Effects of the low lying Dirac modes on baryons and the quark propagator

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In collaboration with L.Ya. Glozman

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Outline

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Why are the lowest Dirac eigenmodes interesting?

The Banks-Casher relation

$$\langle \overline{\psi}\psi \rangle = -\pi\rho(0)$$

directly relates the density of the Dirac modes near the origin $\rho(0)$ to the chiral condensate.

Reducing quark propagators

- We construct *reduced* quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., [DeGrand, Phys. Rev. D 69, 2004]).
- Split the quark propagator S into a low mode (lm) part and a *reduced* (red) part

$$
\begin{aligned}
S &= \sum_{i\leq k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 + \sum_{i>k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 \\
&= S_{\mathrm{lm}(k)} + S_{\mathrm{red}(k)}
\end{aligned}
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$$= S_{\mathrm{lm}(k)} + S_{\mathrm{red}(k)}$$

- In this work we investigate the *reduced* (red) part of the propagator

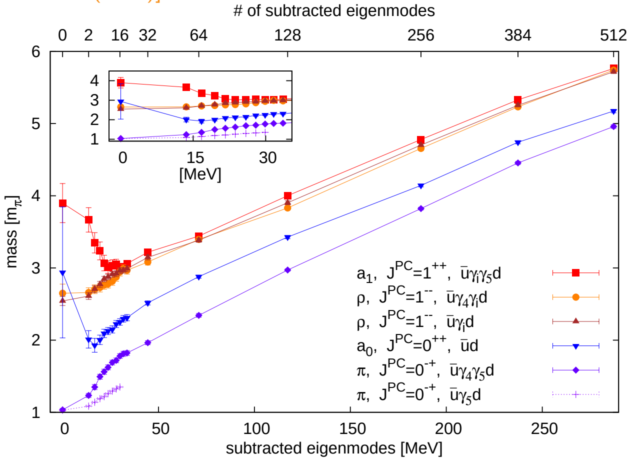
$$S_{\mathrm{red}(k)} = S - S_{\mathrm{lm}(k)}$$

The Setup

- 161 configurations [Gattringer, Hagen, Lang, Limmer, Mohler, Schäfer, Phys. Rev. D 79, 2009]
- size $16^3 \times 32$
- two degenerate flavors of light fermions, $m_\pi = 322(5)\,\mathrm{MeV}$
- lattice spacing $a = 0.1440(12)\,\mathrm{fm}$
- Chirally Improved Dirac operator [Gattringer, Phys. Rev. D 63, 2001] (approximate solution of the Ginsparg-Wilson equation)
- Jacobi smeared quark sources

Reminder: Mesons under Dirac low mode reduction

[C.B.Lang, MS, PRD 84 (2011)]



- the degeneracy of the vector and axial vector current from reduction level 16 on indicates restoration of the chiral symmetry (for $J = 1$ states)

Motivation

- in the constituent quark model, the splitting between the Δ and the nucleon is thought to be (at least partly) due to a Goldstone boson exchange interaction

$$H_\chi \propto -\sum_{i<j} V(\mathbf{r}_{ij}) \boldsymbol{\lambda}_i^F \cdot \boldsymbol{\lambda}_j^F \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

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- what happens to the splitting when we restore the chiral symmetry?
- do the masses of the nucleon and the $N(1535)$ meet?
- how does the constituent quark mass (as seen from the mass function in the quark propagator) change when when we remove the lowest Dirac modes?

Correlators in Lattice QCD

Consider the correlator of an arbitrary operator, e.g., a meson $O(n) = \overline{\psi}_d(n)\Gamma\psi_u(n)$:

$$\left\langle O(n)\overline{O}(m)\right\rangle = \frac{1}{Z}\int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\; O(n)\overline{O}(m)e^{-S_G[U]-S_F[\psi,\overline{\psi},U]}$$

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Fourier-transform and project to zero momentum

$$C(t) \equiv \left\langle \tilde{O}(t)\overline{O}(0)\right\rangle = A_0\,\mathrm{e}^{-tE_0} + \ldots$$

Outline

Motivation and introduction

Baryons

Quark propagator

Conclusions

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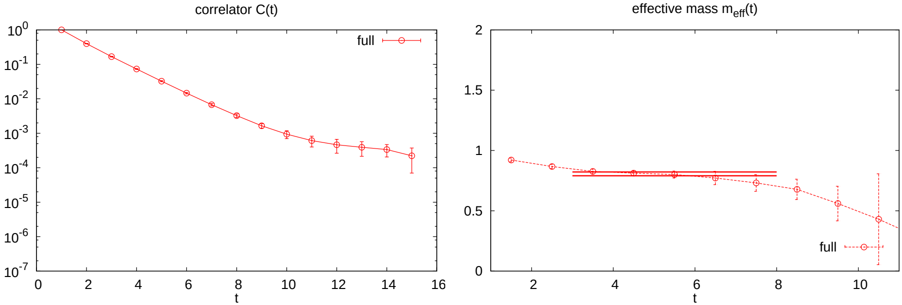

