Symmetries of hadrons after unbreaking the chiral symmetry

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Outline

Key questions to QCD

- How is the hadron mass generated in the light quark sector?
- How important is chiral symmetry breaking for the hadron mass?
- Are confinement and chiral symmetry breaking directly interrelated?
- Is there parity doubling and does chiral symmetry get effectively restored in high-lying hadrons?
- Is there some other symmetry?

The Banks–Casher relation

The lowest eigenmodes of the Dirac operator are related to the quark condensate of the vacuum:

$$\langle \overline{\psi}\psi \rangle = -\pi\rho(0)$$

- $\rho(0)$: density of the lowest quasi-zero eigenmodes of the Dirac operator
- here the sequence of limits is important: $V \to \infty$ then $m_q \to 0$

"Unbreaking" chiral symmetry

- Our goal is to construct hadron correlators out of *reduced* quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., [DeGrand, PRD 69 (2004)]).

- we use the Hermitian Dirac operator $D_5 \equiv \gamma_5 D$ (real eigenvalues)
- split the quark propagator $S \equiv D^{-1}$ into a low mode (lm) part and a *reduced* (red) part

$$
\begin{aligned}
S &= \sum_{i \leq k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 + \sum_{i > k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 \\
&= S_{\mathrm{lm}(k)} + S_{\mathrm{red}(k)}
\end{aligned}
$$

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$$S = \sum_{i \leq k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5 + \sum_{i>k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5$$
$$= S_{\mathrm{lm}(k)} + S_{\mathrm{red}(k)}$$

- in this work we investigate the *reduced* (red) part of the propagator

$$S_{\mathrm{red}(k)} = S - S_{\mathrm{lm}(k)}$$

The setup

- 161 configurations [Gattringer et al., PRD 79 (2009)]
- size $16^3 \times 32$
- two degenerate flavors of light fermions, $m_\pi = 322(5)\,\mathrm{MeV}$
- lattice spacing $a = 0.1440(12)\,\mathrm{fm}$
- Chirally Improved (CI) Dirac operator [Gattringer, PRD 63 (2001)] (approximate solution of the Ginsparg-Wilson equation)
- three different kinds of quark sources: Jacobi smeared narrow (0.27 fm) and wide (0.55 fm) sources and a P wave like derivative source → serves a large operator basis for the variational method.

The lattice quark propagator

The tree-level quark propagator is

$$S_0(p) = \frac{1}{i\not{p} + m_0}$$

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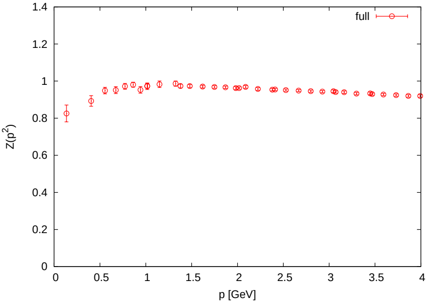
the renormalized quark propagator

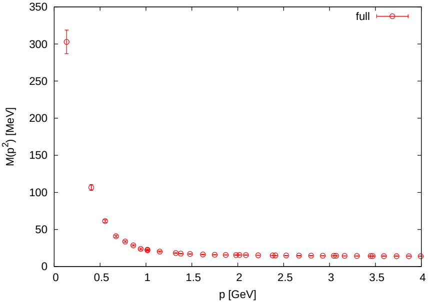
$$S(\mu;p) = \frac{1}{i\not{p} A(\mu;p^2) + B(\mu;p^2)} = \frac{Z(\mu;p^2)}{i\not{p} + M(p^2)}.$$

We calculate $S_{\mathrm{bare}}(a;p)$ in Landau gauge on the lattice and therefrom extract

- the renormalization function $Z(\mu;p^2)$
- the renormalization point independent mass function $M(p^2)$

The quark propagator under eigenmode reduction





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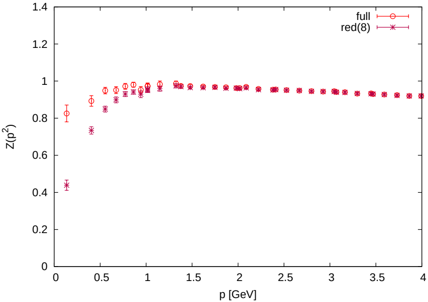

