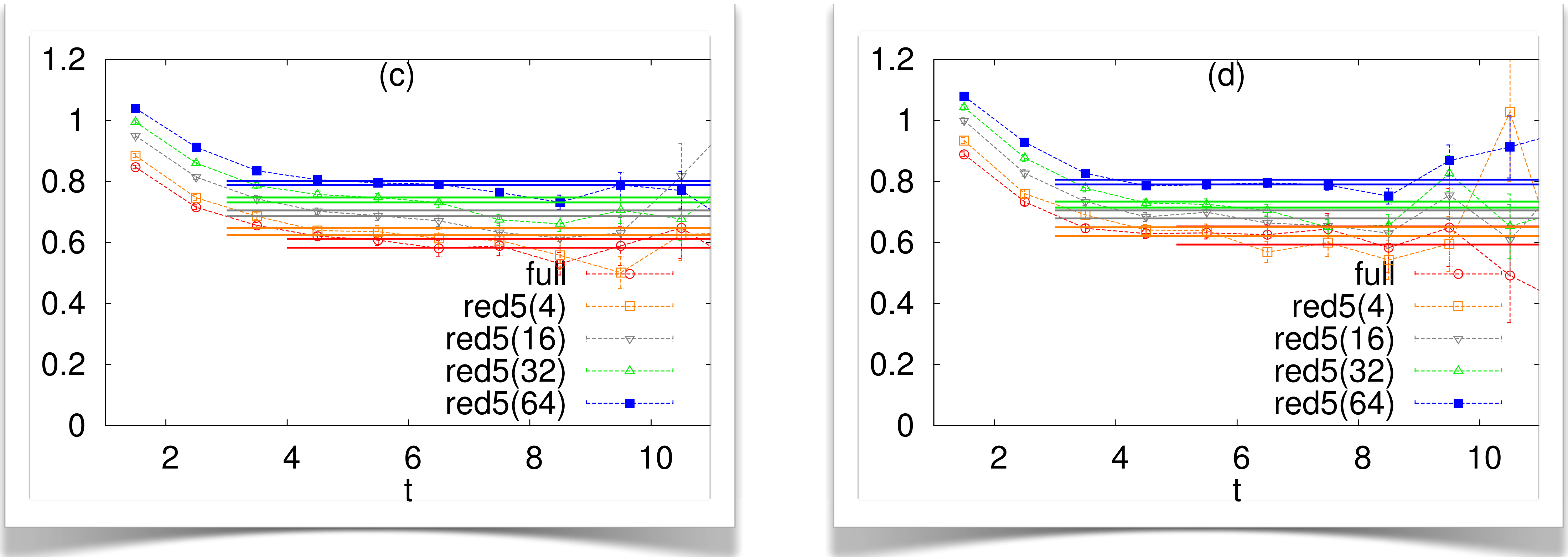
Rho without low-modes



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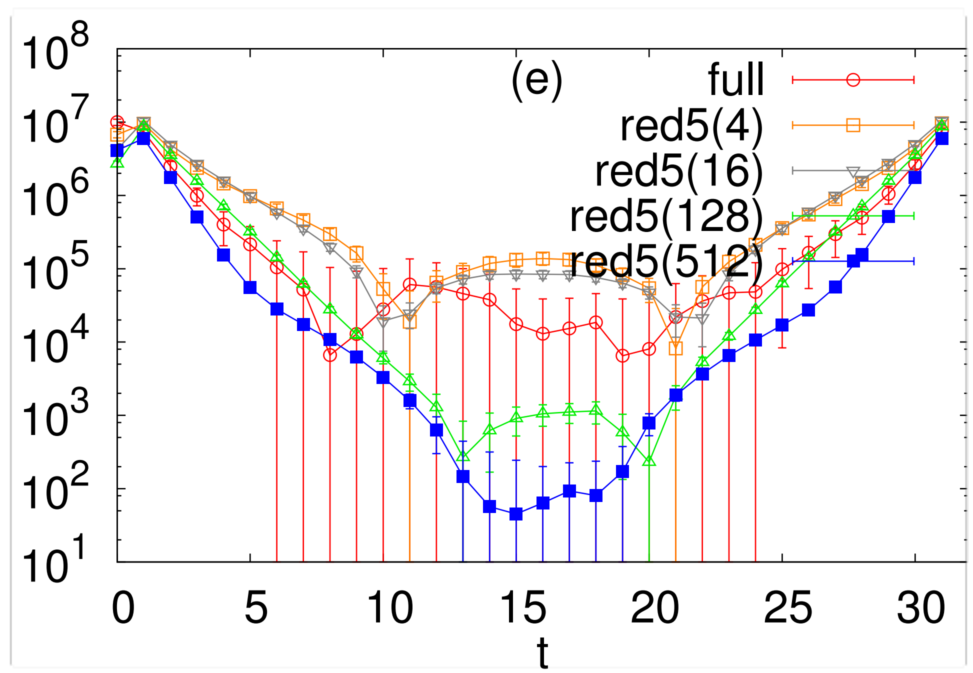
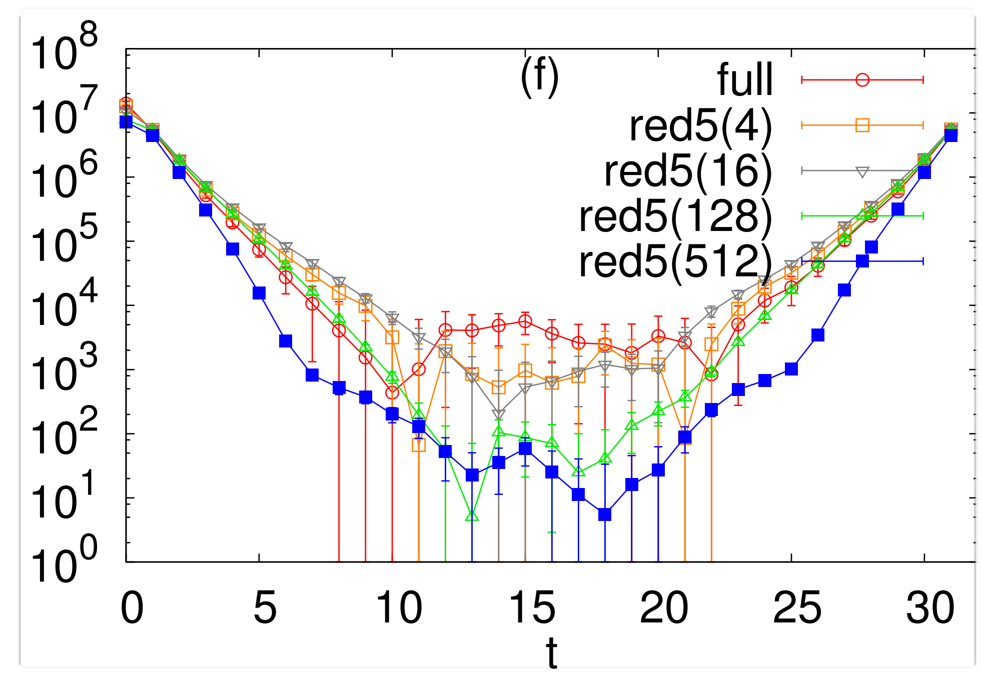
a_0 and a_1 without low-modes





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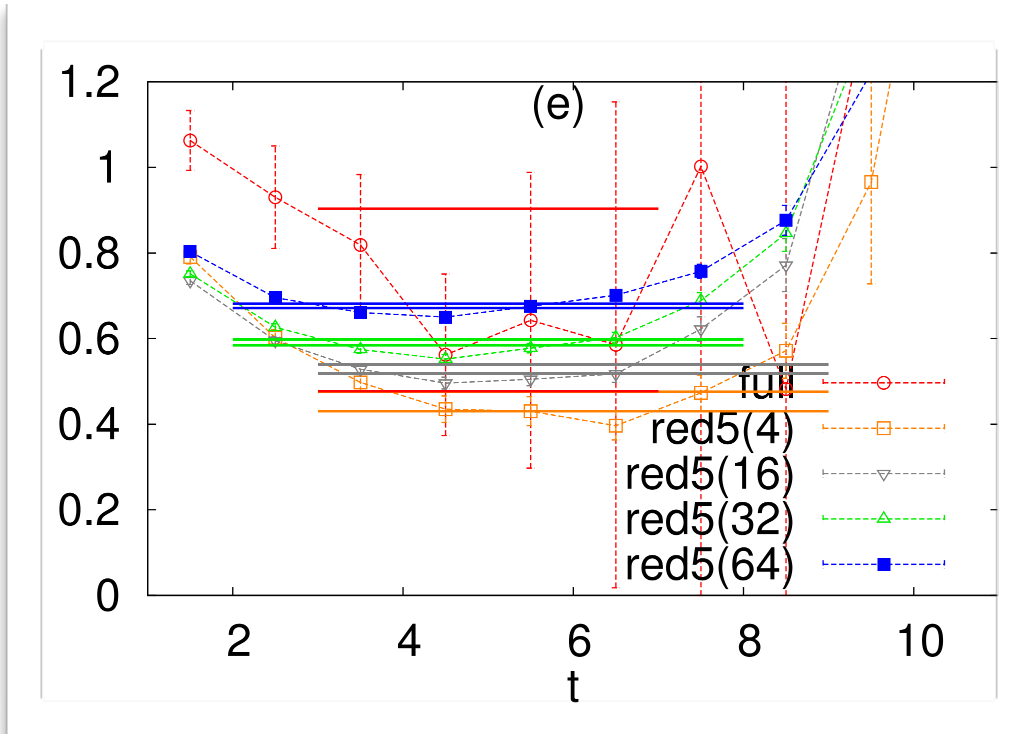
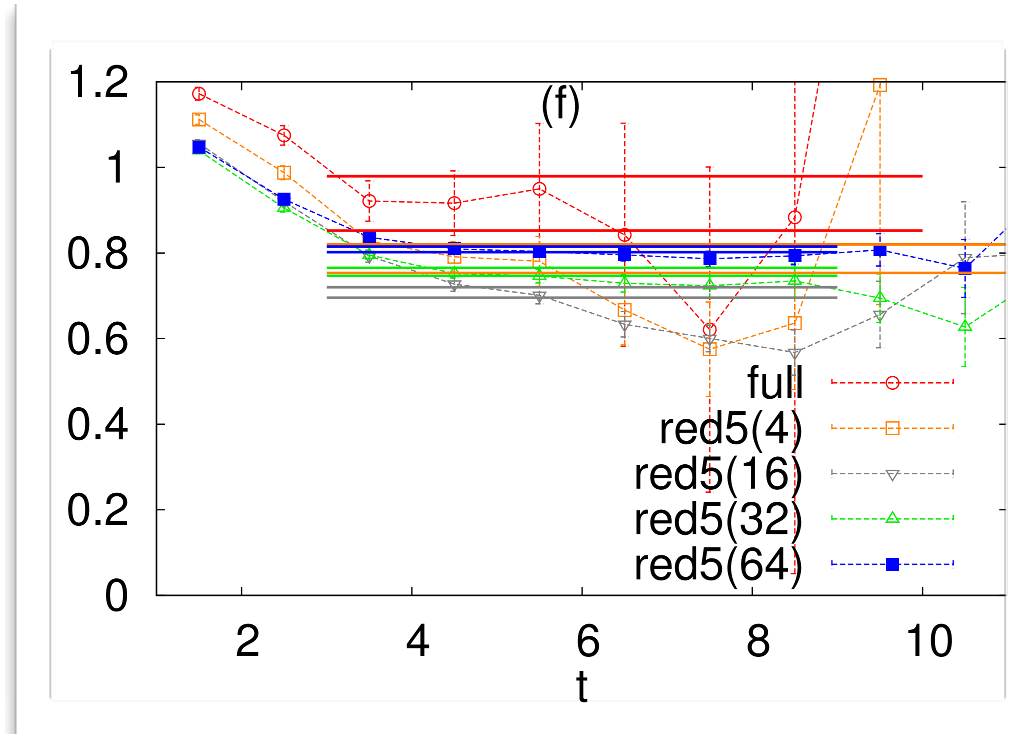
- Low-mode truncated correlators of the $J^{PC} = 0^{++}, 1^{++}$ sector in comparison to the correlators from full propagators