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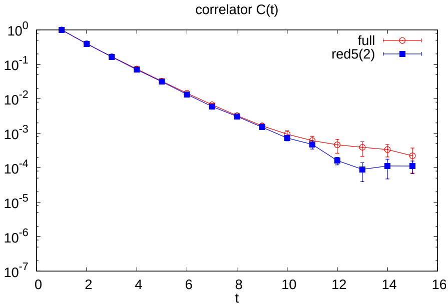
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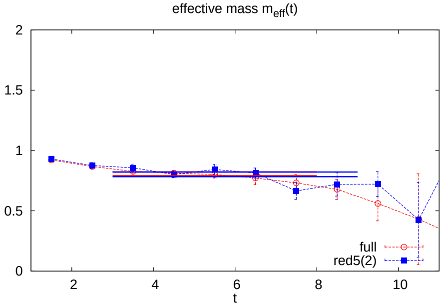
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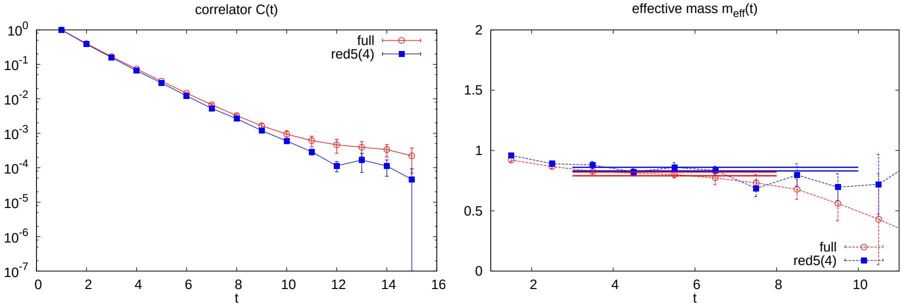
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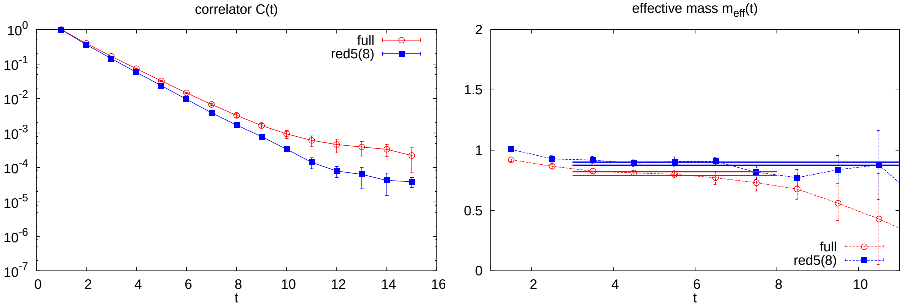
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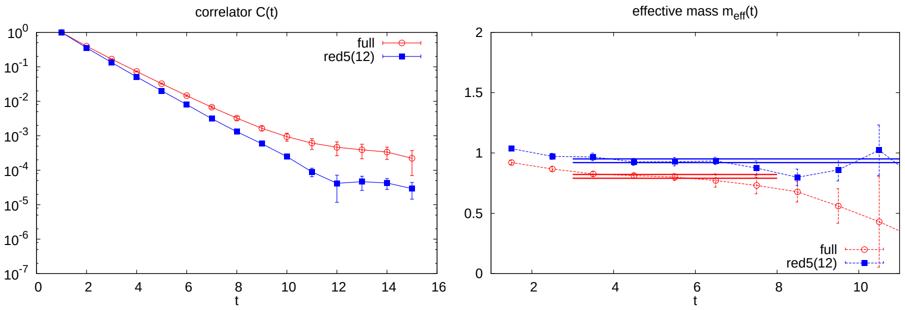
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effective mass m_eff(t)
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2 4 6 8 10
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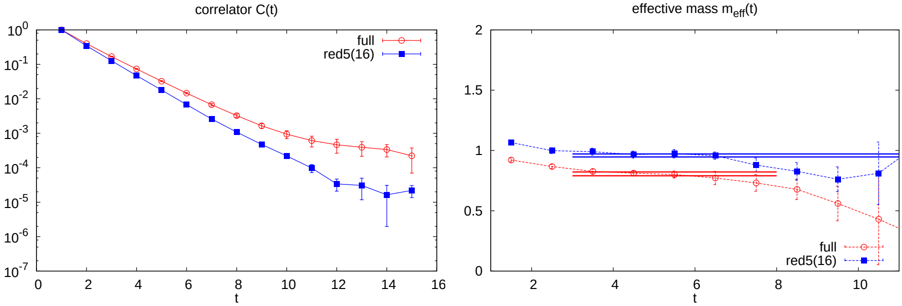
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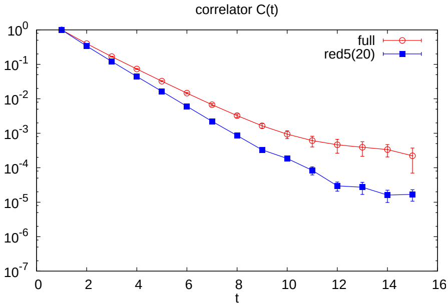
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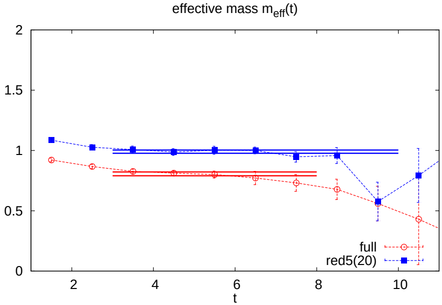
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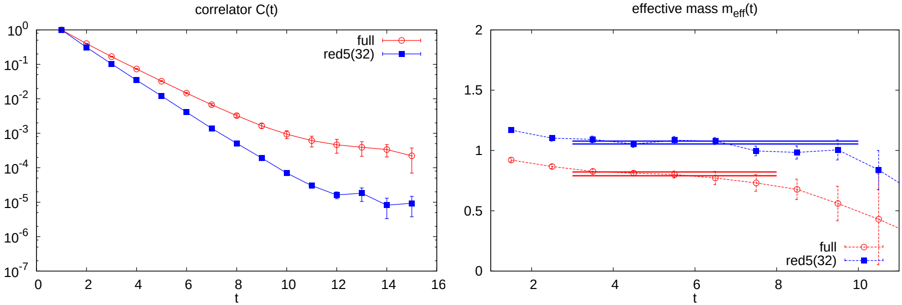
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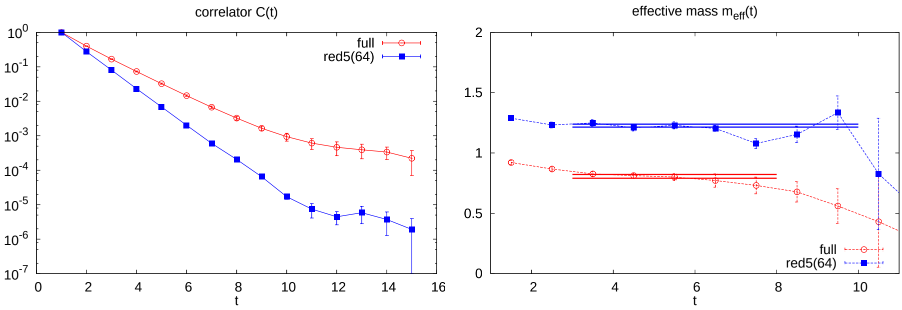
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effective mass $m_{eff}(t)$
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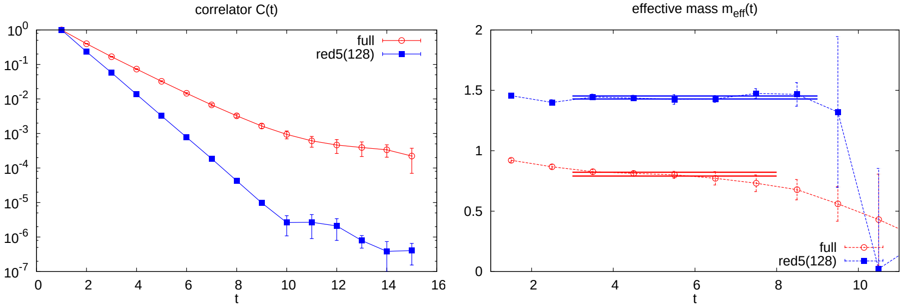
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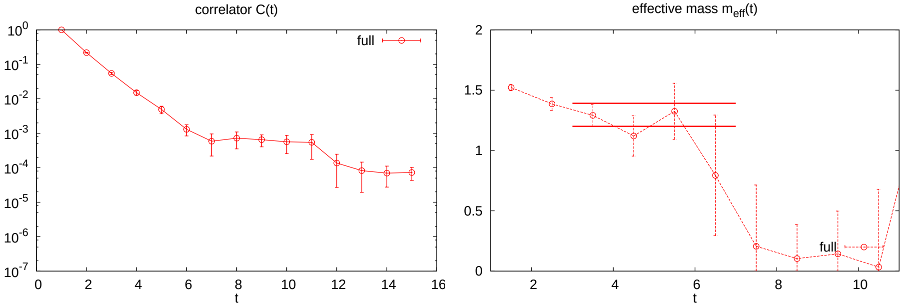