- the dynamically generated mass decreases with the truncation level $\rightarrow$ restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]

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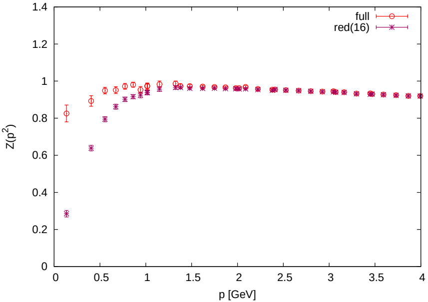

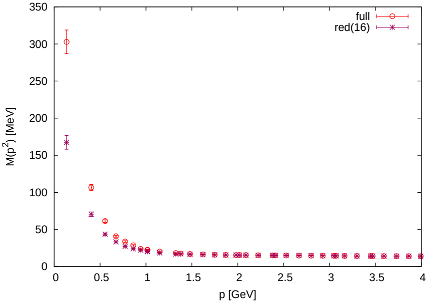

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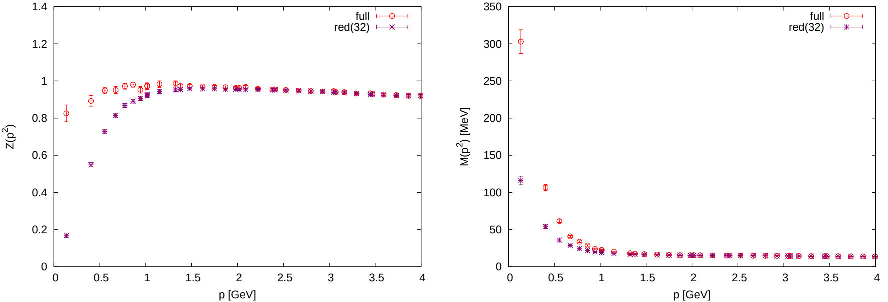

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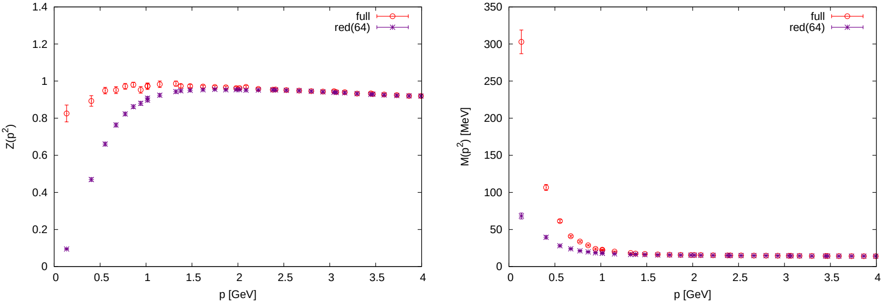

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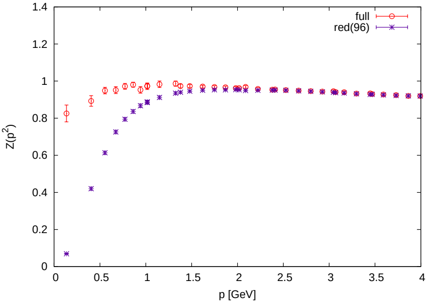

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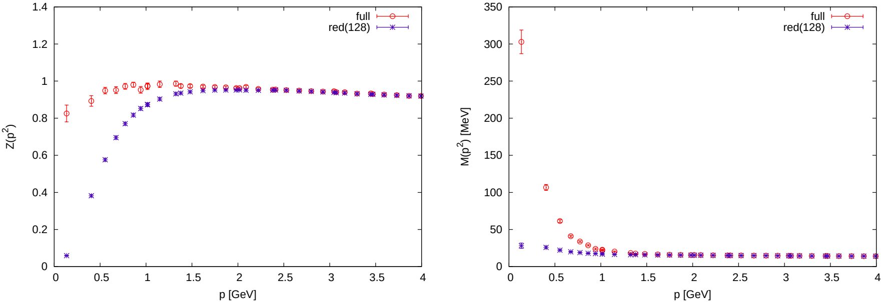

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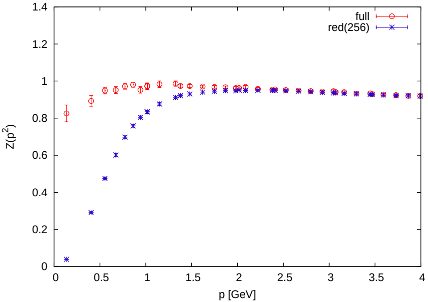