- interpolators: (e) $\bar{u}d$ (f) $\bar{u}\gamma_i\gamma_5 d$

a_0 and a_1 without low-modes





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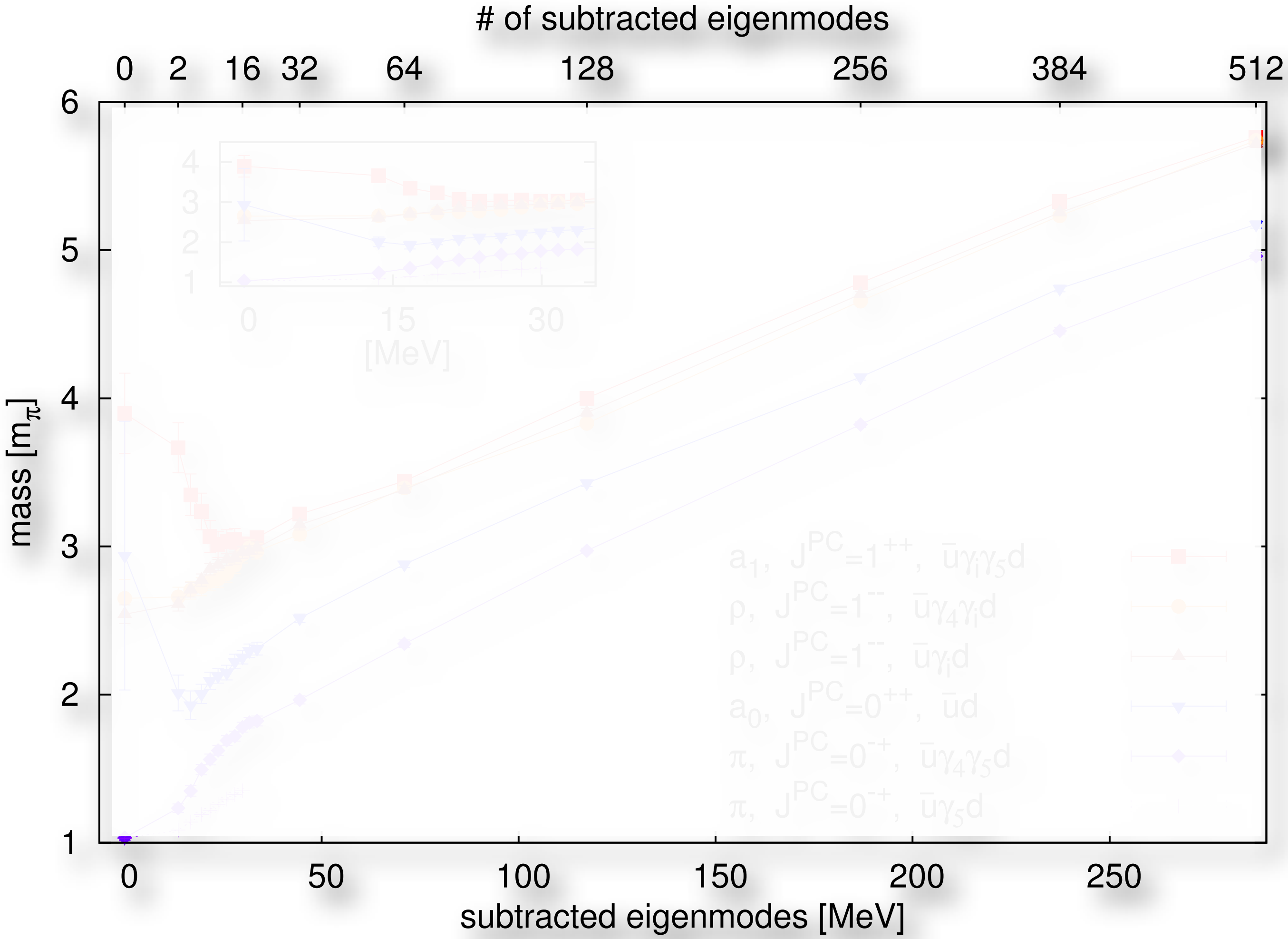
Meson mass evolution



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- degeneracy of rho and a_1: restoration of the chiral symmetry
- nondegeneracy of pion and a_0: $\mathrm{U}(1)_A$ remains broken
- growing of the mesons masses ???

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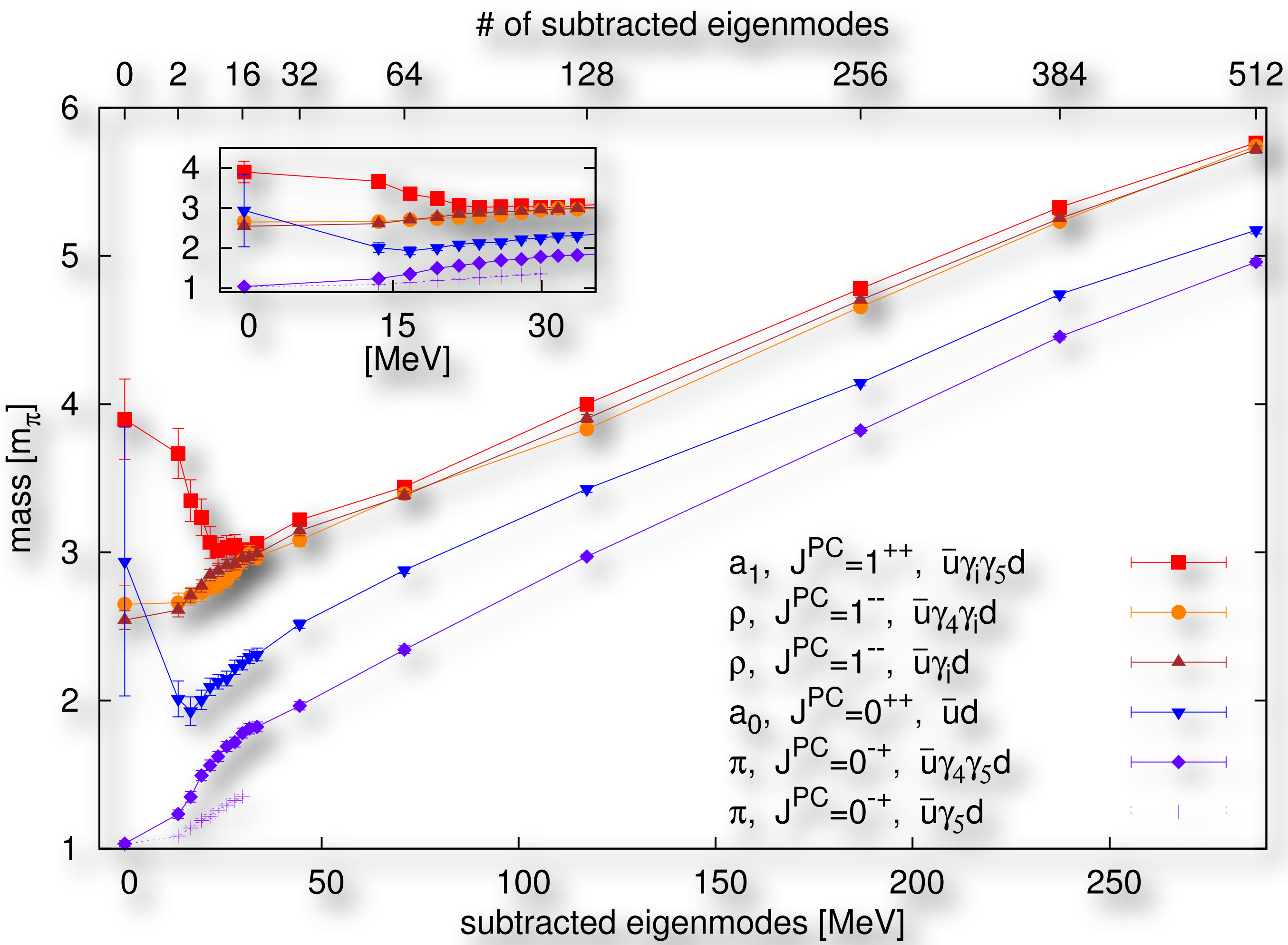

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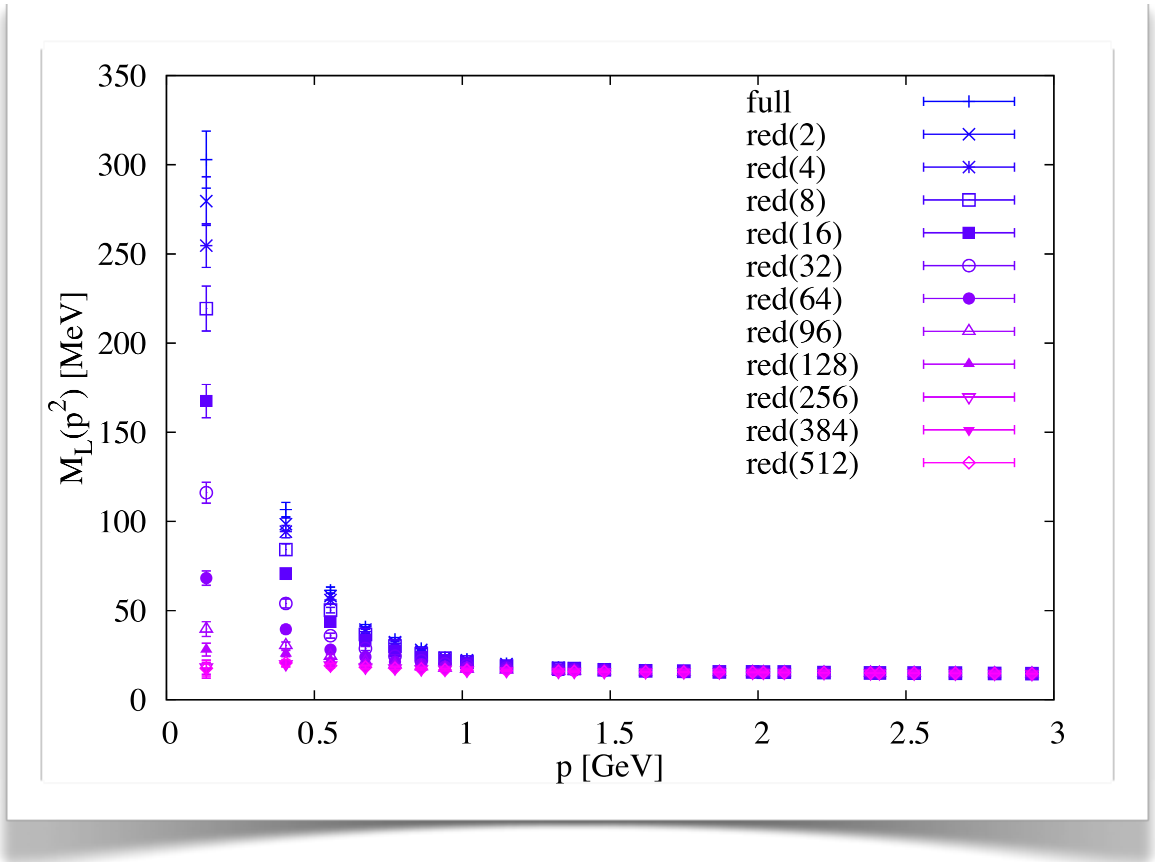
Landau gauge quark propagator