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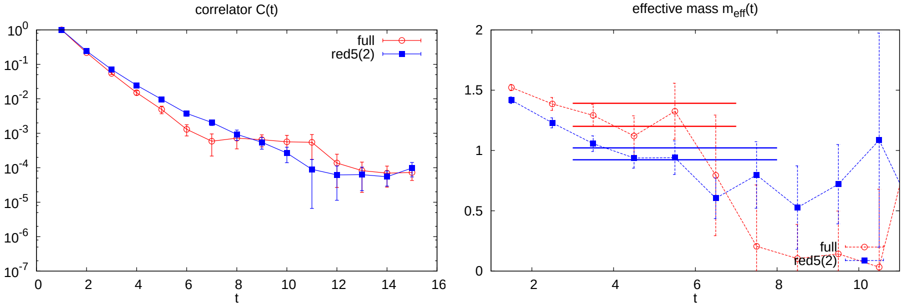
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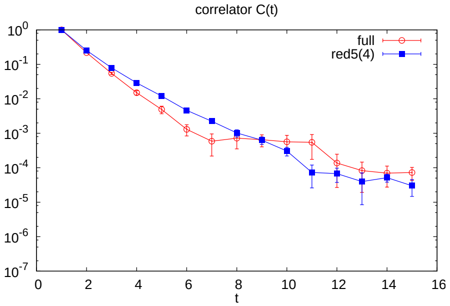
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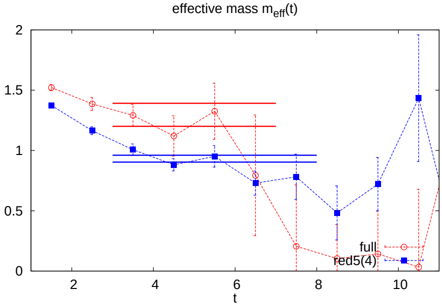
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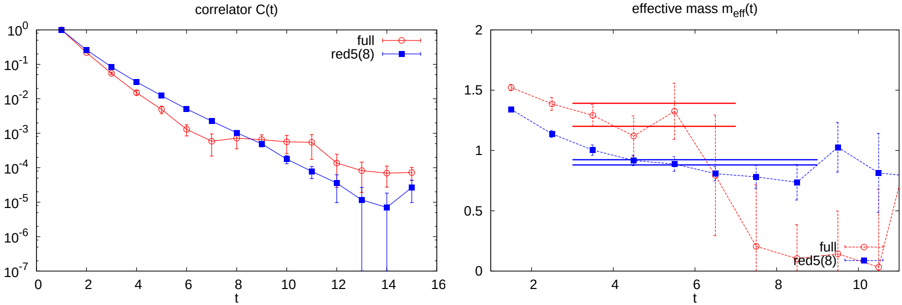
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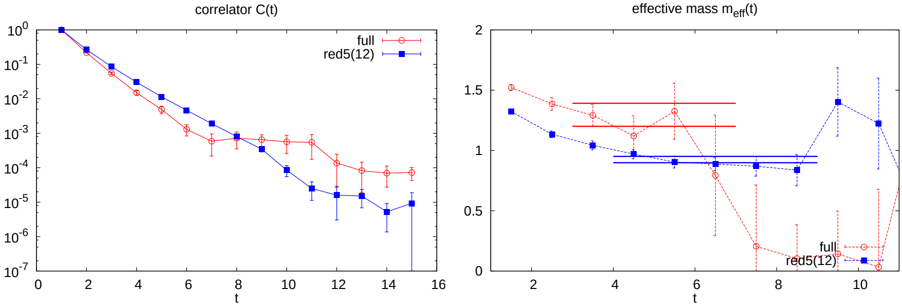
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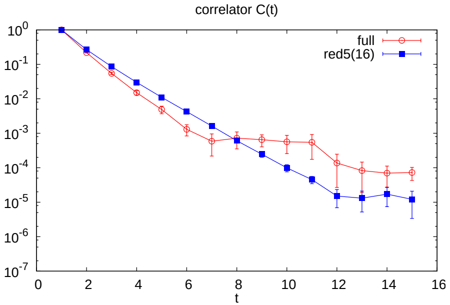
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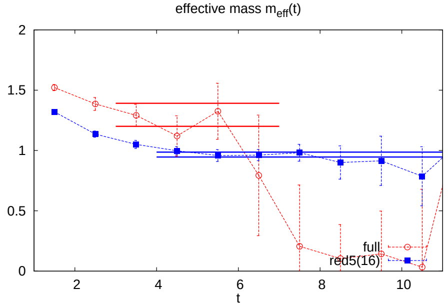
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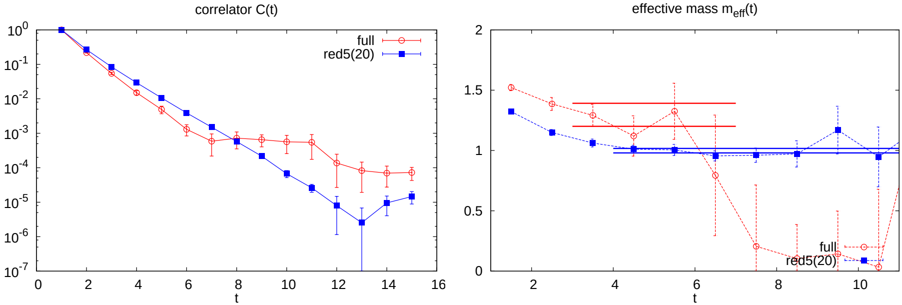
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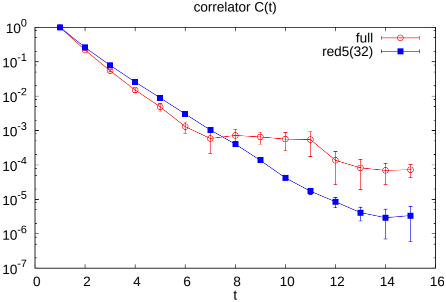
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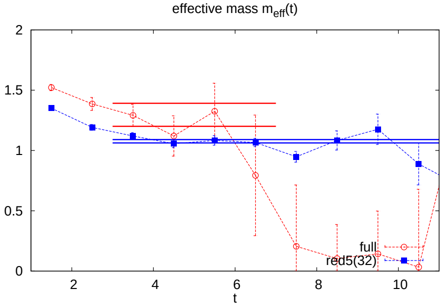
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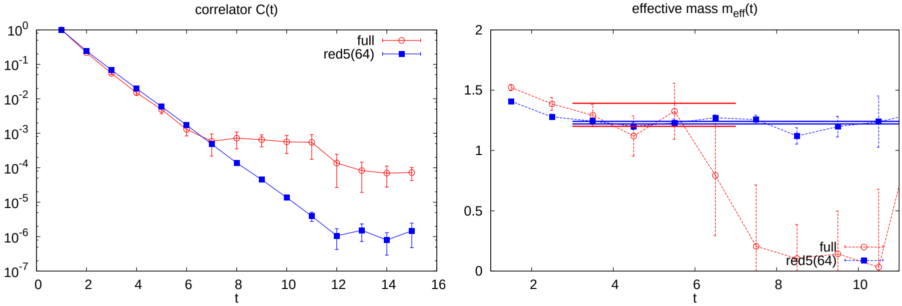
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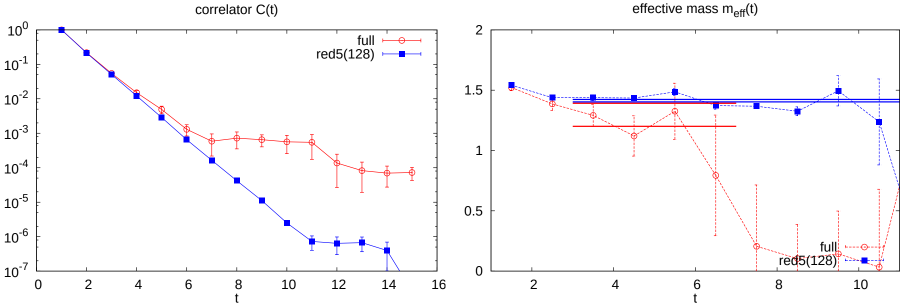
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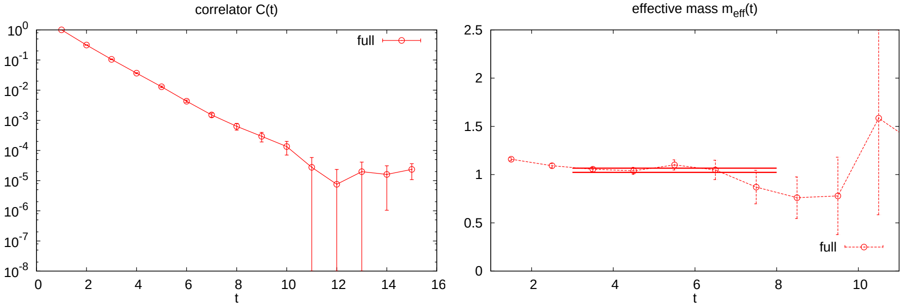