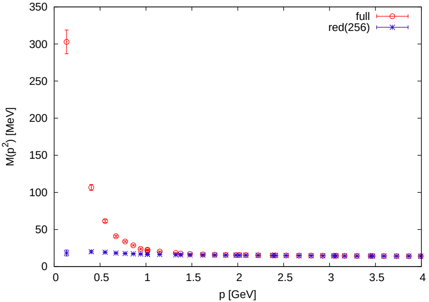

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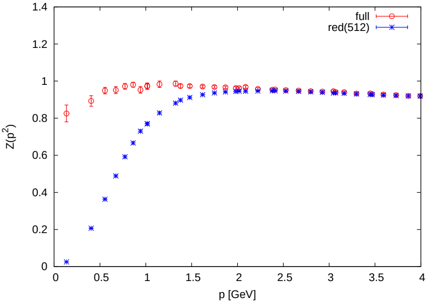

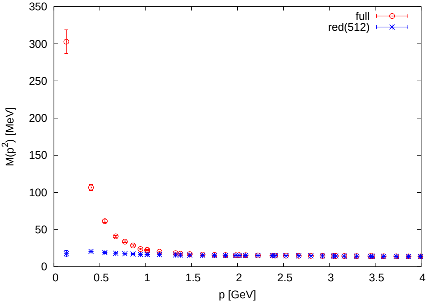

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Reminder: chiral symmetry and its breaking

When neglecting the two lightest quark masses, the QCD Lagrangian becomes invariant under the symmetry group

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

- axial vector part of the $SU(2)_L \times SU(2)_R$ symmetry is broken spontaneously in the vacuum
- vector part is (approximately) preserved
- $U(1)_A$ axial symmetry is not only broken spontaneously but also explicitly (axial anomaly)

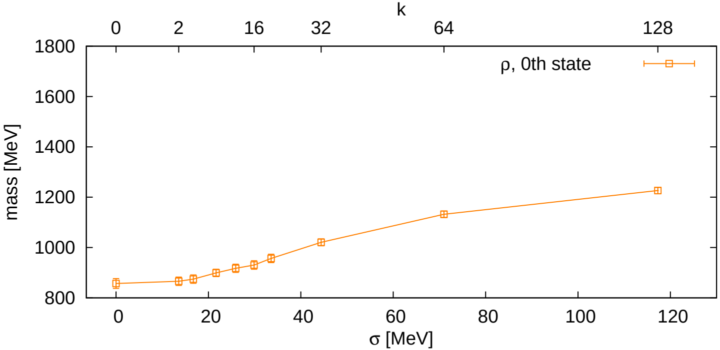
Mesons

We explore the following isovector mesons which, in a chirally symmetric world, would be related via the following symmetries

$SU(2)_L \times SU(2)_R$ (axial)	$U(1)_A$
$\rho \longleftrightarrow a_1$	$\rho \longleftrightarrow b_1$

- can we restore the chiral symmetry and if, what happens to confinement?
- how does the mass of the light mesons change?
- what happens to the $U(1)_A$ axial symmetry?

Meson masses vs. Dirac eigenmode reduction level



- heavy ρ meson: mass not due to dynamical chiral symmetry breaking

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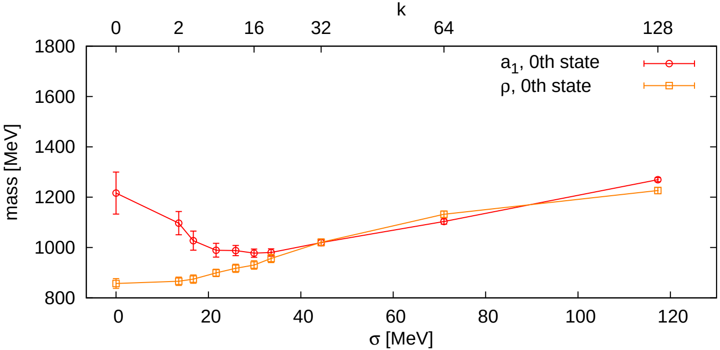