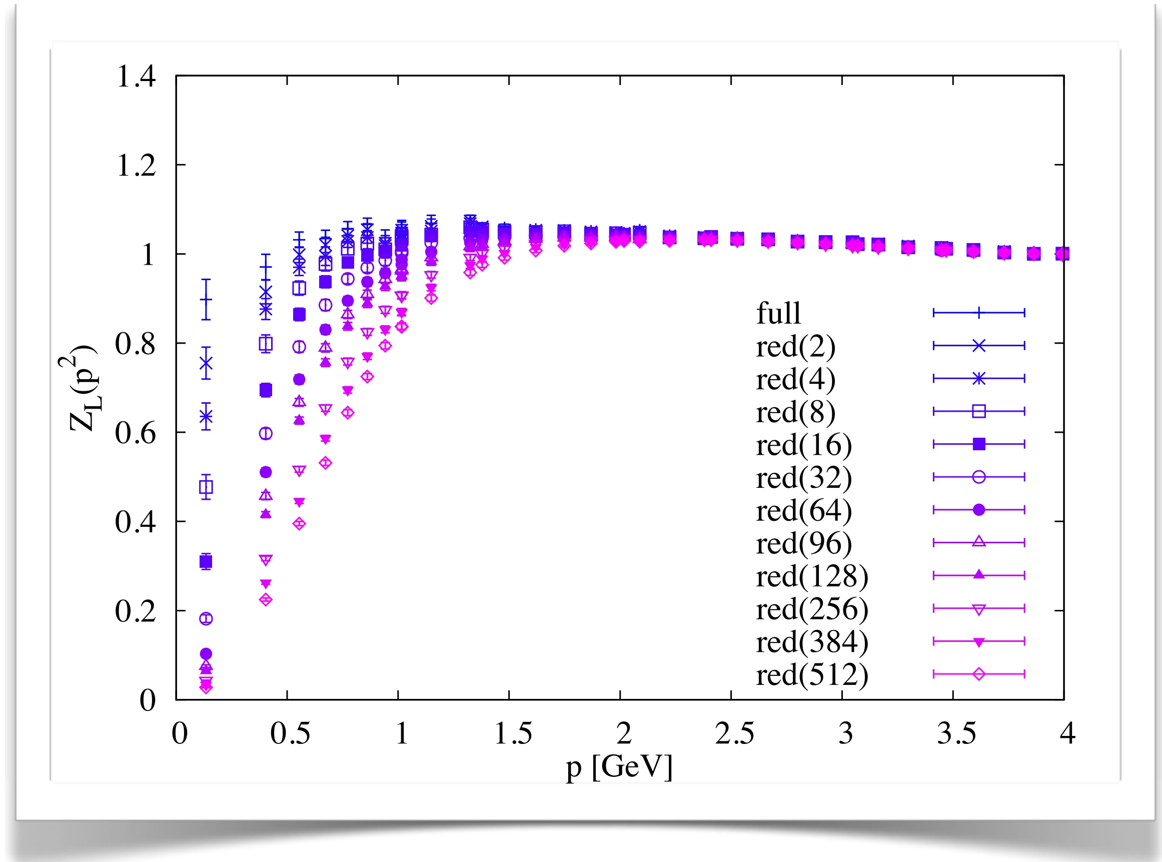
- we study the quark propagator to shed light on the origin of the large meson mass upon Dirac low-mode reduction
- the renormalized quark propagator has the form

$$S(\mu;p^2) = \left(i\not{p}A(\mu;p^2) + B(\mu;p^2)\right)^{-1} = \frac{Z(\mu;p^2)}{i\not{p} + M(p^2)}$$

- we extract the wavefunction renormalization function $Z(\mu;p^2)$ and the mass function $M(p^2)$ from the lattice and study their evolution under low-mode truncation

Truncated quark propagator





[M.S., Phys. Lett. B **711** (2012) 217-224]

- flattening of $M(p^2)$ $\Longleftrightarrow$ vanishing $\langle\,\overline{\psi}\psi\,\rangle$
- IR suppression of $Z(\mu;p^2)$ $\Longleftrightarrow$ suppression of low momentum quarks

Dirac modes and quark momenta

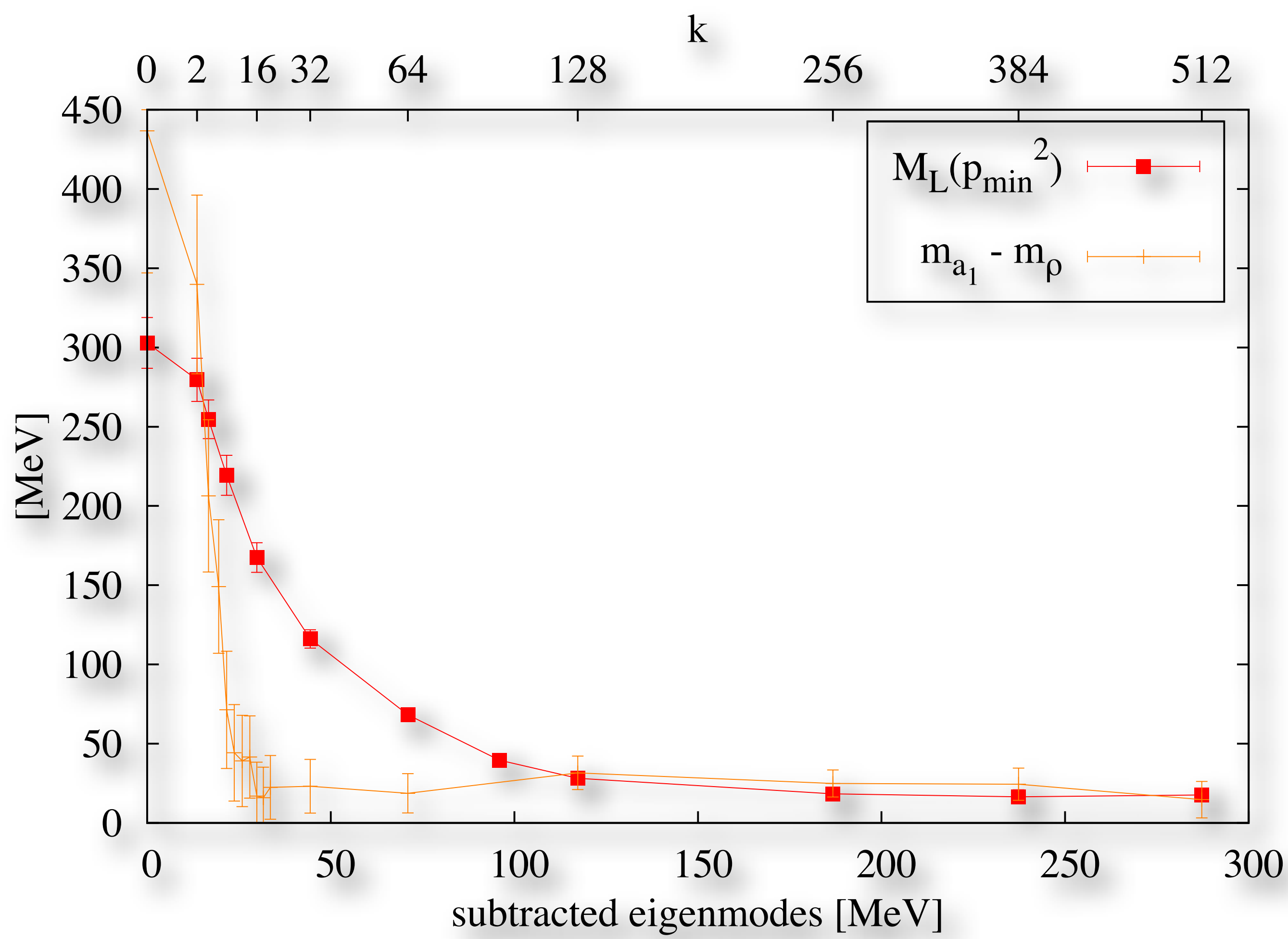
- the eigenvalues of the free Dirac operator can be derived analytically

$$\lambda = s \pm i\,|k|$$

- where $s(p)$ denotes the scalar part of the Dirac operator and $k(p)$ are the lattice momenta
- setting the small eigenvalues to zero makes the low momentum states imaginary and thus unphysical

Increased quark momenta

- 1.) explains growing meson masses
- 2.) chiral restoration in mesons is partially *effective*, compare chiral restoration in mesons with vanishing of the chiral condensate:



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Variational analysis: mesons

- we extend our study by adopting different quark source smearings (Gaussian smearing of different width and a derivative source)
- the variational method than allows the extraction of excited states
- derivative source crucial for tensor meson b_1, which would-be connected

$b_1\,(1^{+-})$ $\rho\,(1^{--})$

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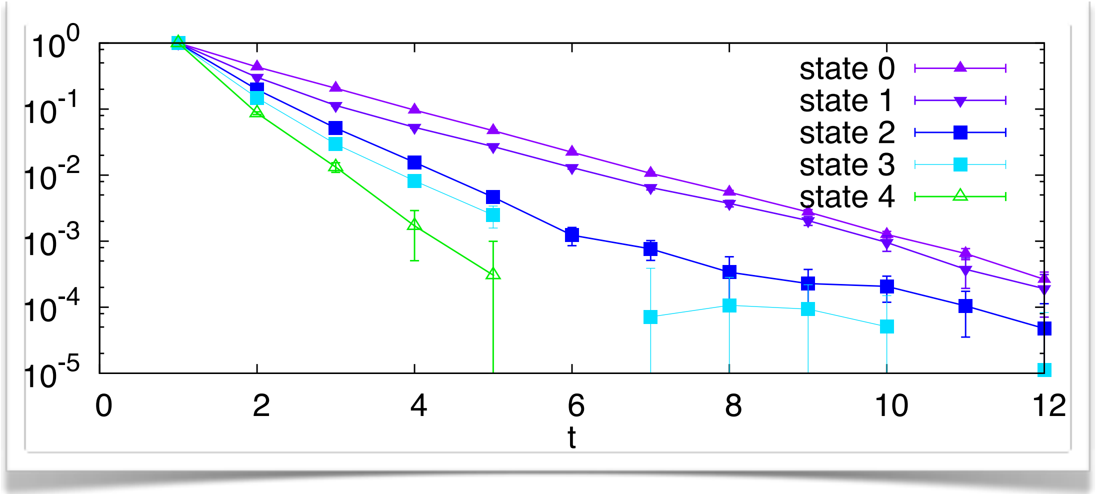
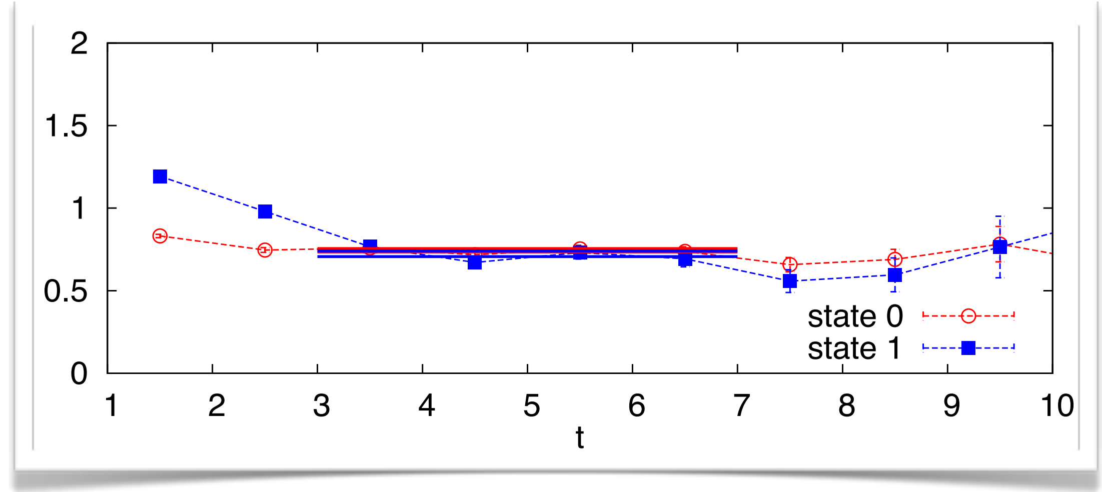
$$b_1\,(1^{+-}) \xleftrightarrow{\quad \mathrm{U}(1)_A \quad} \rho\,(1^{--})$$