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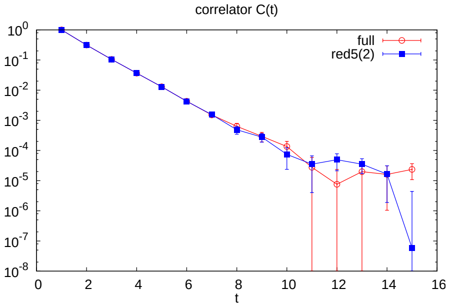
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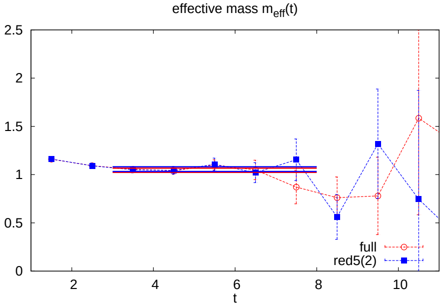
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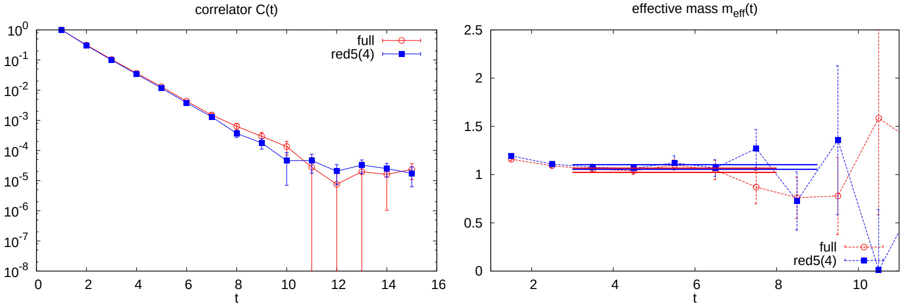
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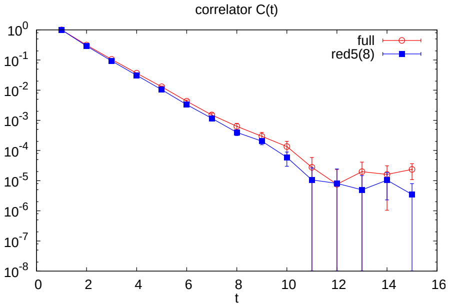
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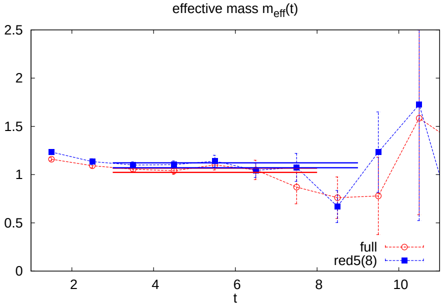
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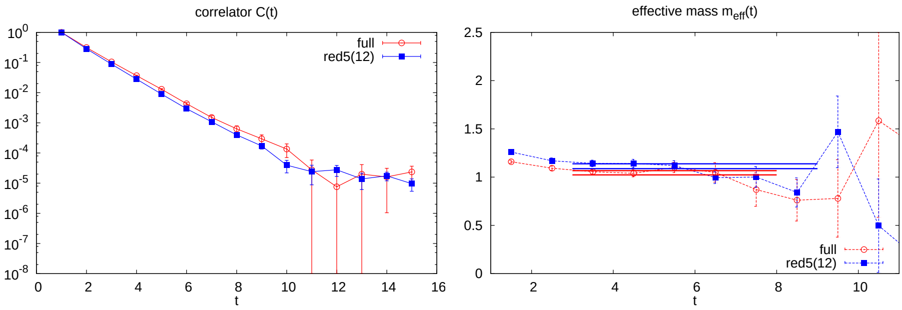
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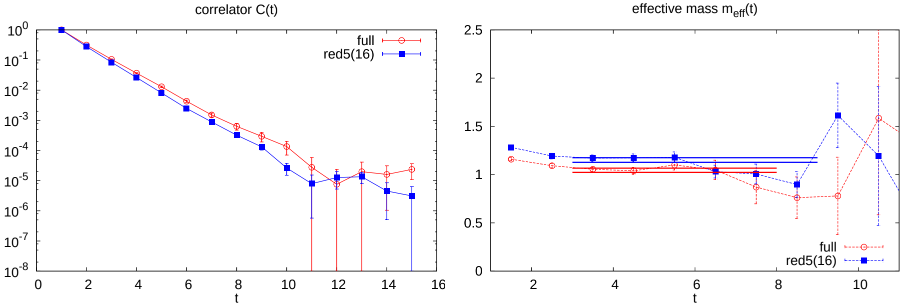
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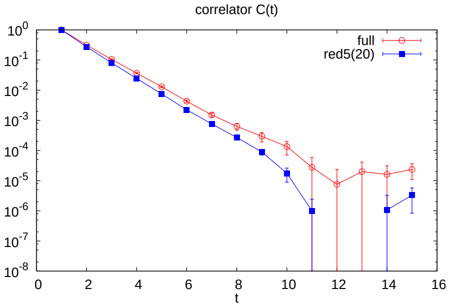
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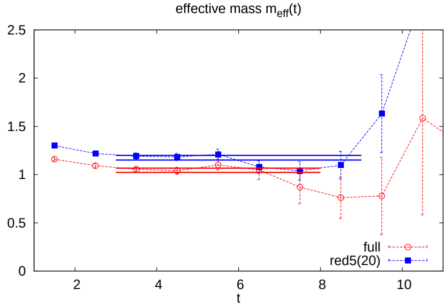
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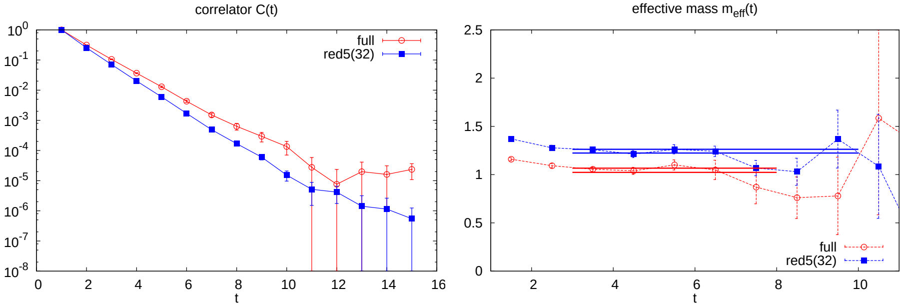
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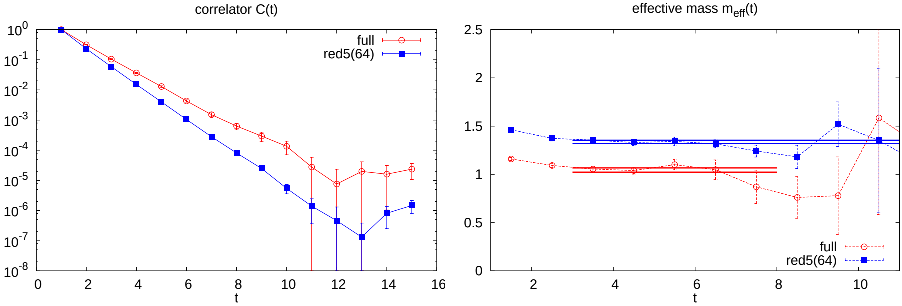
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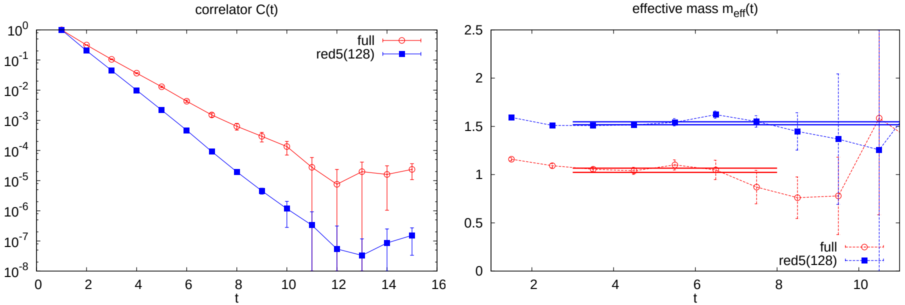
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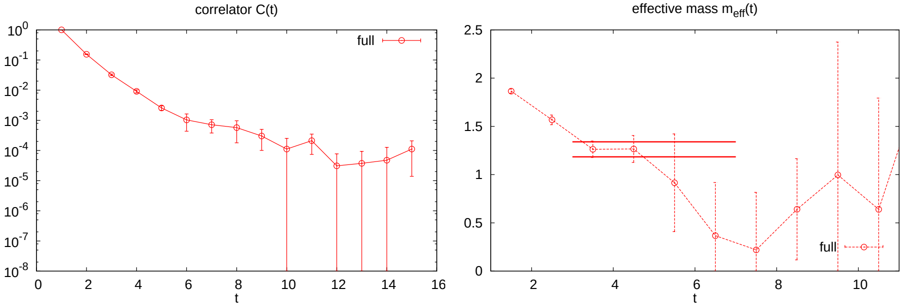

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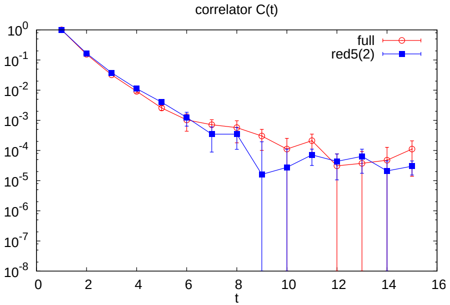
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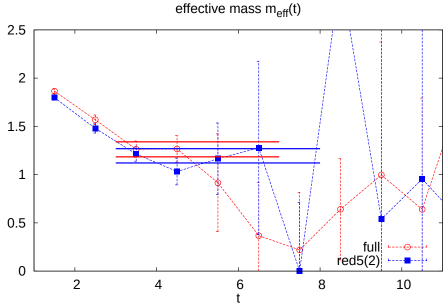
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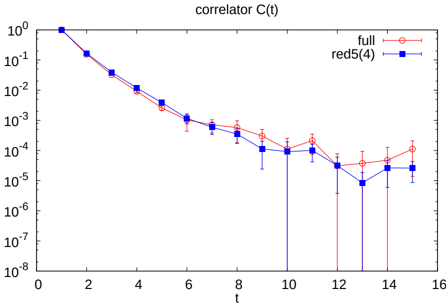
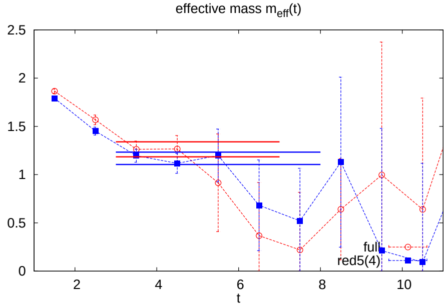
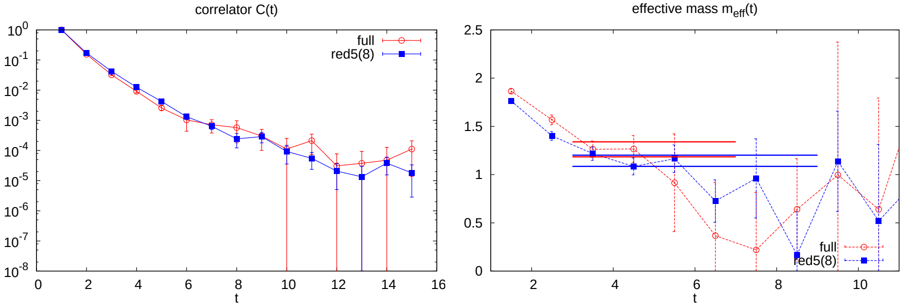
t