- heavy ρ meson: mass not due to dynamical chiral symmetry breaking
- degeneracy of ρ and a_1: restoration of the $SU(2)_L \times SU(2)_R$ chiral symmetry

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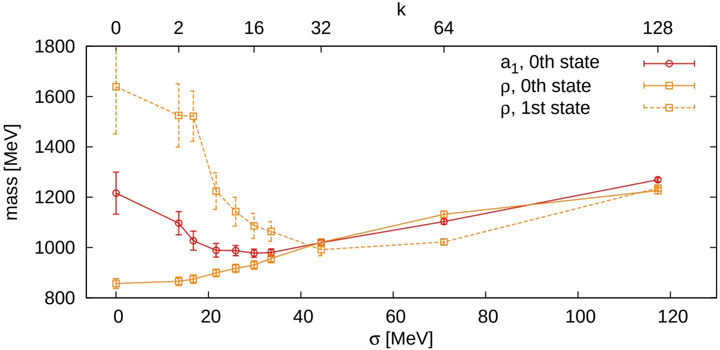

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- degeneracy of ρ and ρ': hint to a higher symmetry which includes $SU(2)_L \times SU(2)_R$ as a subgroup

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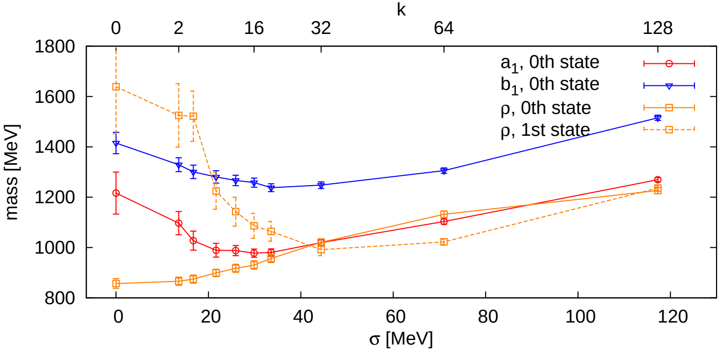

- heavy ρ meson: mass not due to dynamical chiral symmetry breaking
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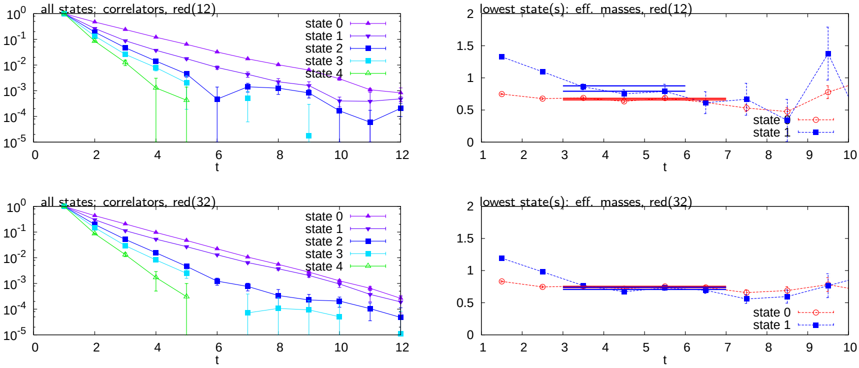
- degeneracy of ρ and ρ': hint to a higher symmetry which includes $SU(2)_L \times SU(2)_R$ as a subgroup
- nondegeneracy of ρ and b_1: $U(1)_A$ remains broken, still existence of confined states

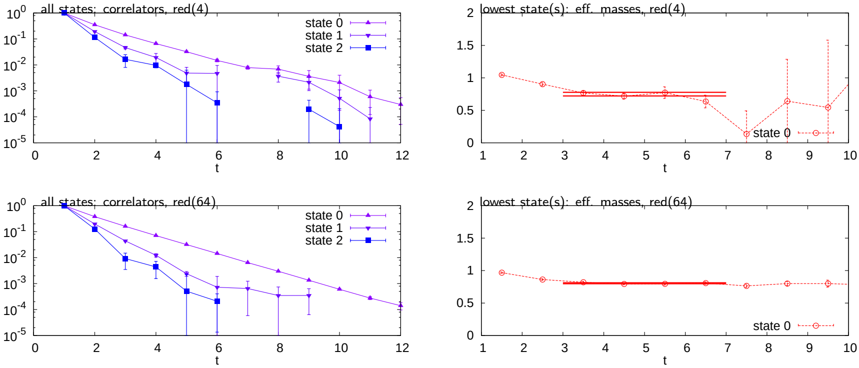
$\rho\,(J^{PC} = 1^{--})$



Do we really still observe exponentially decaying states?