Truncation $k = 64$ of $\rho\ (1^{--})$

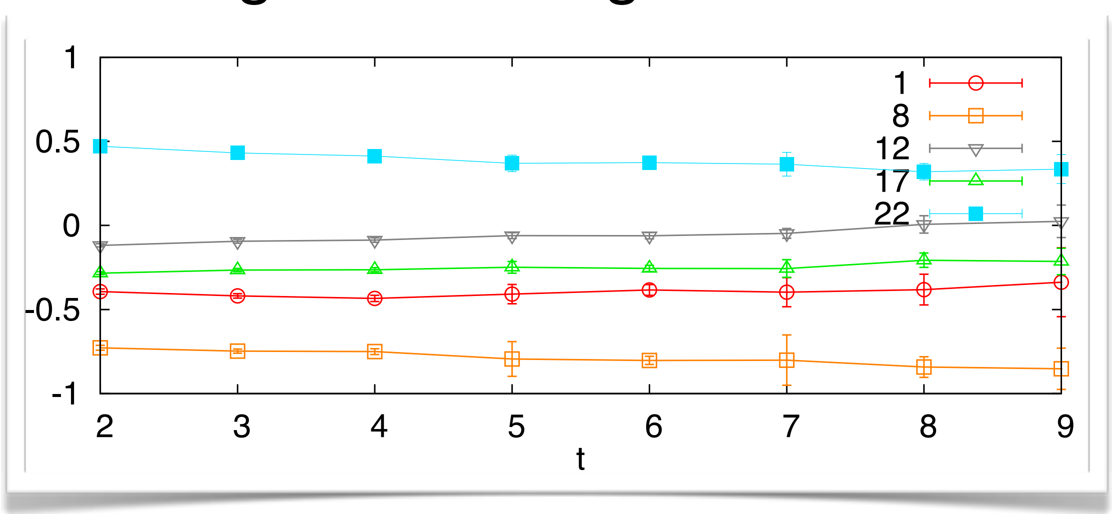
Correlators (eigenvalues) of all states



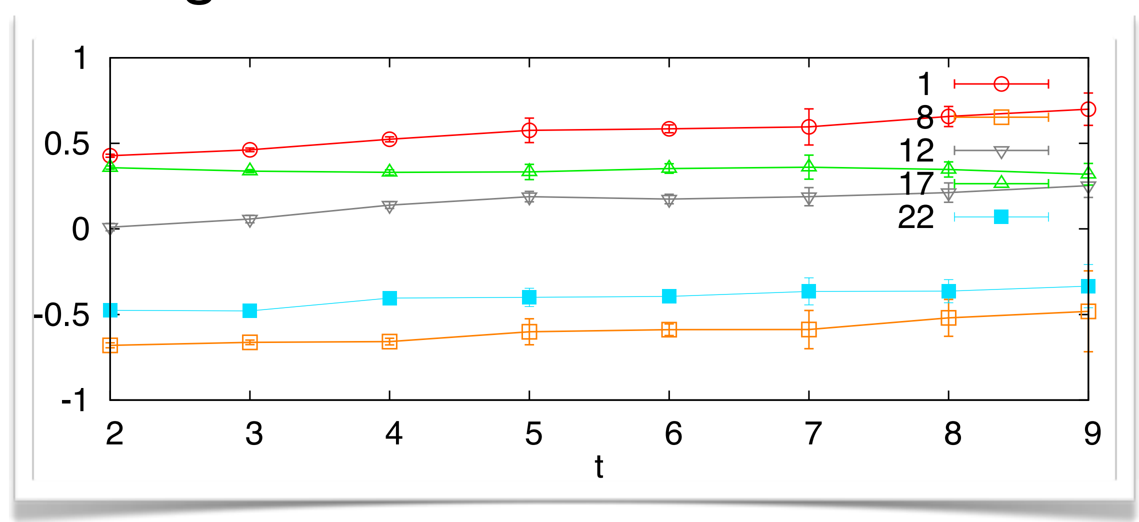
Effective masses of lowest two states



Eigenvectors of ground state



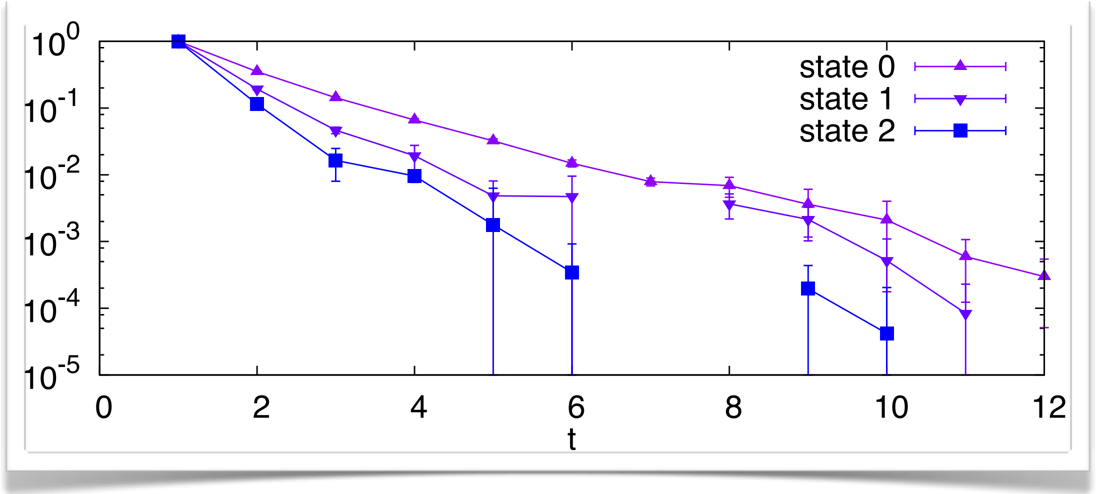
Eigenvectors of first excited state



[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Low-mode truncated $a_1(1^{++})$

Correlators (eigenvalues), k=64



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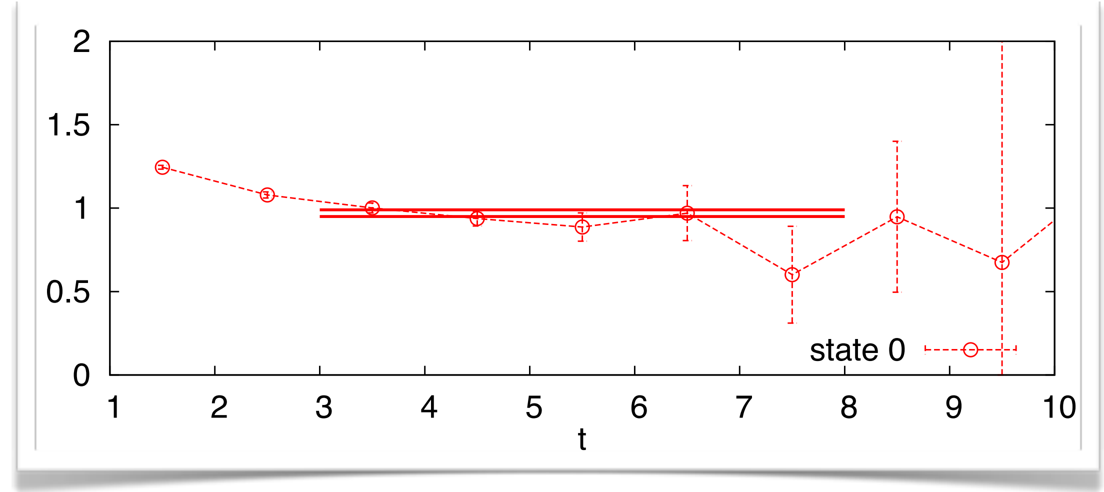
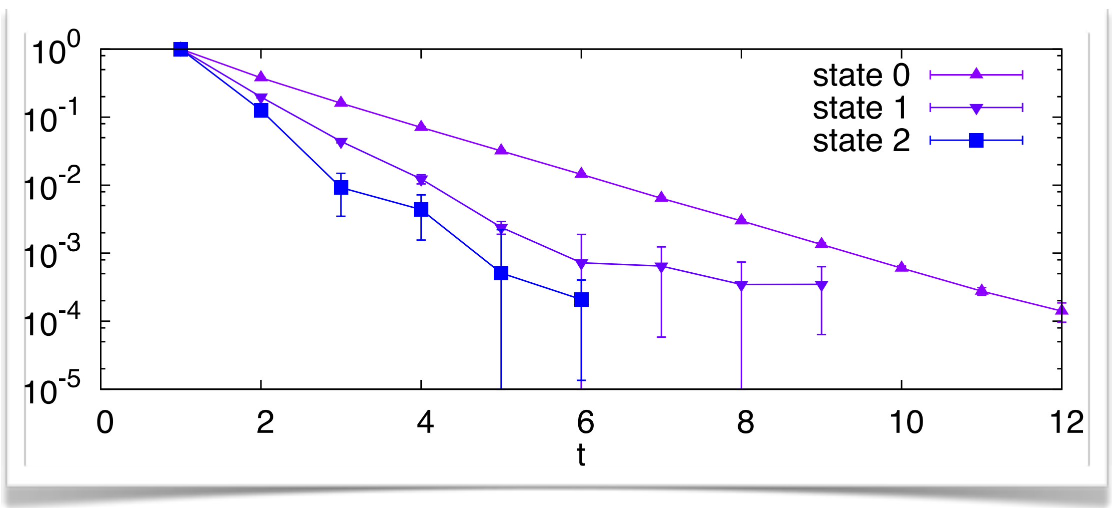
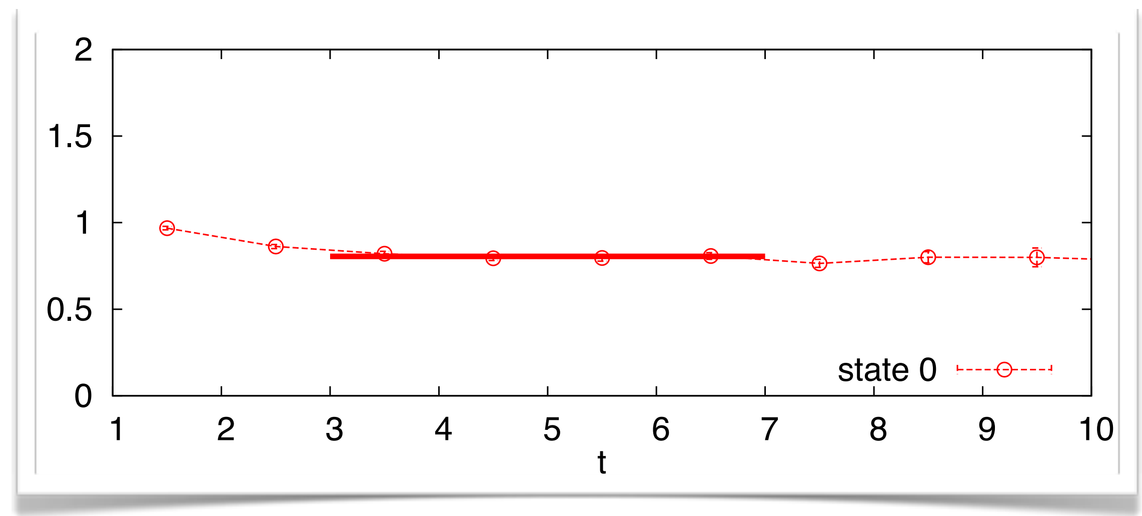


Correlators (eigenvalues), k=4



Eigenvectors of first excited state



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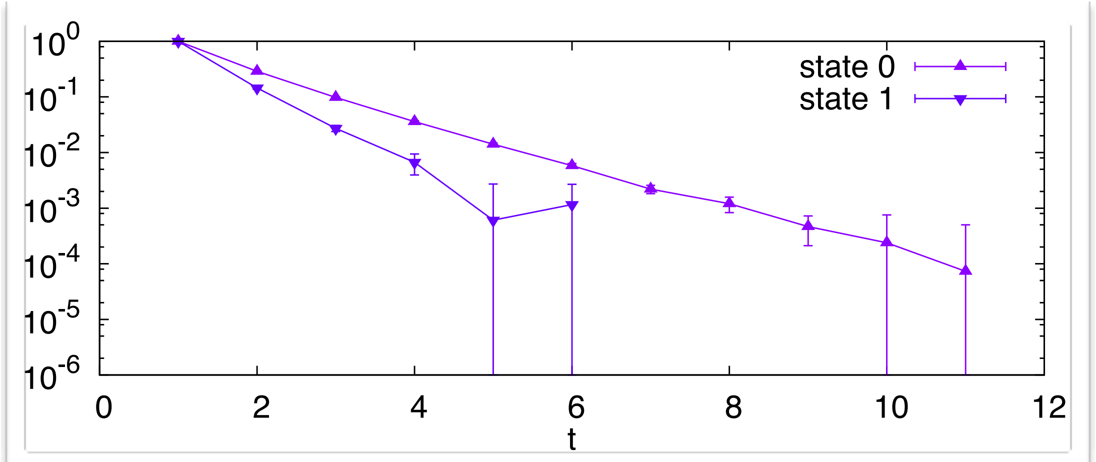
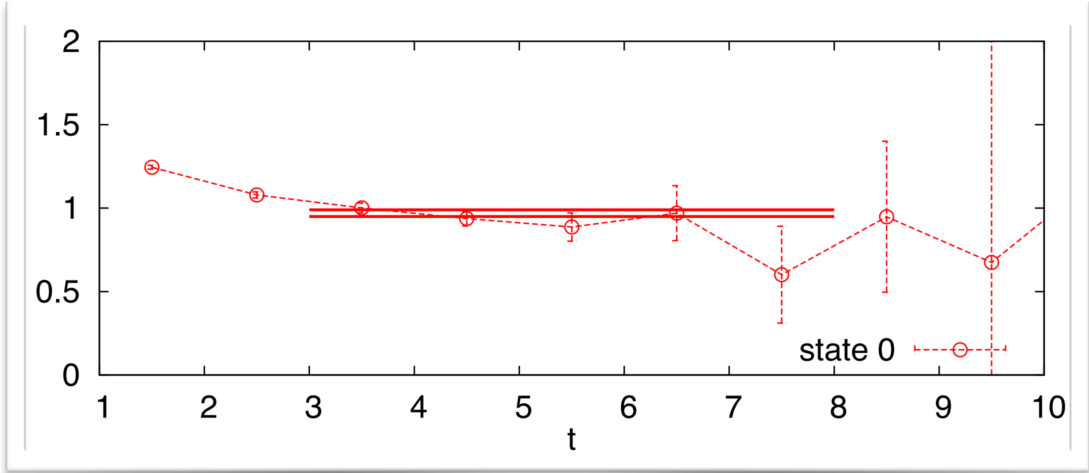
Low-mode truncated $b_1(1^{+-})$