$\Delta(-)$

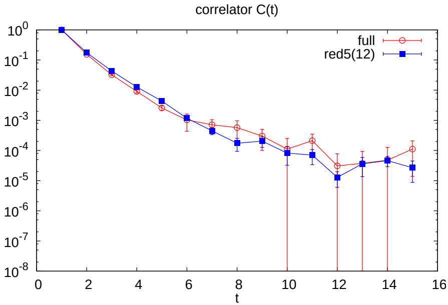

correlator C(t)
full
red5(4)
t

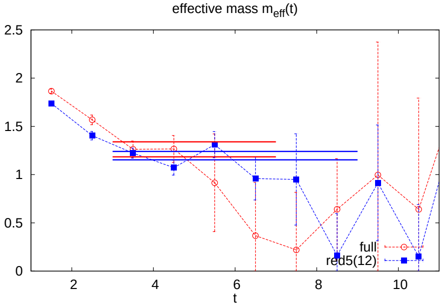

effective mass $m_{eff}(t)$
full
red5(4)
t

$\Delta(-)$

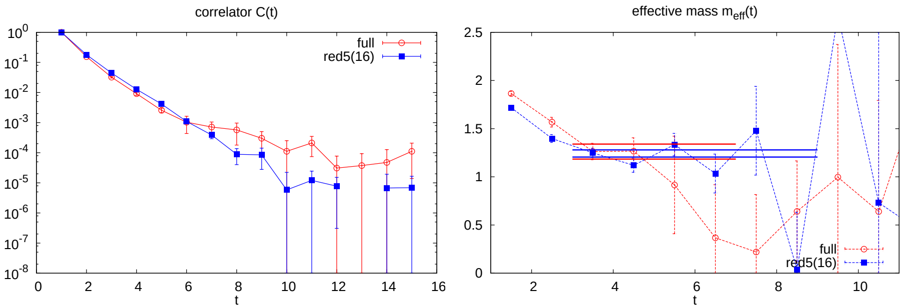

correlator C(t)
full
red5(8)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
2
4
6
8
10
12
14
16
t
effective mass $m_{eff}(t)$
2.5
2
1.5
1
0.5
0
2
4
6
8
10
t
full
red5(8)

$\Delta(-)$

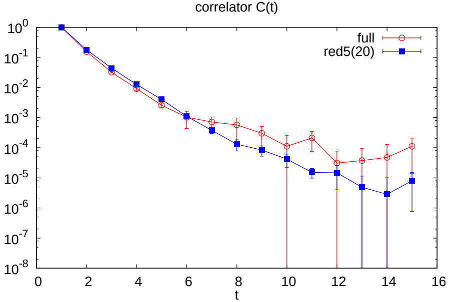

correlator C(t)
full
red5(12)
t

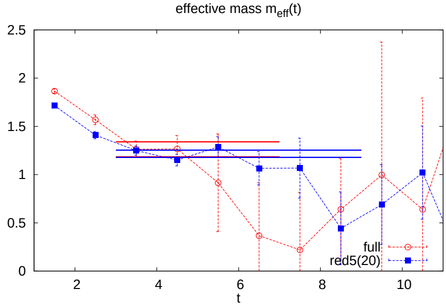

effective mass m_eff(t)
full
red5(12)
t

$\Delta(-)$

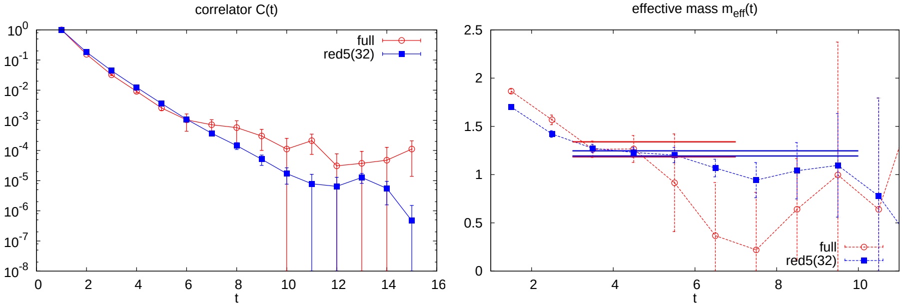

correlator C(t)
full
red5(16)
t
effective mass $m_{eff}(t)$
full
red5(16)
t

$\Delta(-)$

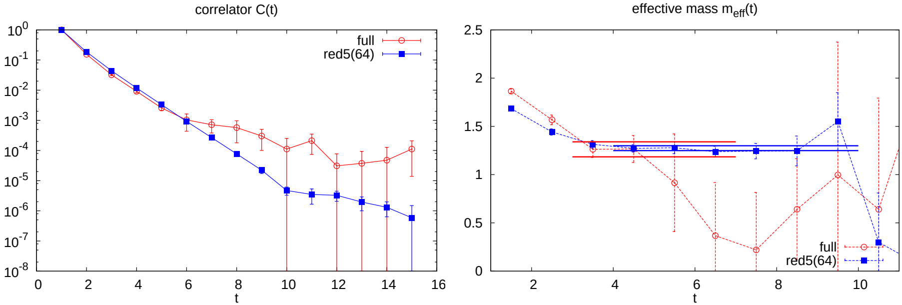

correlator C(t)
full
red5(20)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
2
4
6
8
10
12
14
16
t