- the noise in the correlators (l.h.s.) decreases under Dirac low-mode truncation
- as a consequence the effective mass plots (r.h.s.) become more stable than in full QCD!

a_1 ($J^{PC} = 1^{++}$)



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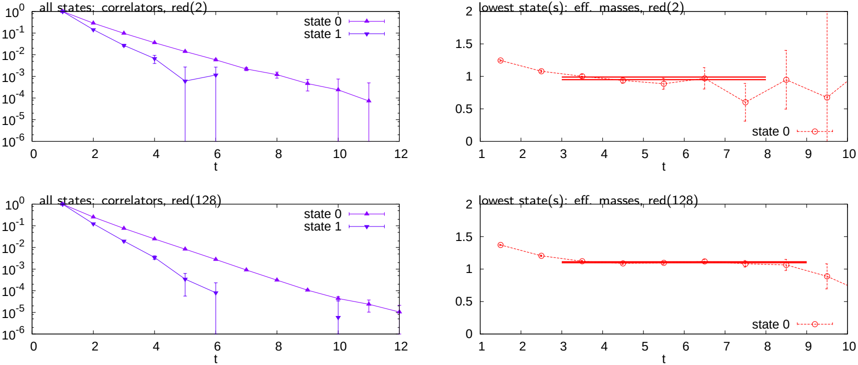
b_1 ($J^{PC} = 1^{+-}$)



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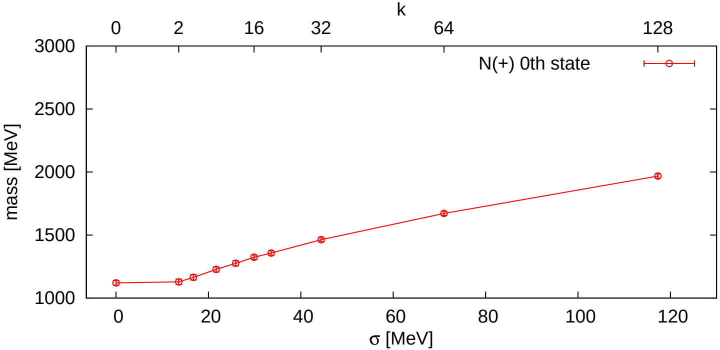
Baryons

The $\Delta - N$ splitting is usually attributed to the hyperfine spin-spin interaction between valence quarks. The realistic candidates for this interaction are

- the spin-spin color-magnetic interaction
- the flavor-spin interaction related to the spontaneous chiral symmetry breaking

What happens to the $\Delta - N$ splitting after restoration of the chiral symmetry?
Do the masses of the nucleon and the $N(1535)$ meet?

Baryon masses vs. Dirac eigenmode reduction level



- heavy $N(+)$: mass not due to dynamical chiral symmetry breaking

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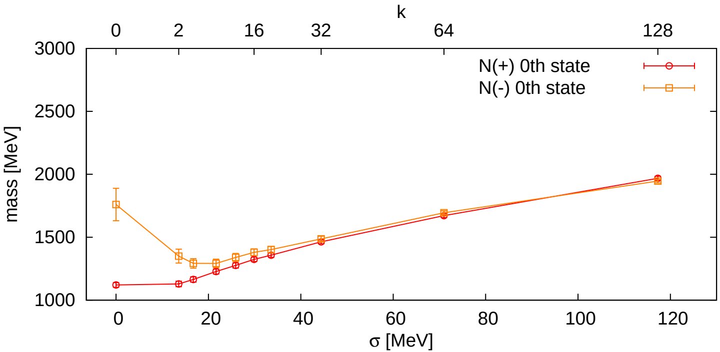
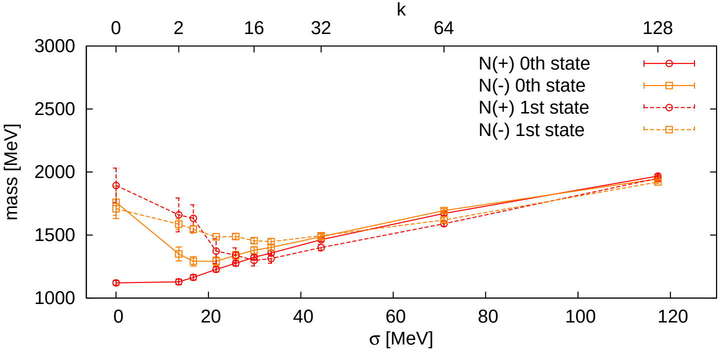


- heavy $N(+)$: mass not due to dynamical chiral symmetry breaking
- parity doubling of $N(+)$ and $N(-)$

Baryon masses vs. Dirac eigenmode reduction level



- heavy $N(+)$: mass not due to dynamical chiral symmetry breaking
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- degeneracy of two $N(+)$ and $N(-)$ states: hint to a higher symmetry which includes $SU(2)_L \times SU(2)_R$ as a subgroup

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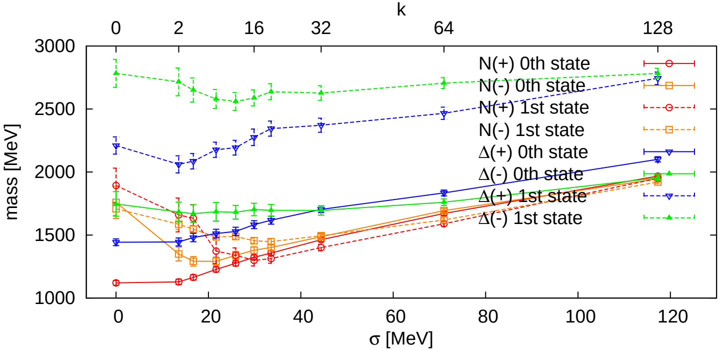

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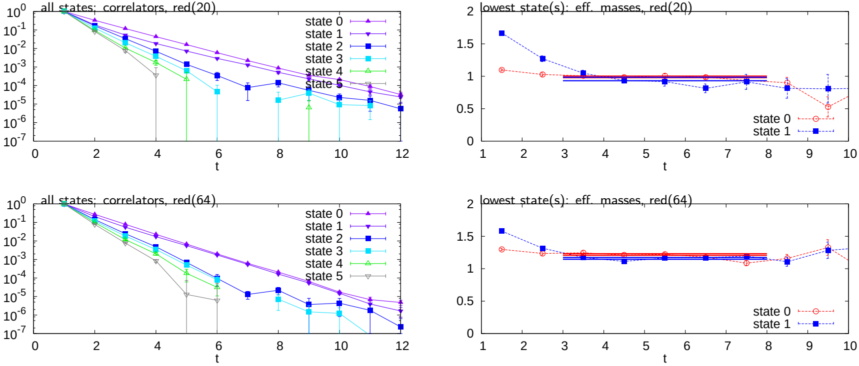
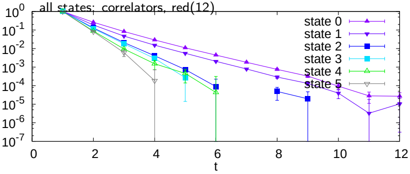
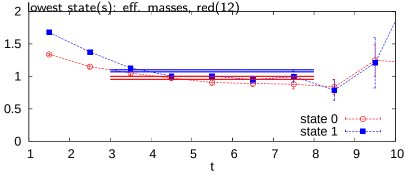
- parity doubling of $N(+)$ and $N(-)$
- degeneracy of two $N(+)$ and $N(-)$ states: hint to a higher symmetry which includes $SU(2)_L \times SU(2)_R$ as a subgroup
- distinguished excited states of $\Delta(+)$ and $\Delta(-)$: confinement persists
- Δ-N splitting reduces to $\approx 50\%$

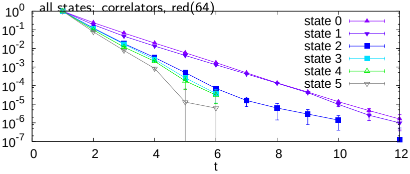
$N\left(J^P = 1/2^+\right)$

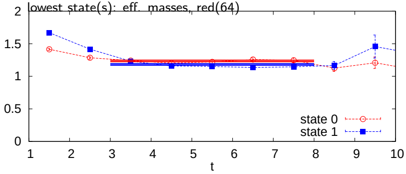


$N\,(J^P = 1/2^-)$


all states; correlators, red(12)
state 0
state 1
state 2
state 3
state 4
state 5
t


lowest state(s): eff. masses, red(12)
state 0
state 1
t


all states; correlators, red(64)
state 0
state 1
state 2
state 3
state 4
state 5
t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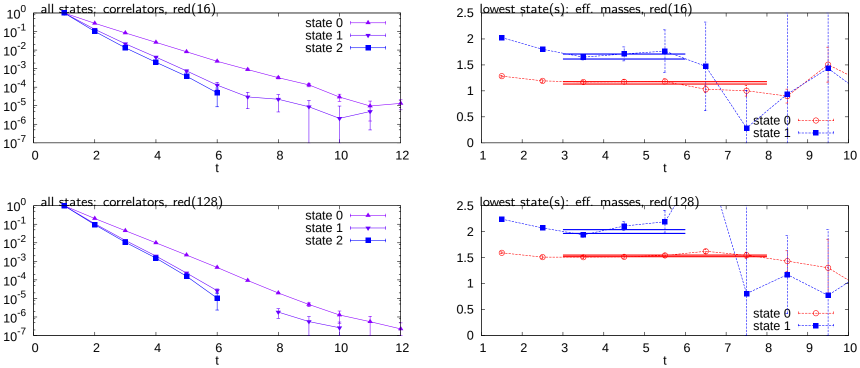
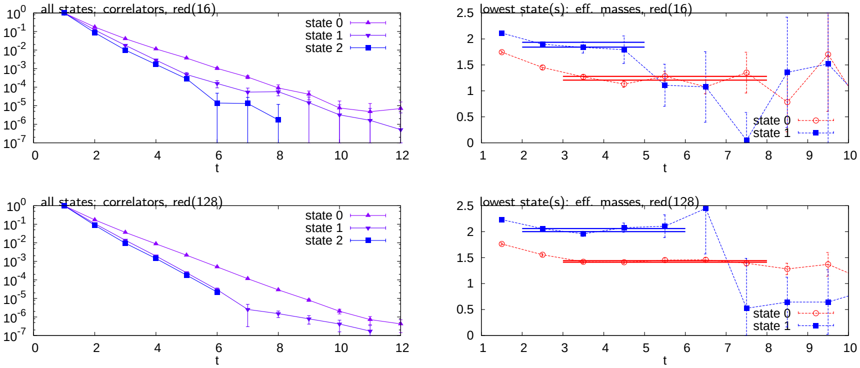
state 0
state 1
t

$\Delta\,(J^P = 3/2^+)$


all states; correlators, red(16)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(16)
state 0
state 1
t
all states; correlators, red(128)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(128)
state 0
state 1
t

$\Delta\,(J^P = 3/2^-)$


all states; correlators, red(16)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(16)
state 0
state 1
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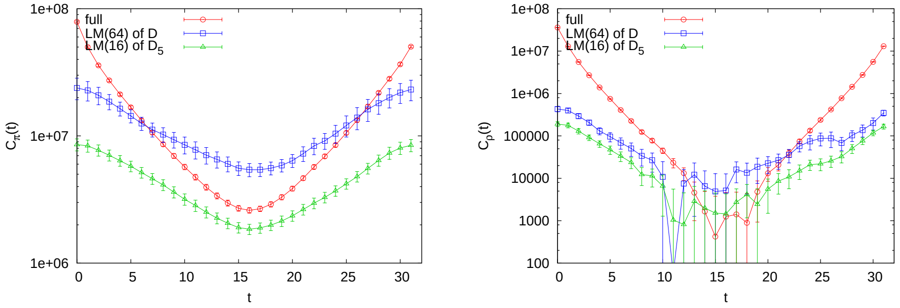
all states; correlators, red(128)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(128)
state 0
state 1
t

Summary

- low lying eigenvalues of the Dirac operator are associated with chiral symmetry breaking
- we have computed hadron propagators while removing increasingly more of the low lying eigenmodes of the Dirac operator
- the confinement properties remain intact, i.e., we still observe clear bound states for all of the studied hadrons
- the mass values of the vector meson chiral partners a_1 and ρ approach each other: restoration of $SU(2)_L \times SU(2)_R$
- no degeneracy between ρ and b_1: $U(1)_A$ axial anomaly untouched
- the nucleon and the $N(1535)$ become degenerate
- the spin-spin color-magnetic interaction and the flavor-spin interaction are of equal importance for the $\Delta - N$ splitting

Low-mode contribution of D and D_5 to the π and ρ correlators



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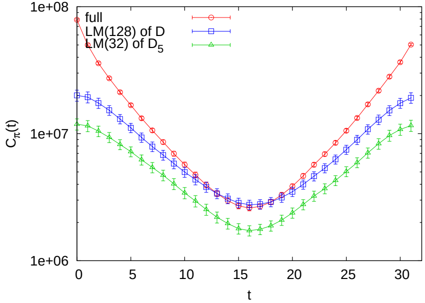

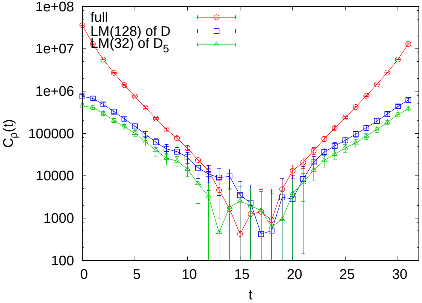

ρ interpolators

$\#_\rho$	interpolator(s)
1	$\overline{a}_n \gamma_k b_n$
8	$\overline{a}_w \gamma_k \gamma_t b_w$
12	$\overline{a}_{\partial_k} b_w - \overline{a}_w b_{\partial_k}$
17	$\overline{a}_{\partial_i} \gamma_k b_{\partial_i}$
22	$\overline{a}_{\partial_k} \epsilon_{ijk} \gamma_j \gamma_5 b_w - \overline{a}_w \epsilon_{ijk} \gamma_j \gamma_5 b_{\partial_k}$

Interpolators for the ρ-meson, $J^{PC} = 1^{--}$. The first column shows the number, the second shows the explicit form of the interpolator. cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]

a_1 interpolators

$\#_{a_1}$	interpolator(s)
1	$\bar{a}_n \gamma_k \gamma_5 b_n$
2	$\bar{a}_n \gamma_k \gamma_5 b_w + \bar{a}_w \gamma_k \gamma_5 b_n$
4	$\bar{a}_w \gamma_k \gamma_5 b_w$

a_1-meson, $J^{PC} = 1^{++}$, cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]

b_1 interpolators

$\#_{b_1}$	interpolator(s)
6	$\overline{a}_{\partial_k}\gamma_5 b_n - \overline{a}_n\gamma_5 b_{\partial_k}$
8	$\overline{a}_{\partial_k}\gamma_5 b_w - \overline{a}_w\gamma_5 b_{\partial_k}$

b_1-meson, $J^{PC} = 1^{+-}$, cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]

N interpolators

- $N^{(i)} = \epsilon_{abc}\, \Gamma_1^{(i)}\, u_a \left(u_b^T\, \Gamma_2^{(i)}\, d_c - d_b^T\, \Gamma_2^{(i)}\, u_c \right)$
- $N(+)$: 1, 2, 4, 14, 15, 18
- $N(-)$: 1, 7, 8, 9

$\chi^{(i)}$	$\Gamma_1^{(i)}$	$\Gamma_2^{(i)}$	smearing	$\#_N$
$\chi^{(1)}$	1	$C\,\gamma_5$	$(nn)n$	1
			$(nn)w$	2
			$(nw)n$	3
			$(nw)w$	4
			$(ww)n$	5
			$(ww)w$	6
$\chi^{(2)}$	γ_5	C	$(nn)n$	7
			$(nn)w$	8
			$(nw)n$	9
			$(nw)w$	10
			$(ww)n$	11
			$(ww)w$	12
$\chi^{(3)}$	$i\,1$	$C\,\gamma_t\,\gamma_5$	$(nn)n$	13
			$(nn)w$	14
			$(nw)n$	15
			$(nw)w$	16
			$(ww)n$	17
			$(ww)w$	18

cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]

Δ interpolators

- $\epsilon_{abc}\, u_a \left(u_b^T \, C \, \gamma_k \, u_c \right)$
- $\Delta(+)$: 1, 2, 3
- $\Delta(-)$: 1, 2, 3

smearing	$\#_\Delta$
$(nn)n$	1
$(nn)w$	2
$(nw)n$	3
$(nw)w$	4
$(ww)n$	5
$(ww)w$	6

cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]