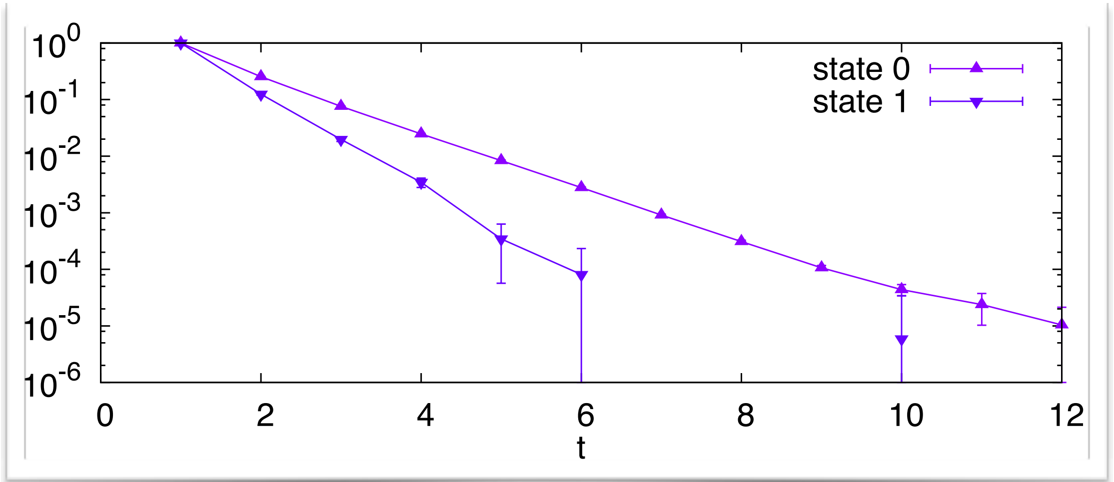
Correlators (eigenvalues), k=2



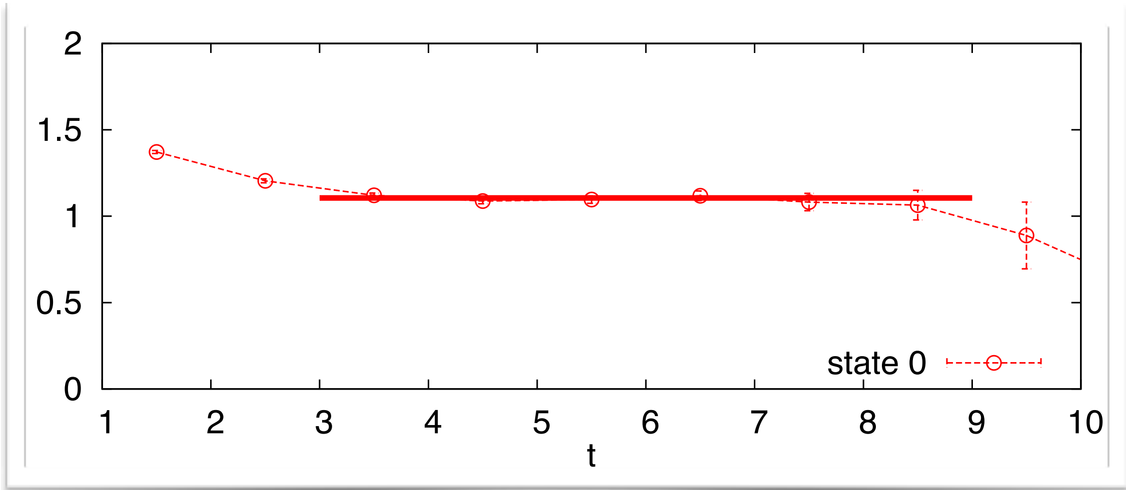
Effective masses of ground state



Correlators (eigenvalues), k=128



Eigenvectors of first excited state



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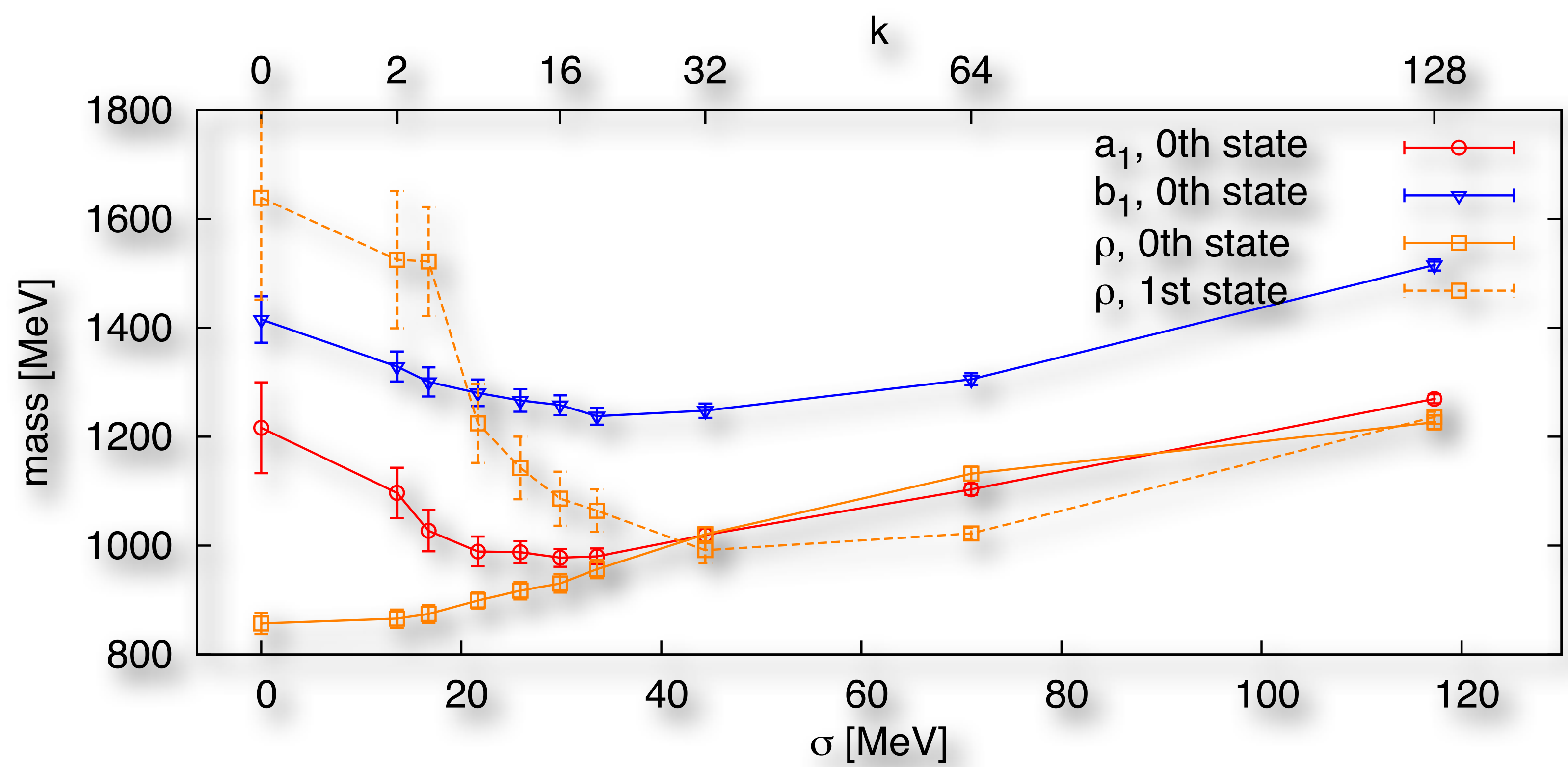
Meson mass evolution



[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

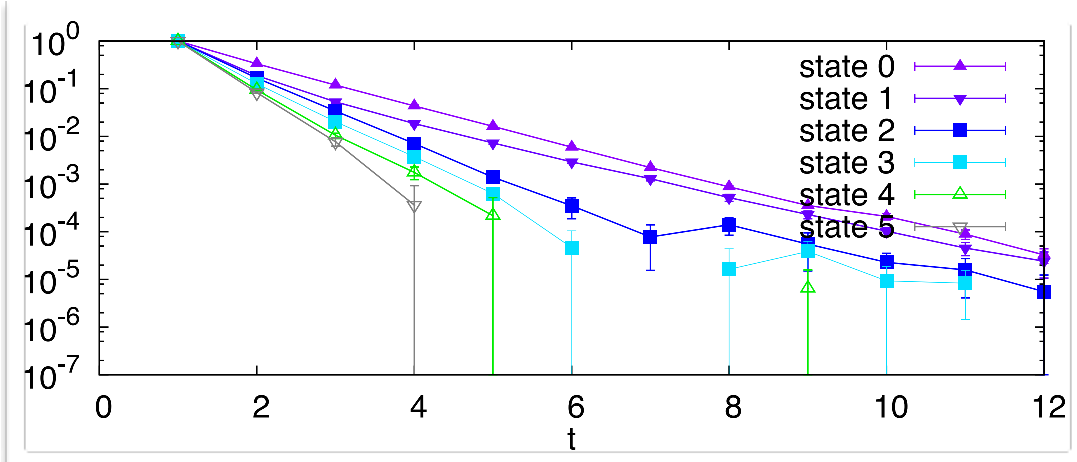
- degeneracy of two lowest rho states
- b_1 mass remains larger then rho mass: confirms that single flavor axial symmetry remains broken

Variational analysis: baryons

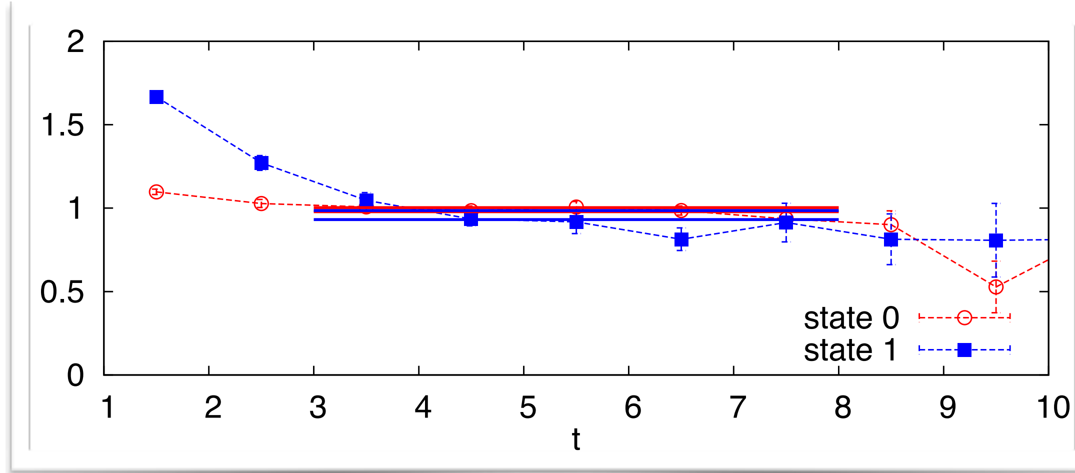
- we study the nucleon and Delta ground and first excited state of positive and negative parity
- can we find parity doubling?
- what happens to the nucleon-Delta splitting?