effective mass $m_{eff}(t)$
full
red5(20)
2.5
2
1.5
1
0.5
0
2
4
6
8
10
t

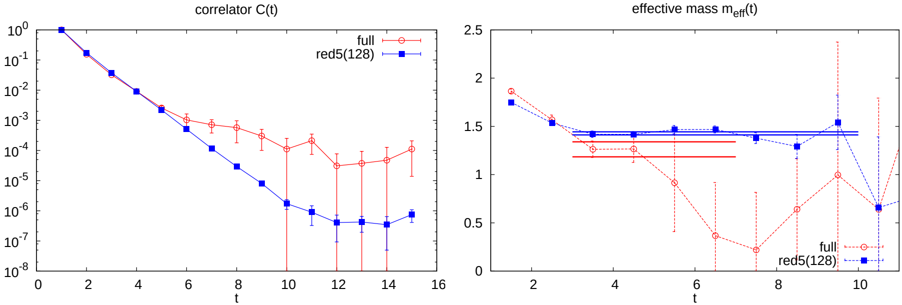
$\Delta(-)$


correlator C(t)
full
red5(32)
t
effective mass $m_{eff}(t)$
full
red5(32)
t

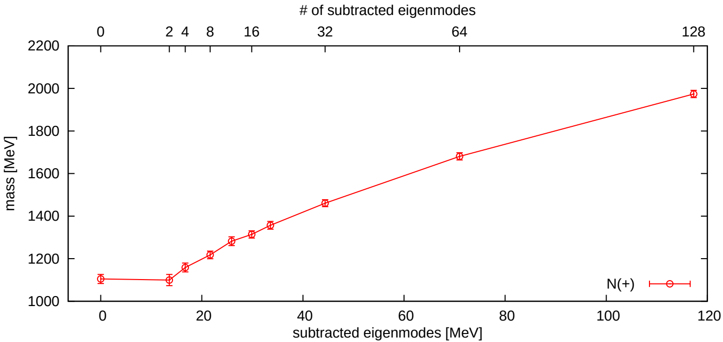
$\Delta(-)$


correlator C(t)
full
red5(64)
t
effective mass m_eff(t)
full
red5(64)
t

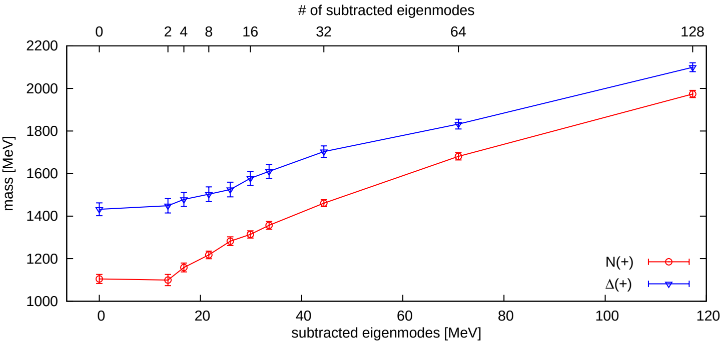
$\Delta(-)$


correlator C(t)
full
red5(128)
t
effective mass $m_{eff}(t)$
full
red5(128)
t

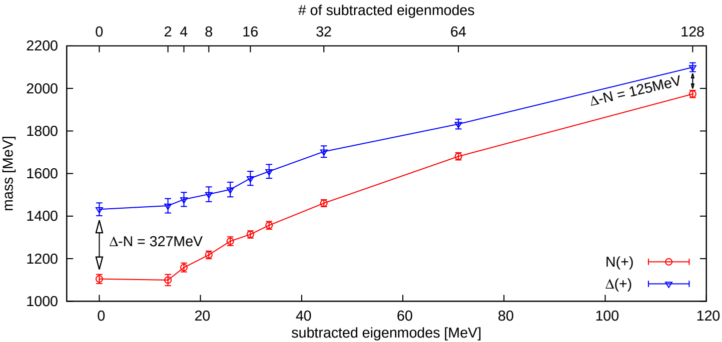
N and Δ vs. eigenmode reduction level



N and Δ vs. eigenmode reduction level

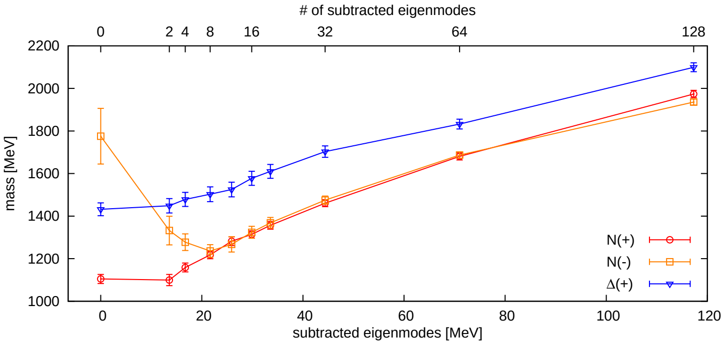


N and Δ vs. eigenmode reduction level

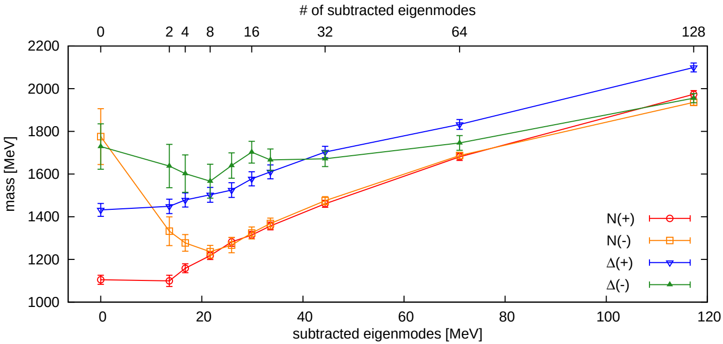


the $\Delta - N$ splitting gets reduced to $\approx 40\%$

N and Δ vs. eigenmode reduction level



N and Δ vs. eigenmode reduction level



Outline

The lattice quark propagator

The tree-level quark propagator is

$$S_0(p) = \frac{1}{i\not{p} + m_0}$$

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the renormalized quark propagator

$$S(\mu; p) = \frac{1}{i\not{p} A(\mu; p^2) + B(\mu; p^2)} = \frac{Z(\mu; p^2)}{i\not{p} + M(p^2)}.$$

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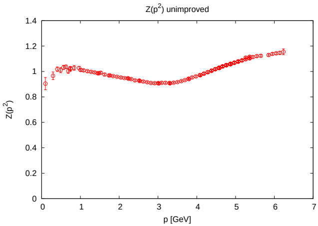
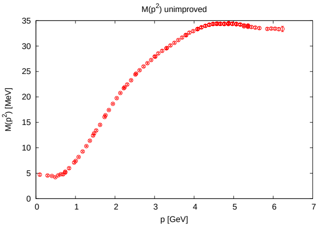
We calculate $S_{\mathrm{bare}}(a;p)$ in Landau gauge on the lattice and therefrom extract

- the renormalization function $Z(\mu;p^2)$
- the renormalization point independent mass function $M(p^2)$

Improving the lattice quark propagator





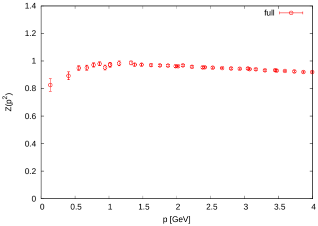
To improve the functions we perform

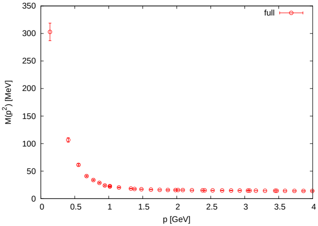
- tree-level improvement to reduce $\mathcal{O}(a)$ errors of off-shell quantities [Skullerud, Williams (2001) hep-lat/0007028, Heatlie et al. (1991)]

$$S_I(x,y) = \langle S_I(x,y;U)\rangle \equiv \langle (1+b_q am)S(x,y;U) - a\lambda\delta(x-y)\rangle$$

- tree-level correction: $Z \to Z/Z_0$ and $M \to M/(M_0 + m_{\mathrm{ren}})$
- a data cut at $4\,\mathrm{GeV}$ since for larger momenta lattice artifacts dominate

Improving the lattice quark propagator





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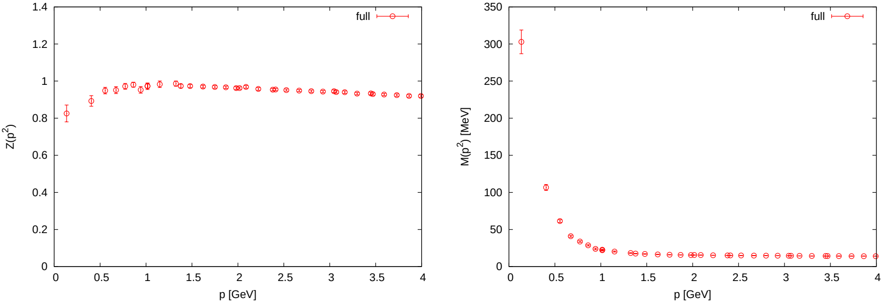
The quark propagator under eigenmode reduction



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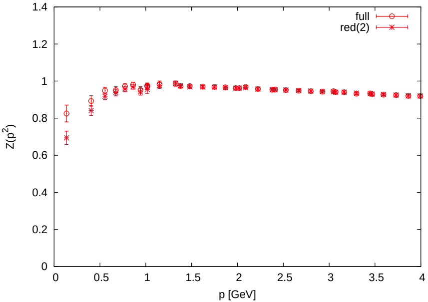

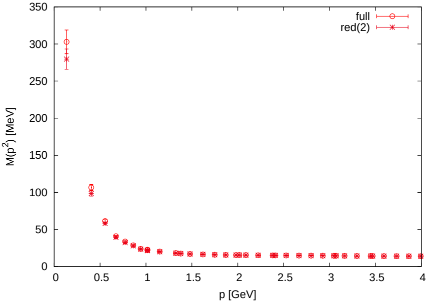

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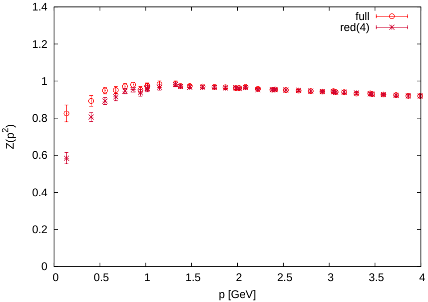

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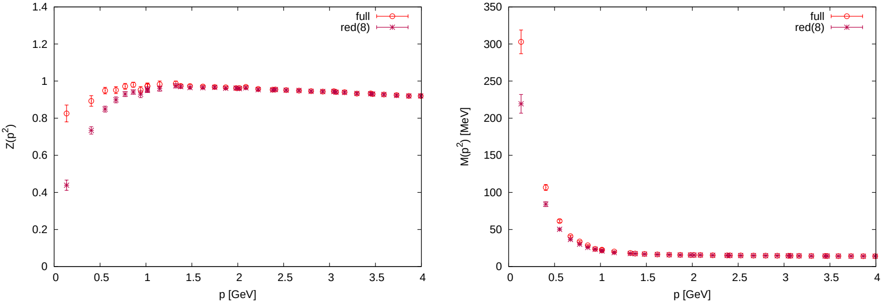

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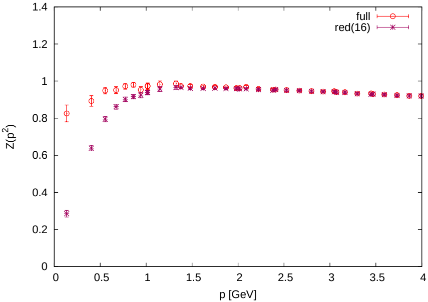

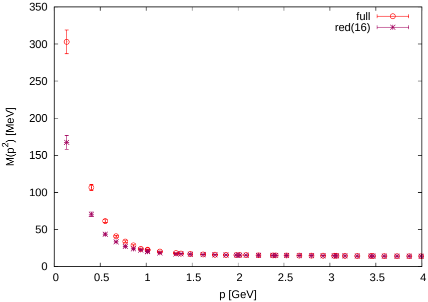

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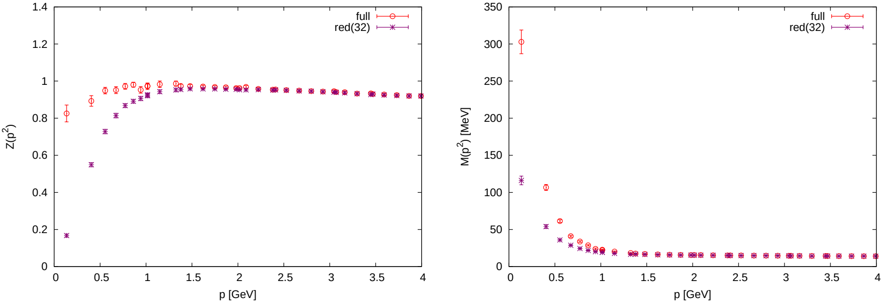

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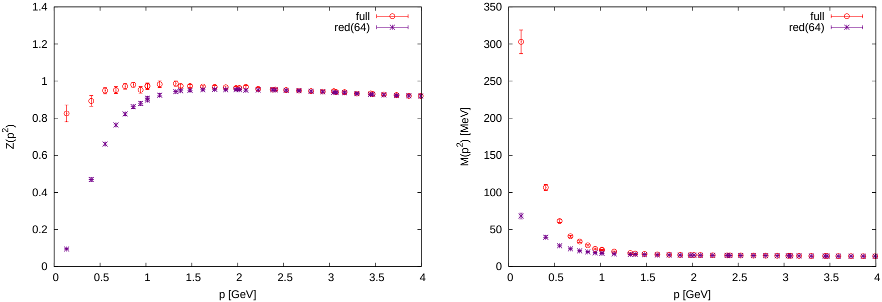

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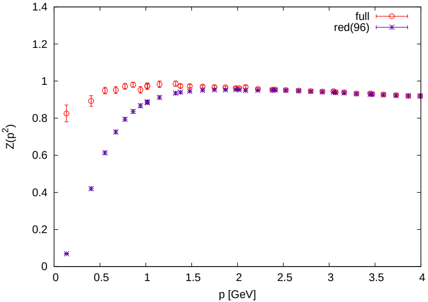

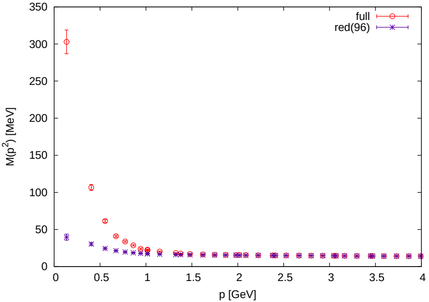

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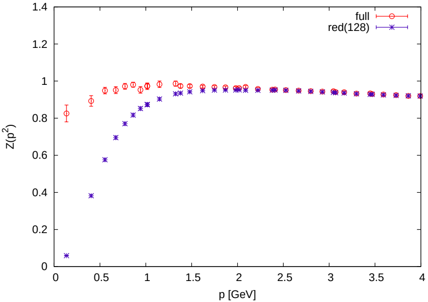

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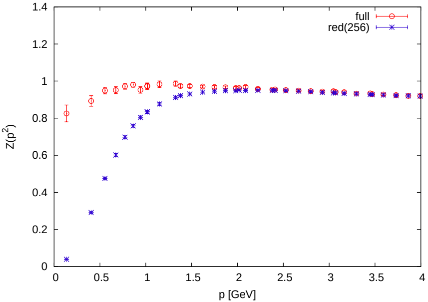

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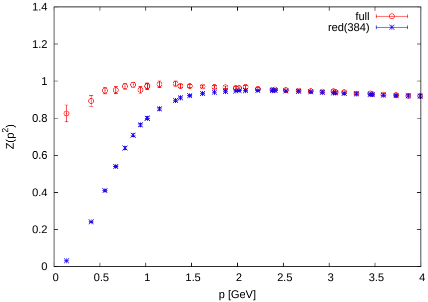

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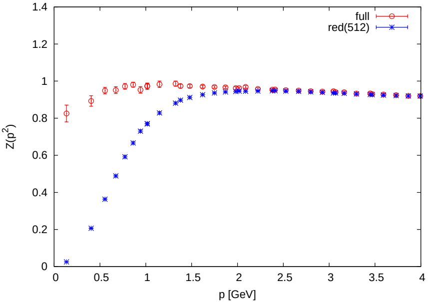
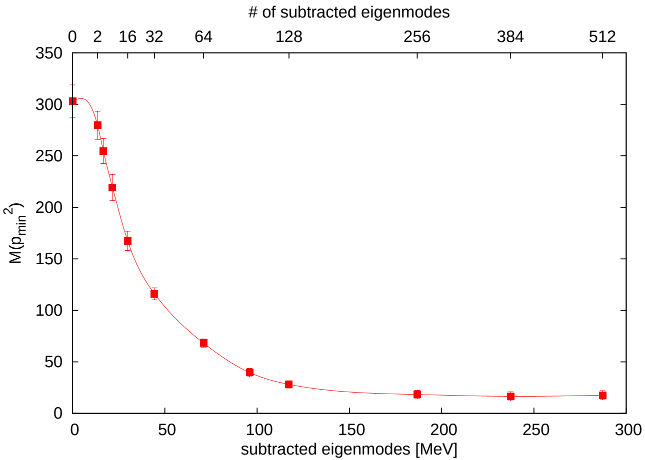


“Constituent quark mass” vs. eigenmode reduction level



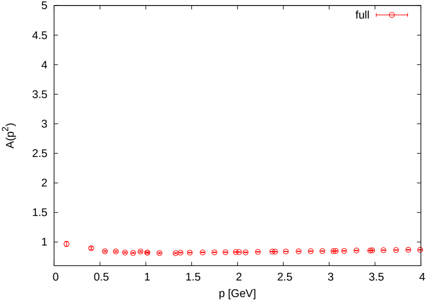
Outline

Conclusions

We showed that when removing an increasing number of the lowest Dirac eigenmodes

- the $\Delta - N$ splitting decreases to $\approx 40\%$
- the nucleon and the $N(1535)$ become degenerate
- the dynamical mass generation of quarks as seem from the IR behavior of $M(p^2)$ decreases towards zero, although it is still at roughly 55% when removing 16 eigenmodes of D_5 which is where we found chiral symmetry to be restored
- the quark wave-function renormalization function $Z(p^2)$ becomes strongly suppressed in the IR
- the quark propagator above $\sim 2.5\,\mathrm{GeV}$ is not affected.