Truncation $k = 20$ of $N(1/2^+)$

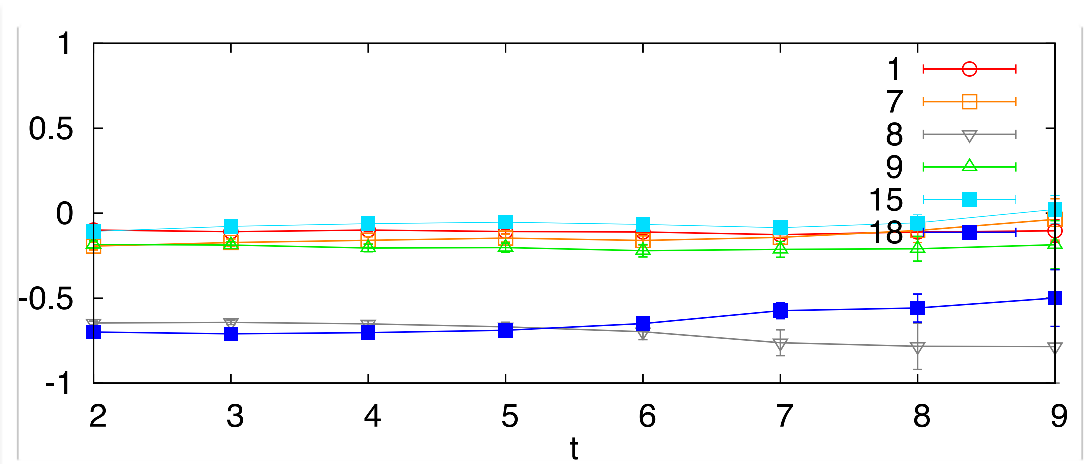
Correlators (eigenvalues) of all states



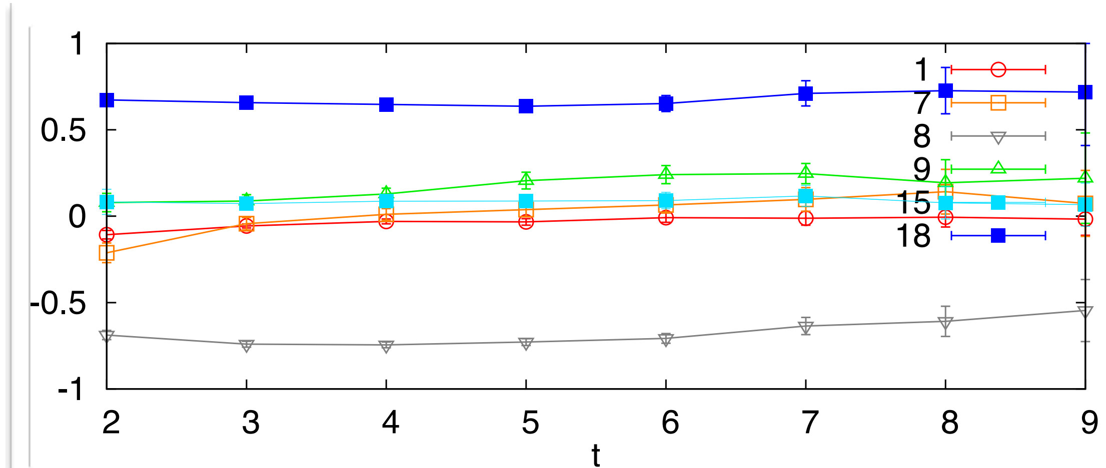
Effective masses of lowest two states



Eigenvectors of ground state



Eigenvectors of first excited state



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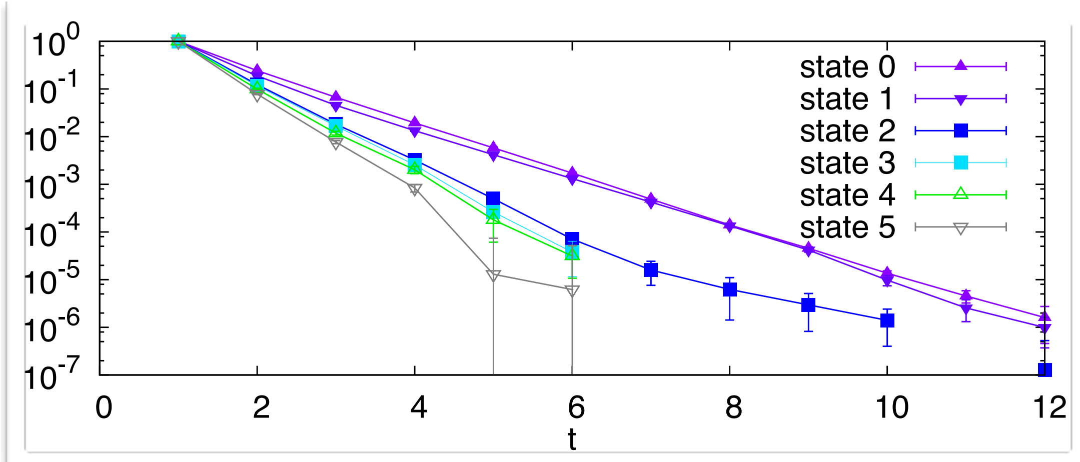
Truncation $k = 64$ of $N(1/2^-)$

Correlators (eigenvalues) of all states



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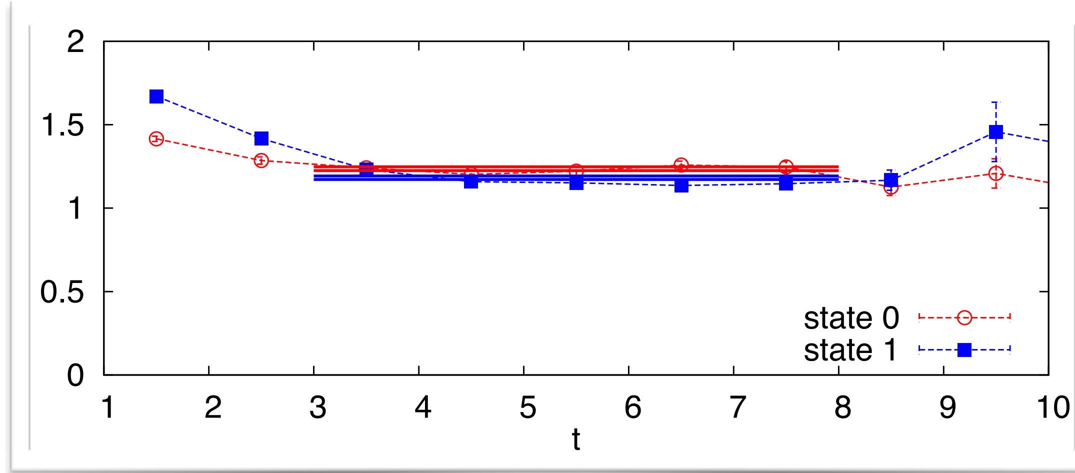
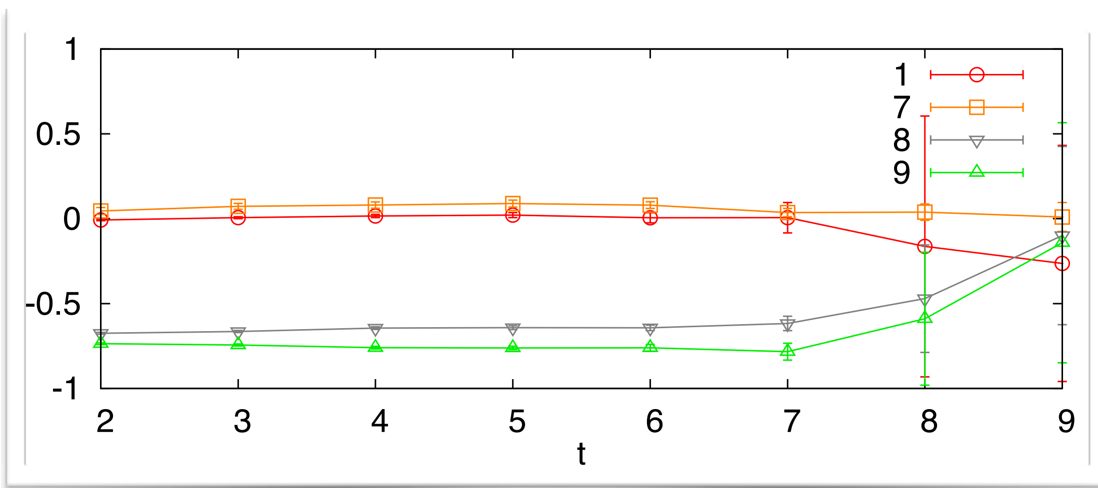
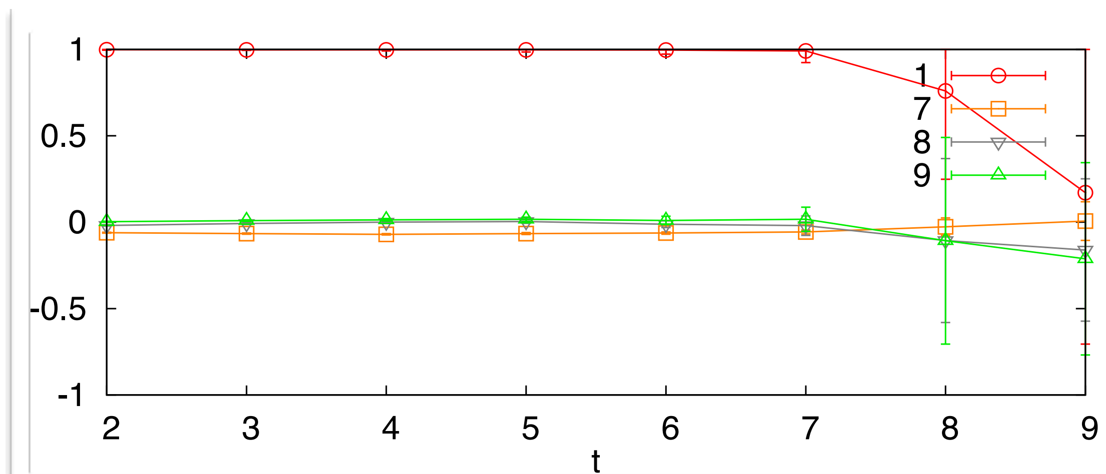


Eigenvectors of ground state



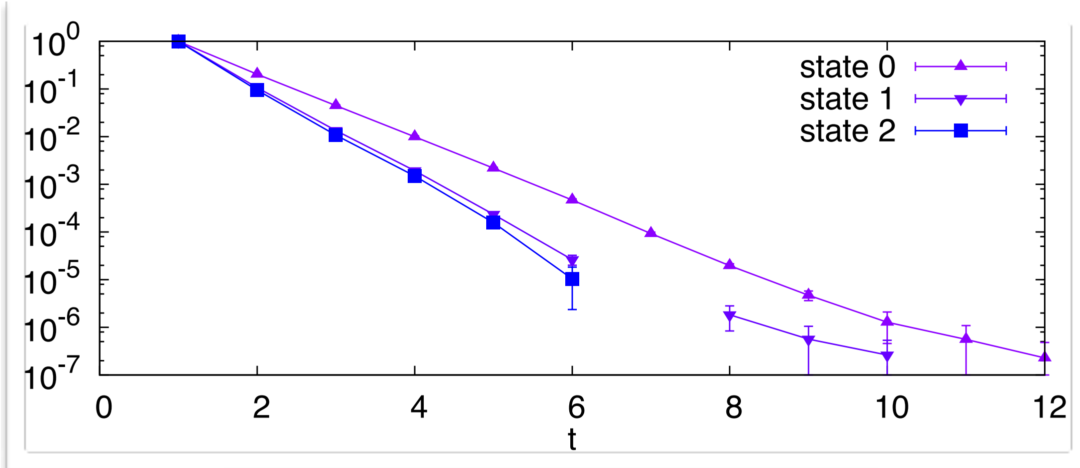
Eigenvectors of first excited state



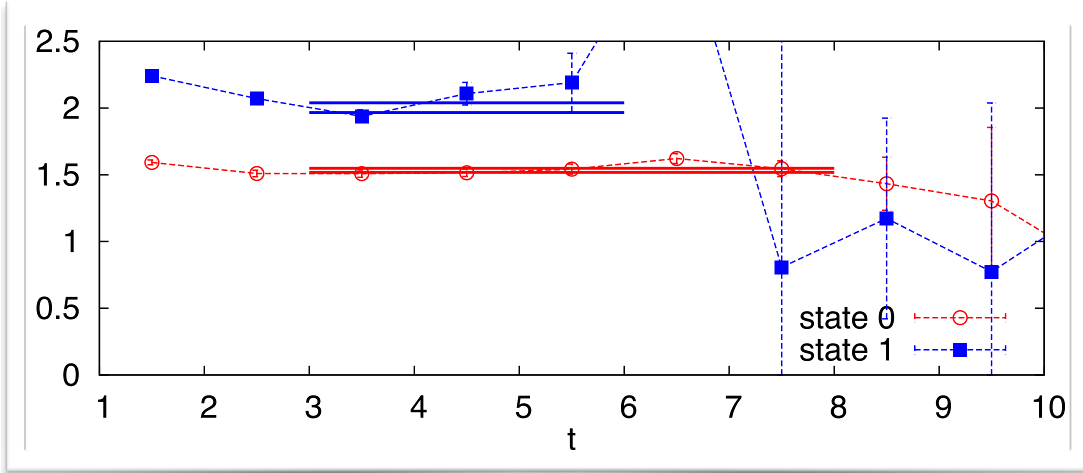
[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Truncation $k = 128$ of $\Delta(1/2^+)$