$A(p^2)$ and $B(p^2)$ of the quark propagator under eigenmode reduction



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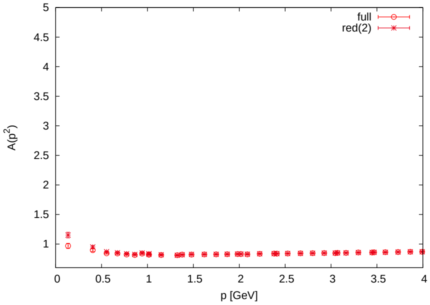
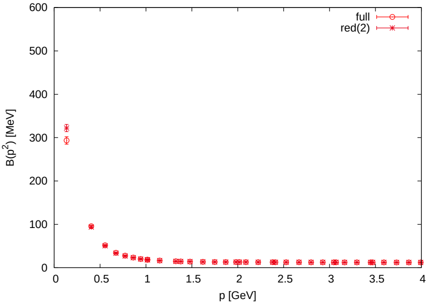
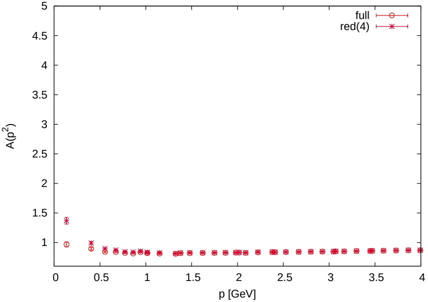
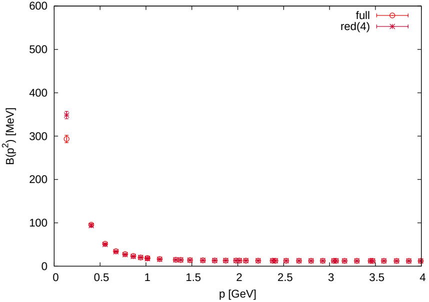




$A(p^2)$ and $B(p^2)$ of the quark propagator under eigenmode reduction





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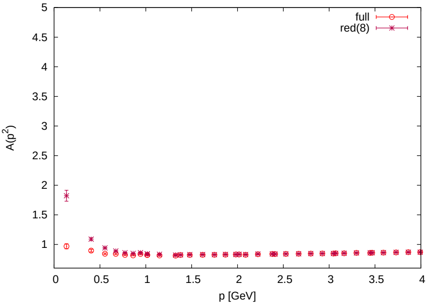

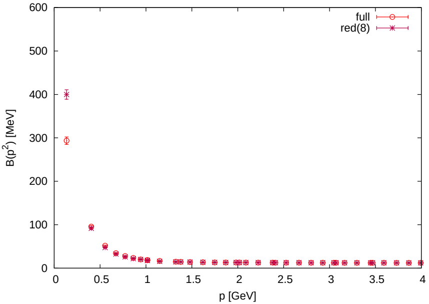

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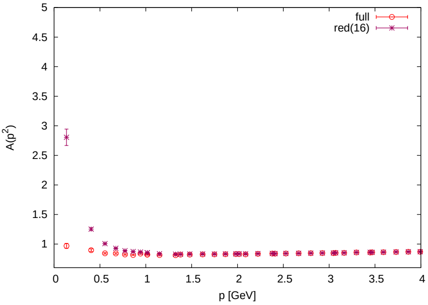

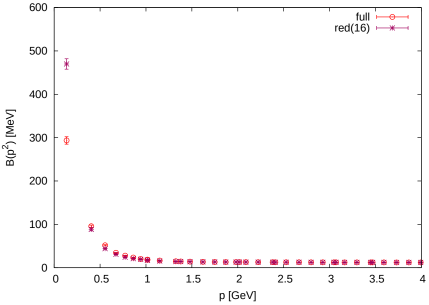

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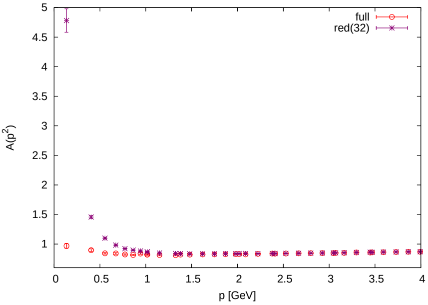

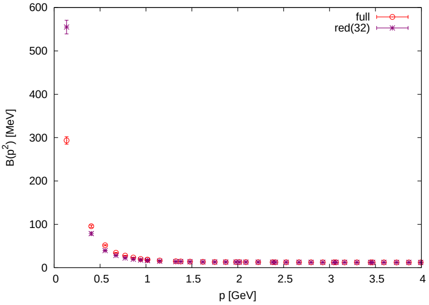

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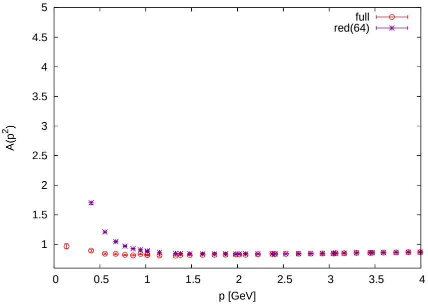

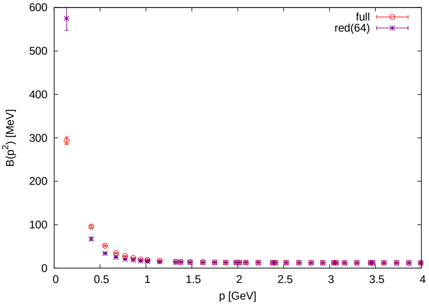

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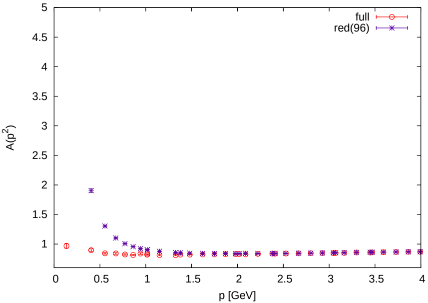

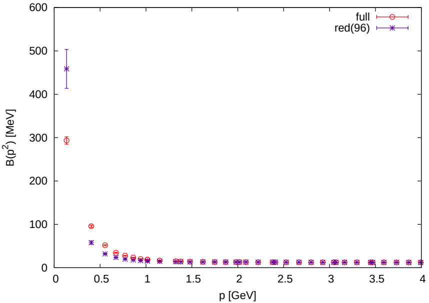

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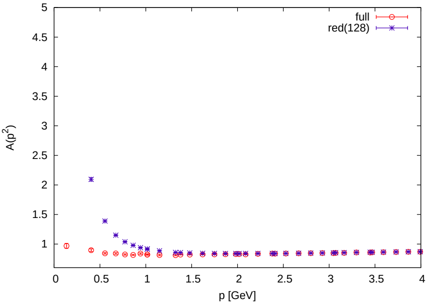

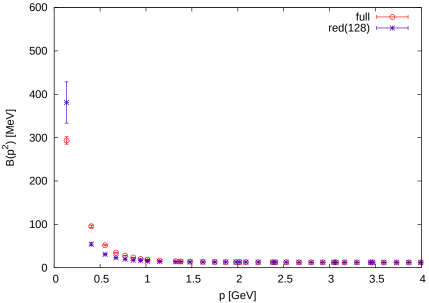

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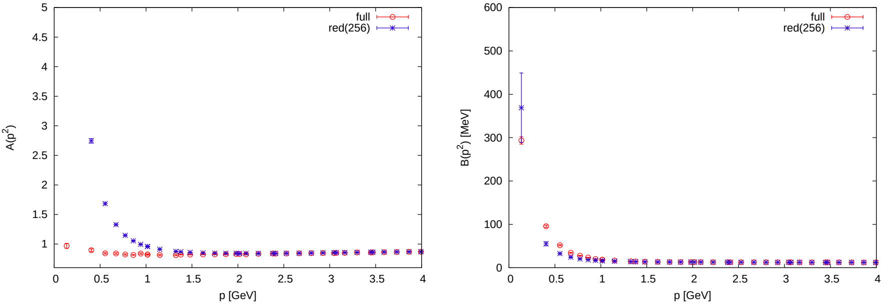

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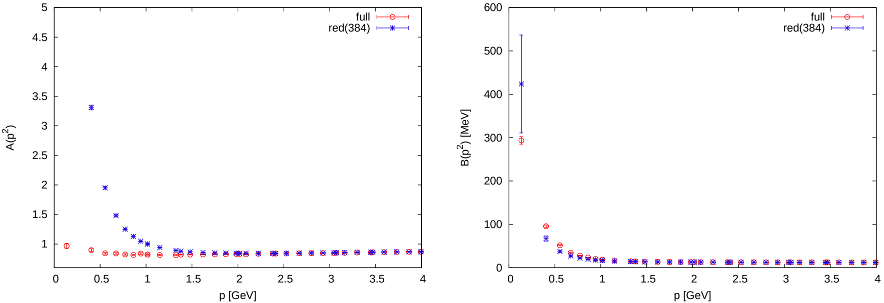

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