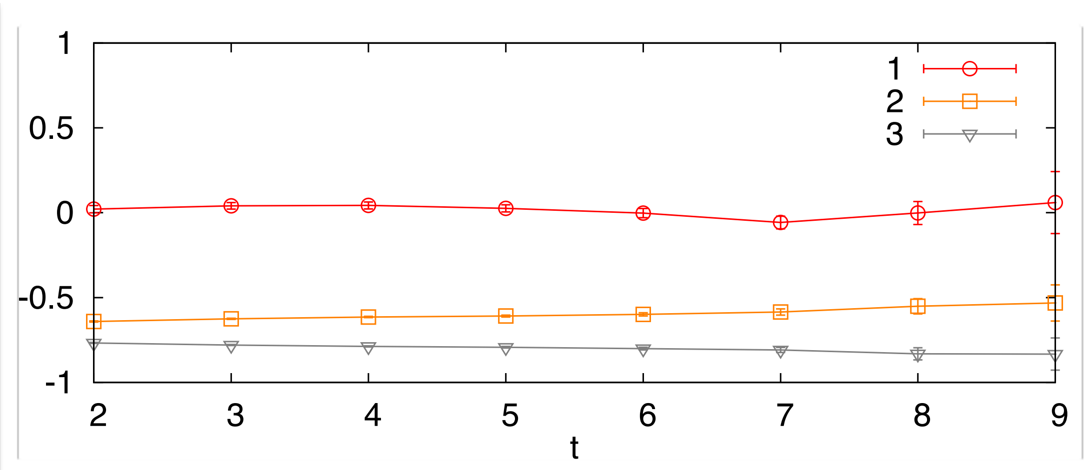
Correlators (eigenvalues) of all states



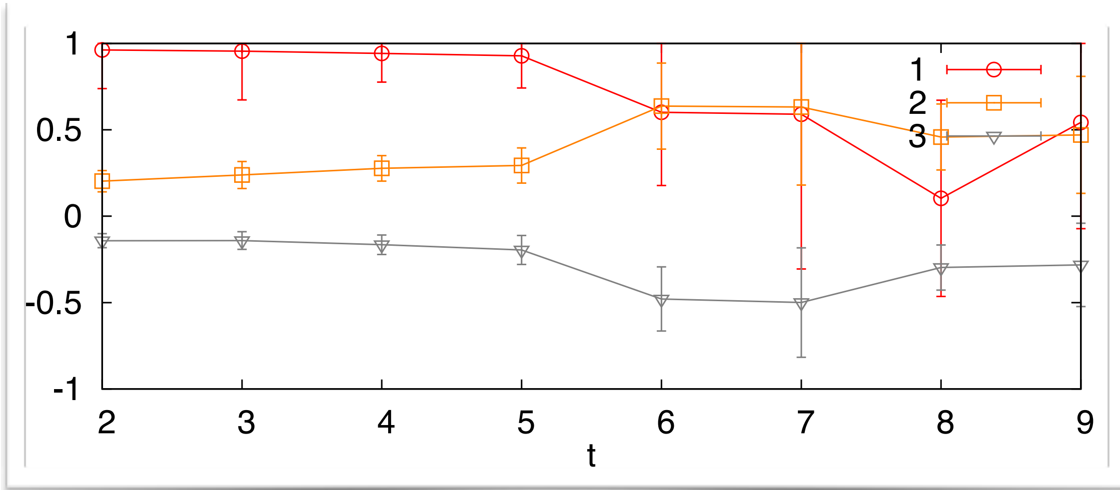
Effective masses of lowest two states



Eigenvectors of ground state



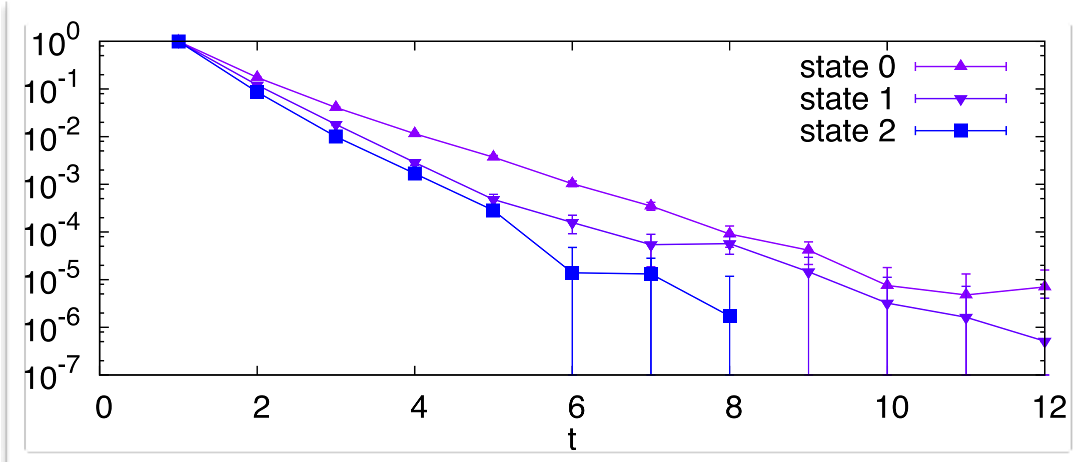
Eigenvectors of first excited state



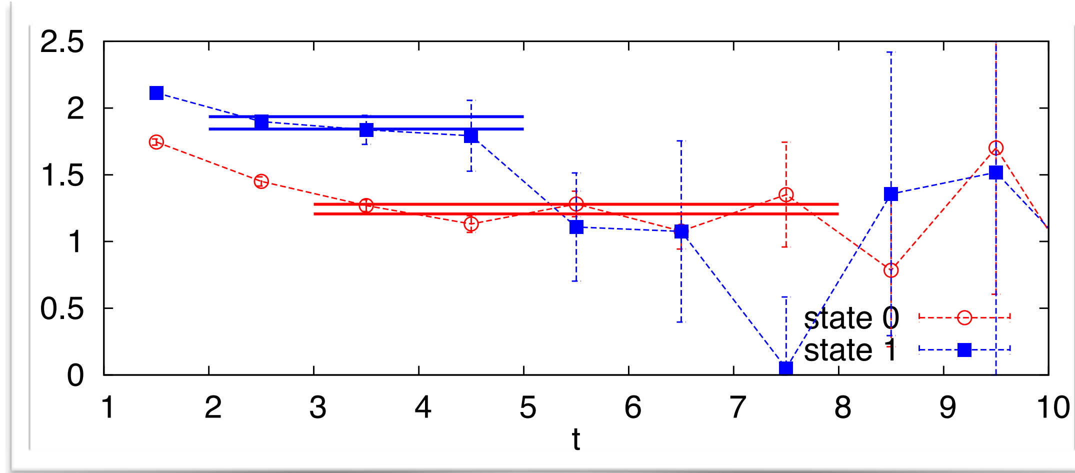
[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Truncation $k = 16$ of $\Delta(1/2^-)$

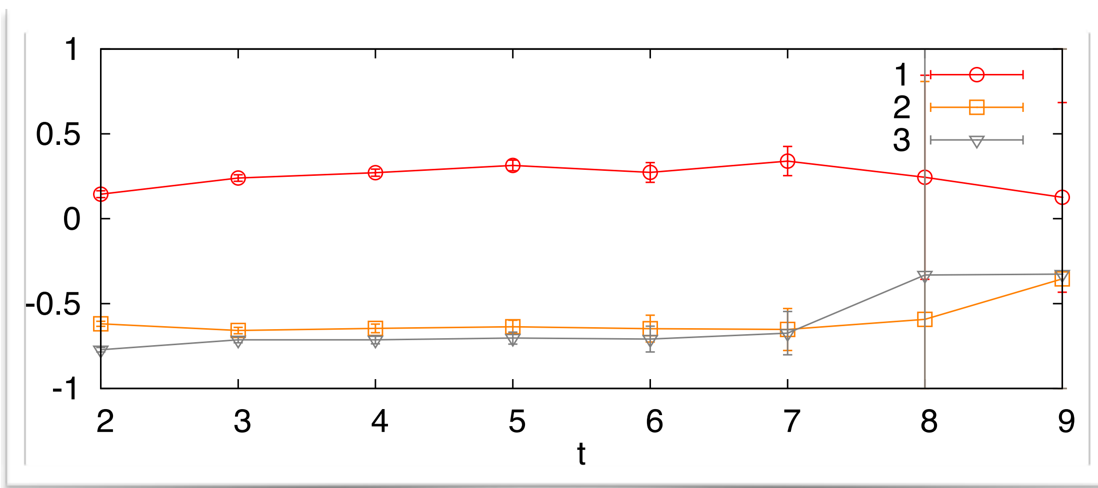
Correlators (eigenvalues) of all states



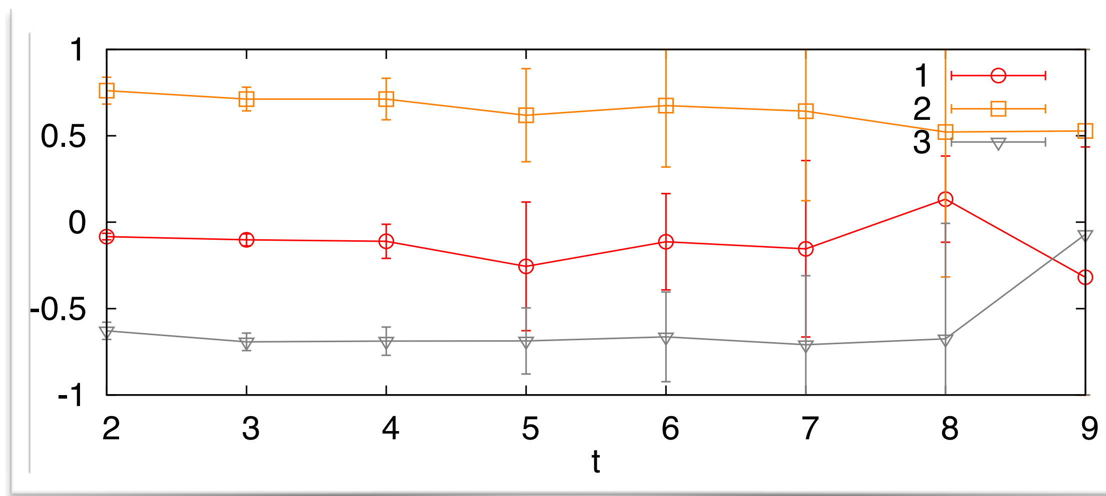
Effective masses of lowest two states



Eigenvectors of ground state

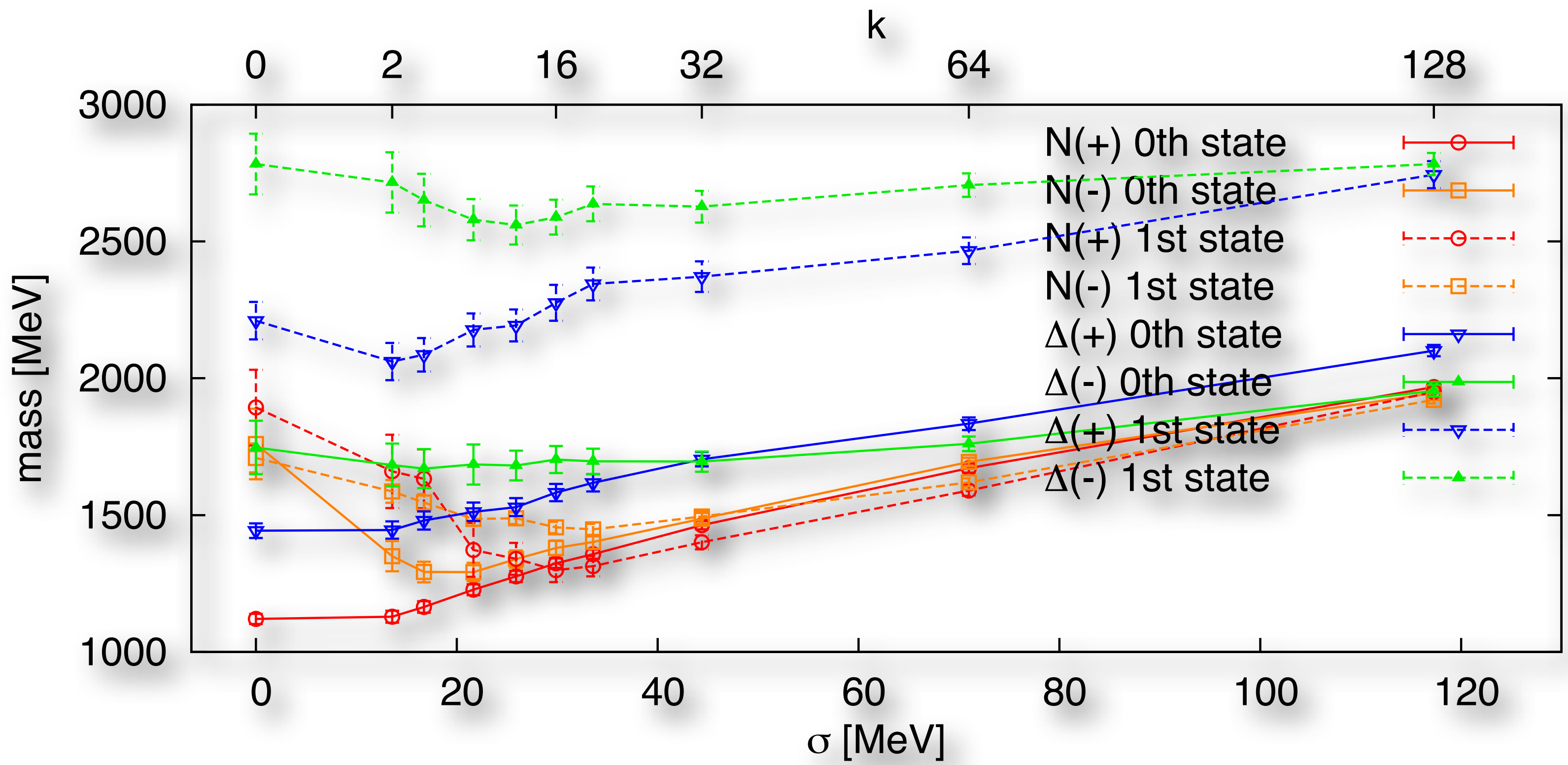


Eigenvectors of first excited state



[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

Baryon mass evolution

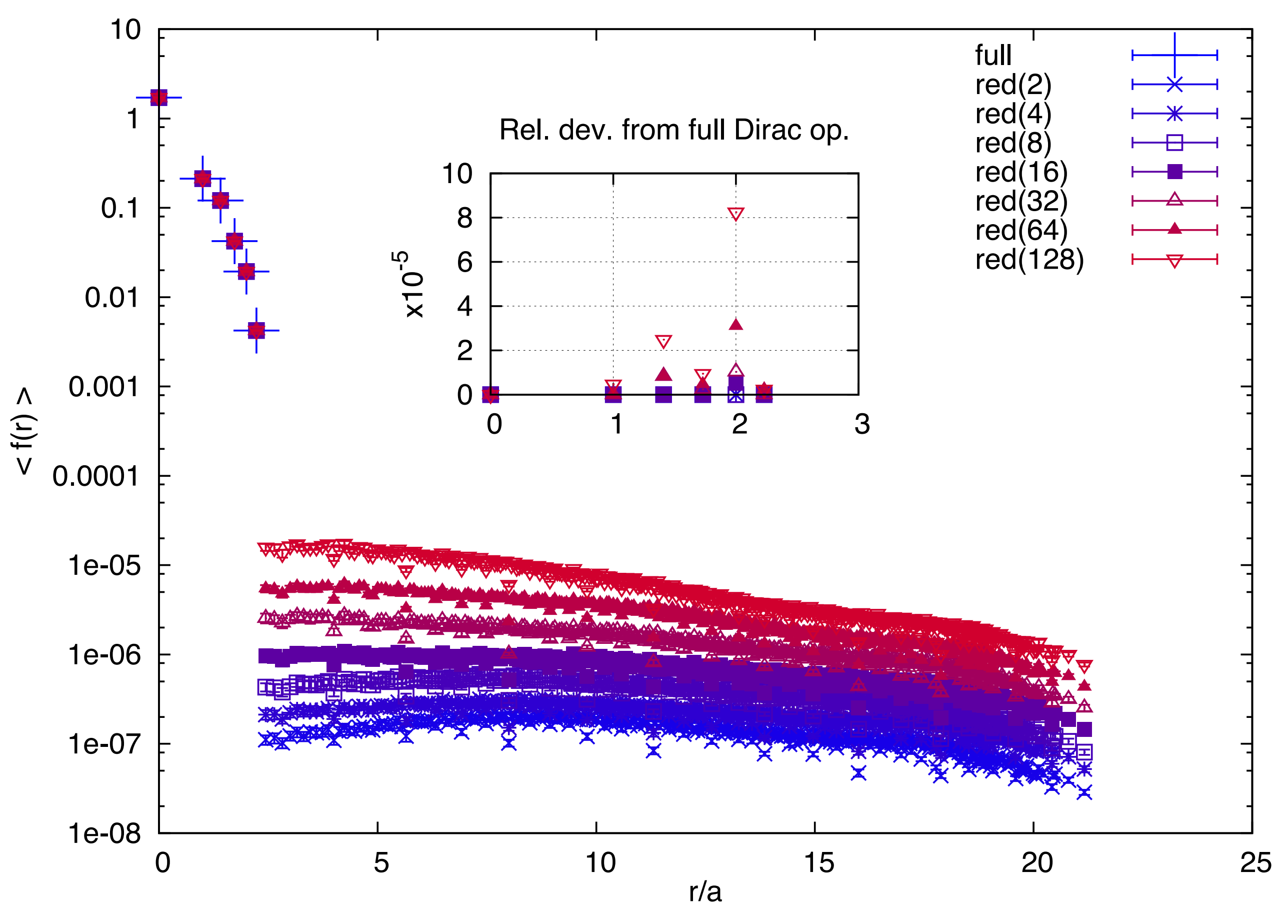


[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

- parity doubling in the $J = 1/2$ and $J = 3/2$ channels
- degeneracy of nucleon ground and exited states
- splitting of Delta ground vs. excited states remains

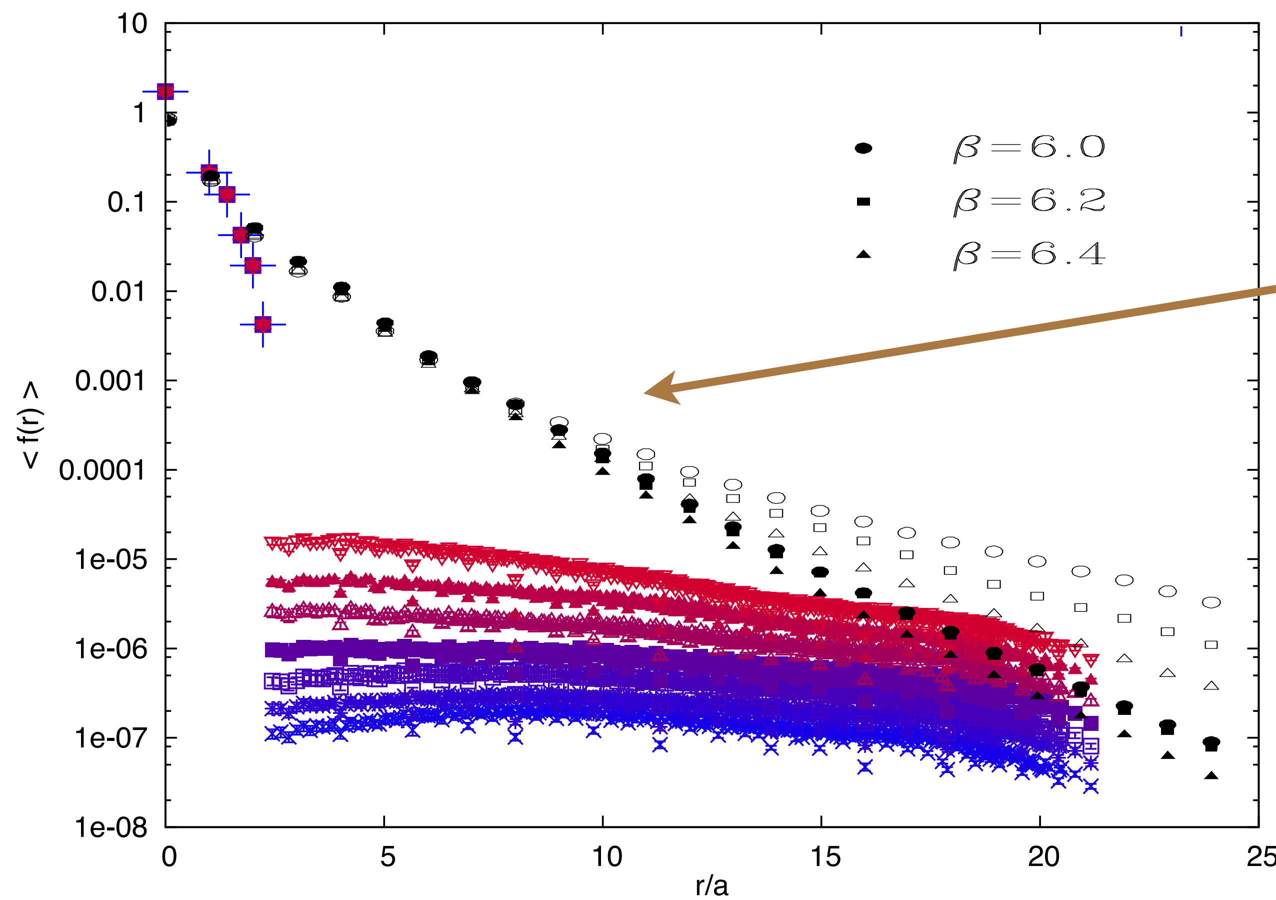
Locality properties

- it is not a priori clear to what extent the locality of the Dirac operator is violated by the low-mode truncation
- the evolution of the contributions of a column of the Dirac matrix serves as a measure of locality



Locality properties

- it is not a priori clear to what extent the locality of the Dirac operator is violated by the low-mode truncation
- the evolution of the contributions of a column of the Dirac matrix serves as a measure of locality



(non)locality of the overlap operator

[Hernandez et al., Nucl. Phys. B **552** (1999) 363–378]

Summary

We removed the lowest lying Dirac eigenmodes of the valence quark sector and found the following effects thereupon

- on the quarks:
  - vanishing of the dynamically generated mass
  - no effect on the bare quark mass
  - increasing of the quark momenta

Summary

We removed the lowest lying Dirac eigenmodes of the valence quark sector and found the following effects thereupon

- on the hadron spectrum:
  - persistence of confinement
  - matching of chiral partners
  - no restoration of $\mathrm{U}(1)_A$

Summary

We removed the lowest lying Dirac eigenmodes of the valence quark sector and found the following effects thereupon

- on the hadron mass:
  - no significant drop of the hadron masses after chiral restoration (except some excited states)
  - hadron mass increases with the truncation level, due to the increased quark momenta

Summary

We removed the lowest lying Dirac eigenmodes of the valence quark sector and found the following effects thereupon

- on the interrelation of DχSB and confinement: