ROMA TRE UNIVERSITÀ DEGLI STUDI

INFN Istituto Nazionale di Fisica Nucleare

Overlap quark propagator in Coulomb gauge: chiral symmetry breaking and confinement

Mario Schröck

in collaboration with

Ydalia Delgado and Markus Pak (Uni Graz)

Odense, December 12, 2014

Outline

- Motivation
- Introduction
  - Dirac low-modes
  - Hadrons under low-mode truncation
- Coulomb gauge and confinement
- Overlap quark propagator
- Conclusions

Motivation

Can *confinement* persist in a world without *dynamical chiral symmetry breaking*?

Eigenvalues of the Dirac operator

- the difference of left- and right-handed zero modes of the Dirac operator accounts for the *topological charge* which is responsible for the axial anomaly

[Atiyah, Singer, Ann. Math. **93** (1971) 139]

- the density of the smallest nonzero eigenvalues is related to the chiral condensate

$$\langle \overline{\psi}\psi \rangle = -\pi\rho(0)$$

[Banks, Casher, Nucl. Phys. B **169** (1980) 103]

Artificially restoring chiral symmetry

- we subtract the Dirac low-mode contribution from the valence quark propagators

$$S_{\mathrm{red}(k)} = S_{\mathrm{full}} - \sum_{i=1}^{k} \mu_i^{-1} \left| w_i \right\rangle \left\langle w_i \right| \gamma_5$$

- $\mu_i, \; |w_i\rangle$ are the eigenvalues and vectors of the hermitian Dirac operator $D_5 = \gamma_5 D$ and k denotes the truncation level

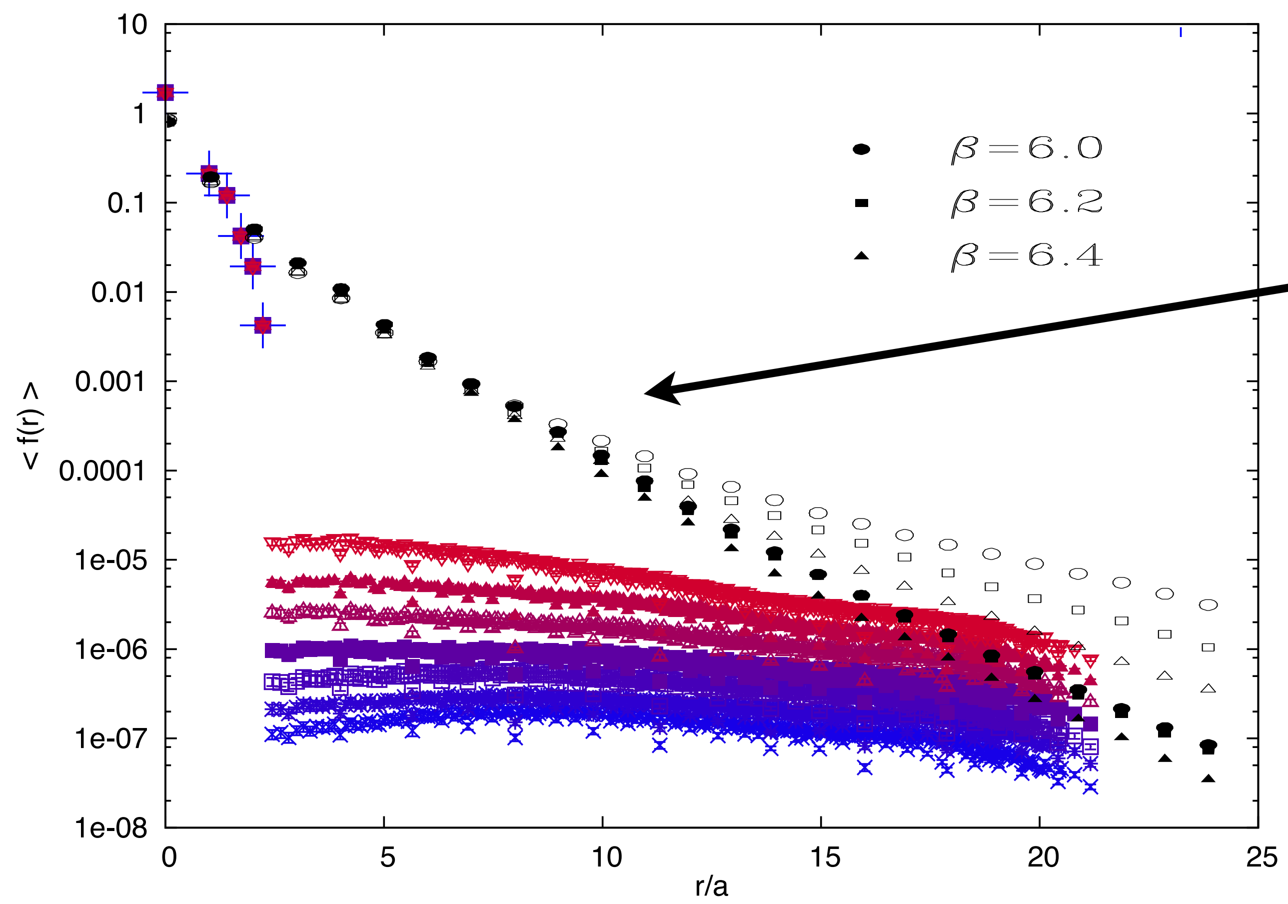
- this truncation corresponds to removing the chiral condensate of the valence quark sector by hand

Locality

- to what extent is the locality of the low-mode truncated Dirac operator violated?

$$\psi(x)^{[x_0,\alpha_0,a_0]} = \sum_y D_5(x,y)\,\eta(y)^{[x_0,\alpha_0,a_0]}$$

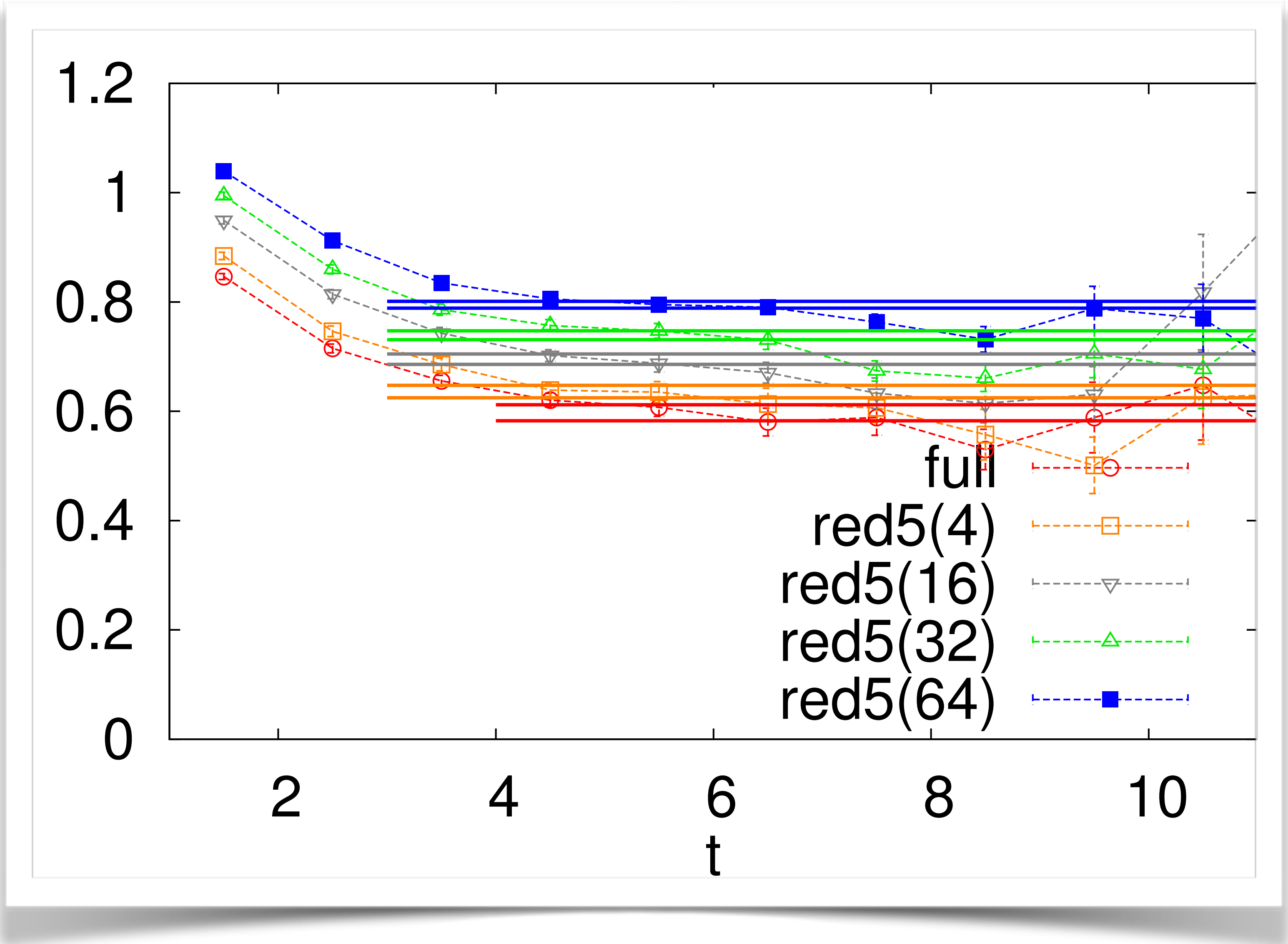
$$f(r) = \max_{x,\alpha_0,a_0} \{\|\psi(x)\| \mid \| x \| = r\}$$



(non)locality of the overlap operator

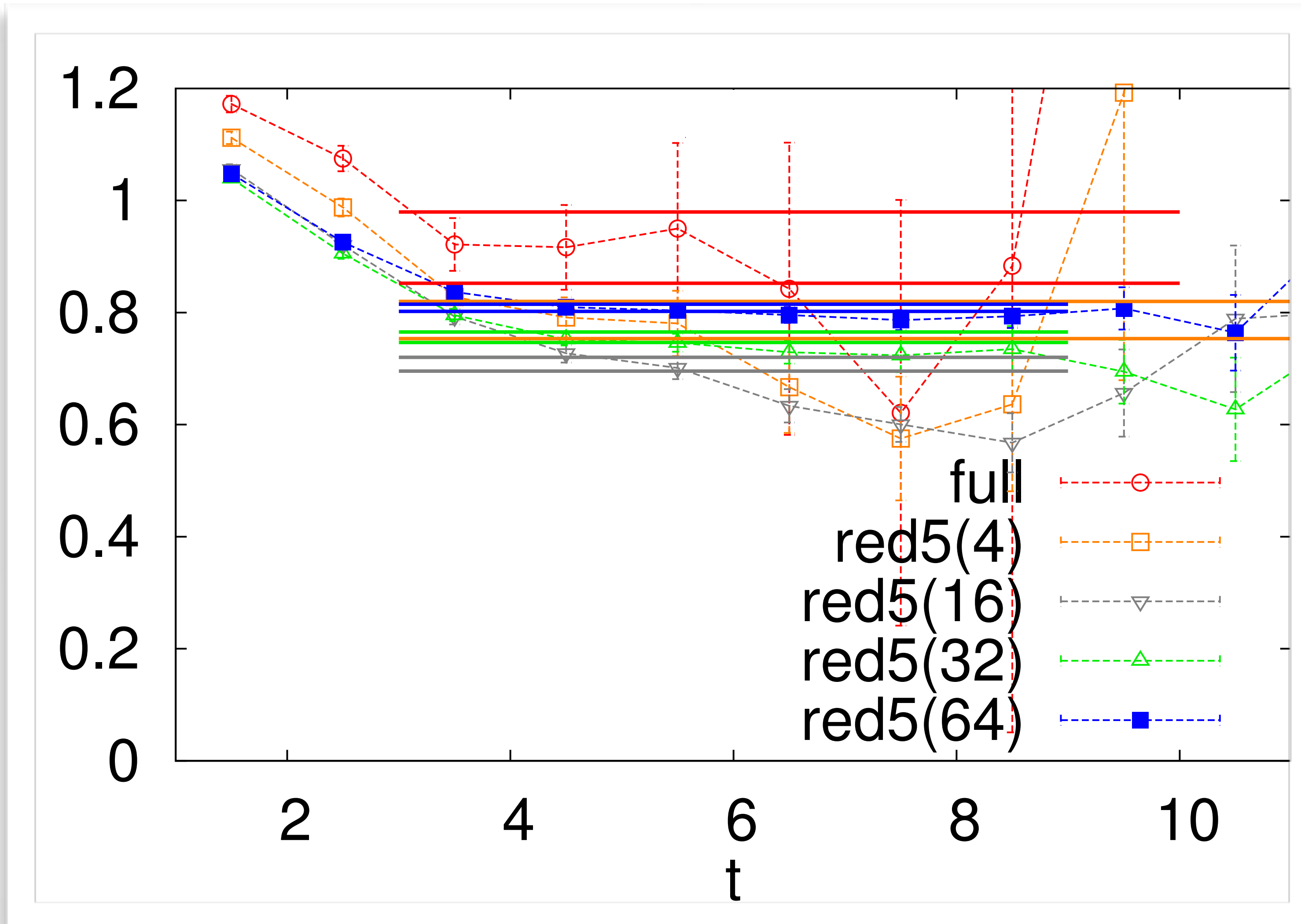
[Hernandez, Jansen, Lüscher, Nucl. Phys. B **552** (1999)]

Rho without low-modes: eff. masses



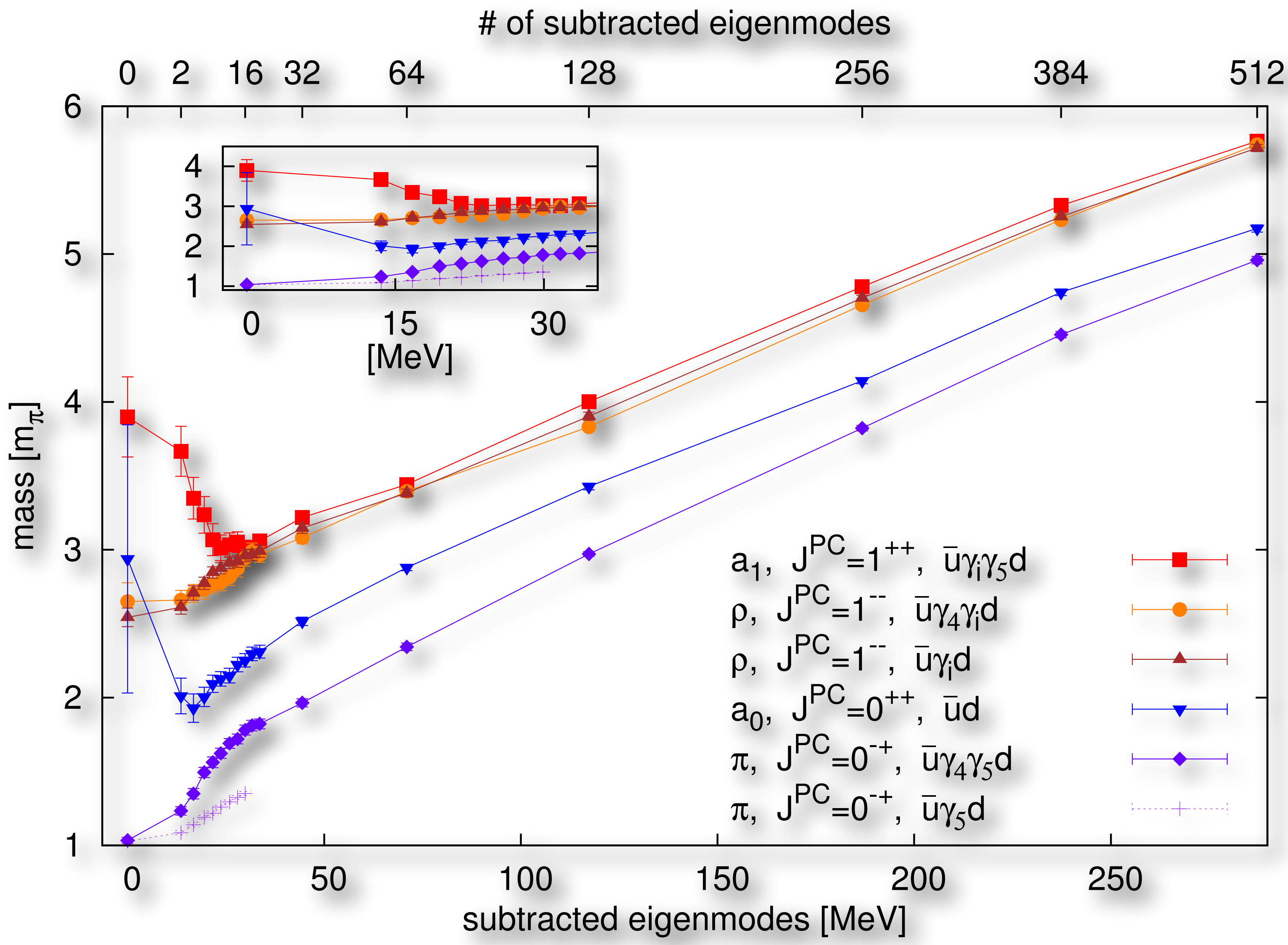
- Low-mode truncated effective masses of the $J^{PC} = 1^{--}$ sector in comparison to the eff. masses from full propagators

a_1 without low-modes



- Low-mode truncated effective mass of the axial vector current

Meson mass evolution

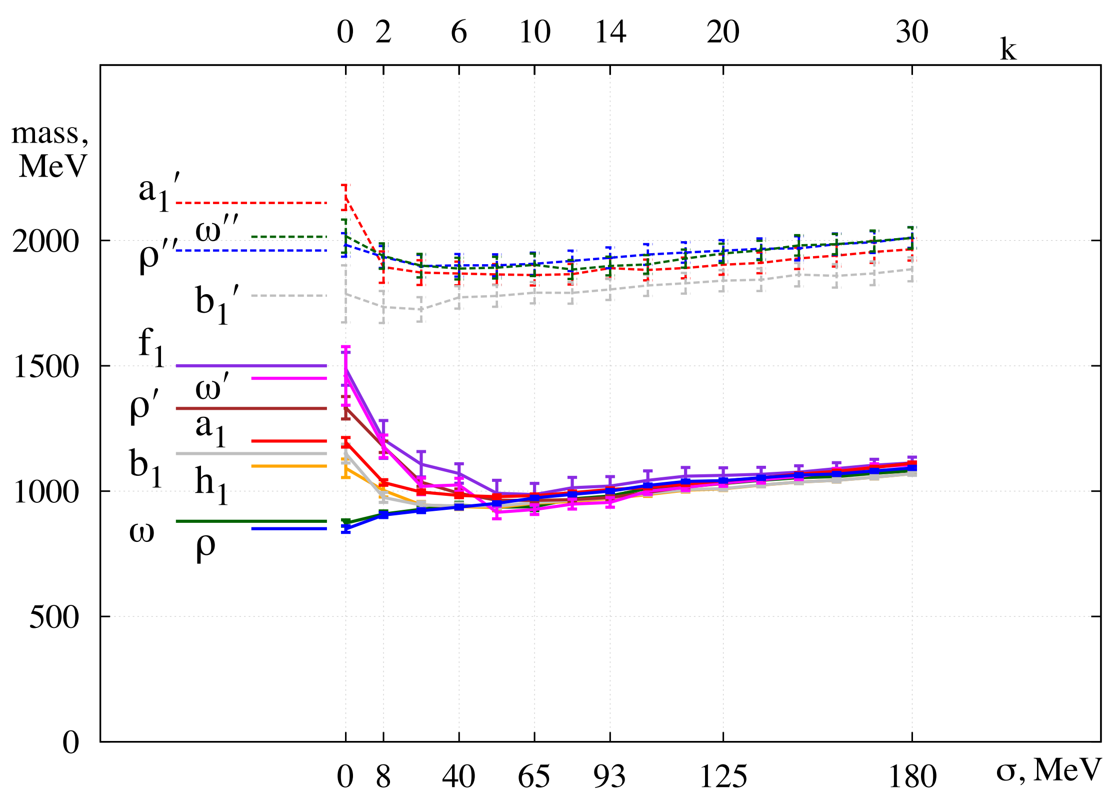


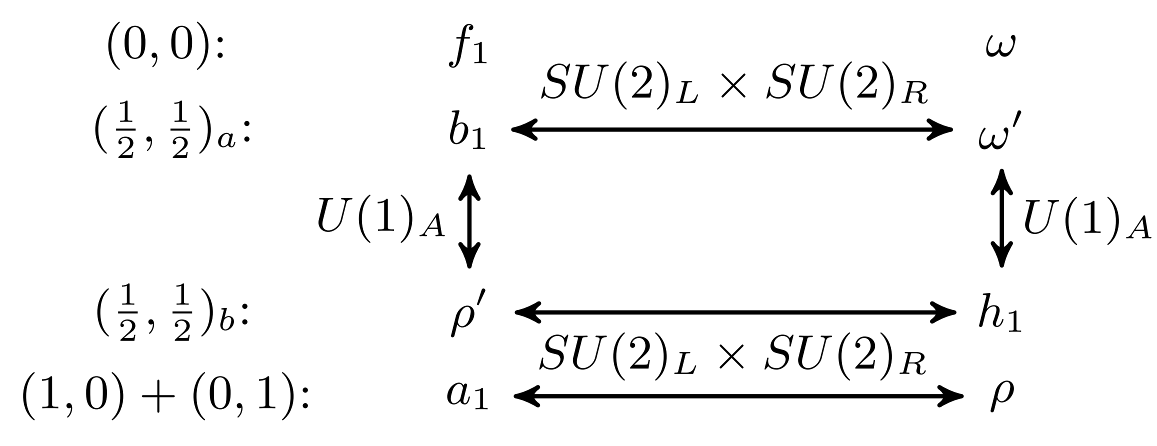
[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- degeneracy of rho and a_1: restoration of the chiral symmetry

Hadrons under low mode truncation

- isovectors [C.B. Lang, M.S., *Phys. Rev. D* ***84*** (2011)]
- baryons [Glozman, Lang, M.S., *Phys. Rev. D* ***86*** (2012)]
- with overlap quarks (Nf=2) [Denissenya, Glozman, Lang, *Phys. Rev. D* ***89*** (2014)]
- isoscalars [Denissenya, Glozman, Lang, *arXiv:1410.8751*]





Symmetry relations among $J = 1$ mesons.

➜ restoration of chiral and U(1) axial symmetry

So chiral symmetry is restored…
but what happens to
confinement?

Quark confinement is the non-existence of single quark entities in the physical spectrum.

Quarks in Coulomb gauge

- Quark Propagator has four independent dressing functions

$$S^{-1}(\boldsymbol{p}, p_4) = i\gamma_i p_i A_s(\boldsymbol{p}) + i\gamma_4 p_4 A_t(\boldsymbol{p}) + \gamma_4 p_4 \gamma_i p_i A_d(\boldsymbol{p}) + B(\boldsymbol{p})$$

spatial temporal mixed scalar

- All dressing functions seem to be independent of p_4
- Mixed component seems to vanish non-perturbatively
- Spatial and scalar components seem to diverge in the infrared
- Dynamical quark mass is finite in the infrared, cancellation of divergencies

$$M(\boldsymbol{p}) = \frac{B(\boldsymbol{p})}{A(\boldsymbol{p})}$$

Quark confinement in Coulomb gauge

- Divergence of $A_s(\boldsymbol{p})$ in the infrared leads to quark confinement
- Explanation: Integrate free quark propagator over p_4

$$S(\boldsymbol{p}) = \int \frac{dp_4}{2\pi} \frac{1}{i\boldsymbol{\gamma}\cdot\boldsymbol{p} + i\gamma_4 p_4 + m_0} = \frac{m_0 - i\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2\omega(\boldsymbol{p})}$$

$$\omega(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + m^2}$$ dispersion relation for free quark

- Perform same integration over non-perturbative quark propagator

$$S(\boldsymbol{p}) = \frac{B(\boldsymbol{p}) - i\boldsymbol{\gamma}\cdot\boldsymbol{p}A_s(\boldsymbol{p})}{2\omega(\boldsymbol{p})}$$

$$\omega(\boldsymbol{p}) = A_t(\boldsymbol{p})A_s(\boldsymbol{p})\sqrt{\boldsymbol{p}^2 + M^2(\boldsymbol{p})}$$ dispersion relation for confined quark

Massless overlap propagator

- Massless Overlap Dirac operator: $D(0) = \rho\left(1 + \gamma_5 \mathrm{sign}\left[H_{\mathrm{W}}(-\rho)\right]\right)$
- Fullfills Ginsparg-Wilson equation: $\{D(0), \gamma_5\} = \frac{1}{\rho} D(0)\gamma_5 D(0)$
- Free massless Overlap quark propagator:

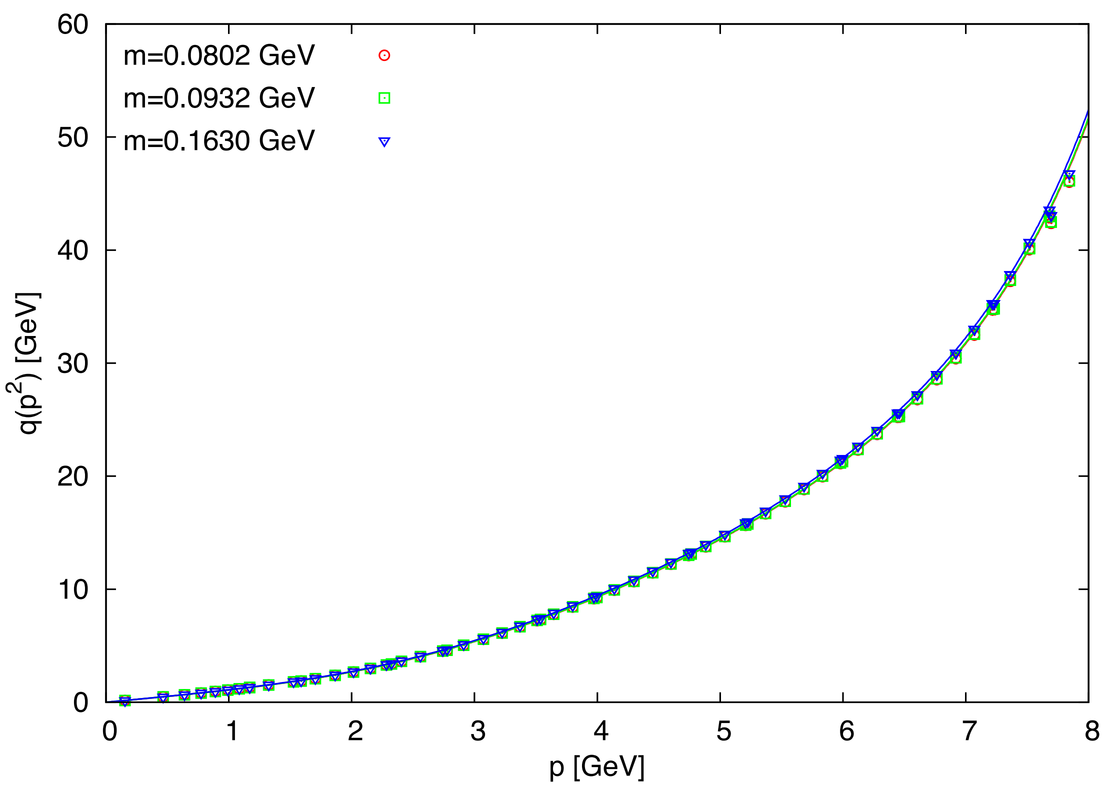
$$S^{(0)}(p) = -i\gamma_\mu \mathcal{C}_\mu(p) + \frac{1}{2\rho}, \quad \mathcal{C}_\mu(p) = \frac{1}{2\rho}\frac{k_\mu}{\sqrt{k_\mu^2 + A^2} + A}, \quad A = \frac{1}{2}\hat{k}_\mu^2 - a\rho$$

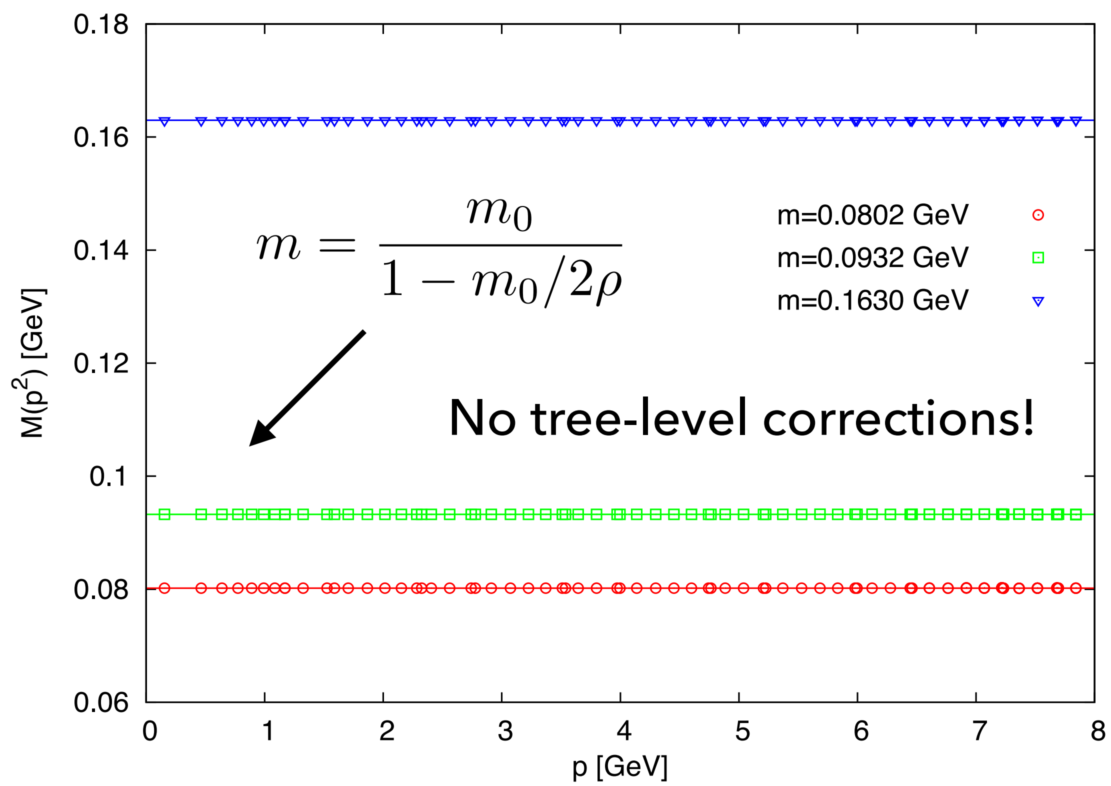
- Basic step to make contact with continuum quark propagator:

$$\widetilde{S} = S - \frac{1}{2\rho} \Longrightarrow \{\widetilde{S}, \gamma_5\} = 0$$

Massive overlap propagator

- Massive Overlap Dirac operator: $D(m_0) = \left(1 - \frac{m_0}{2\rho}\right) D(0) + m_0$
- Free massive Overlap quark propagator: $\left(S^{(0)}\right)^{-1}(p) = i\gamma_\mu q_\mu + m$
- Identify Overlap lattice momenta q_μ and current quark mass m





Lattice setup

- effect of dynamical quarks on Coulomb gauge quark propagator small

  [Burgio, M.S., Reinhardt, Quandt, Phys. Rev. D **86** (2012)]

- Quenched Lüscher-Weisz gauge field configurations on 20×20 lattice with $a = 0.2$ fm

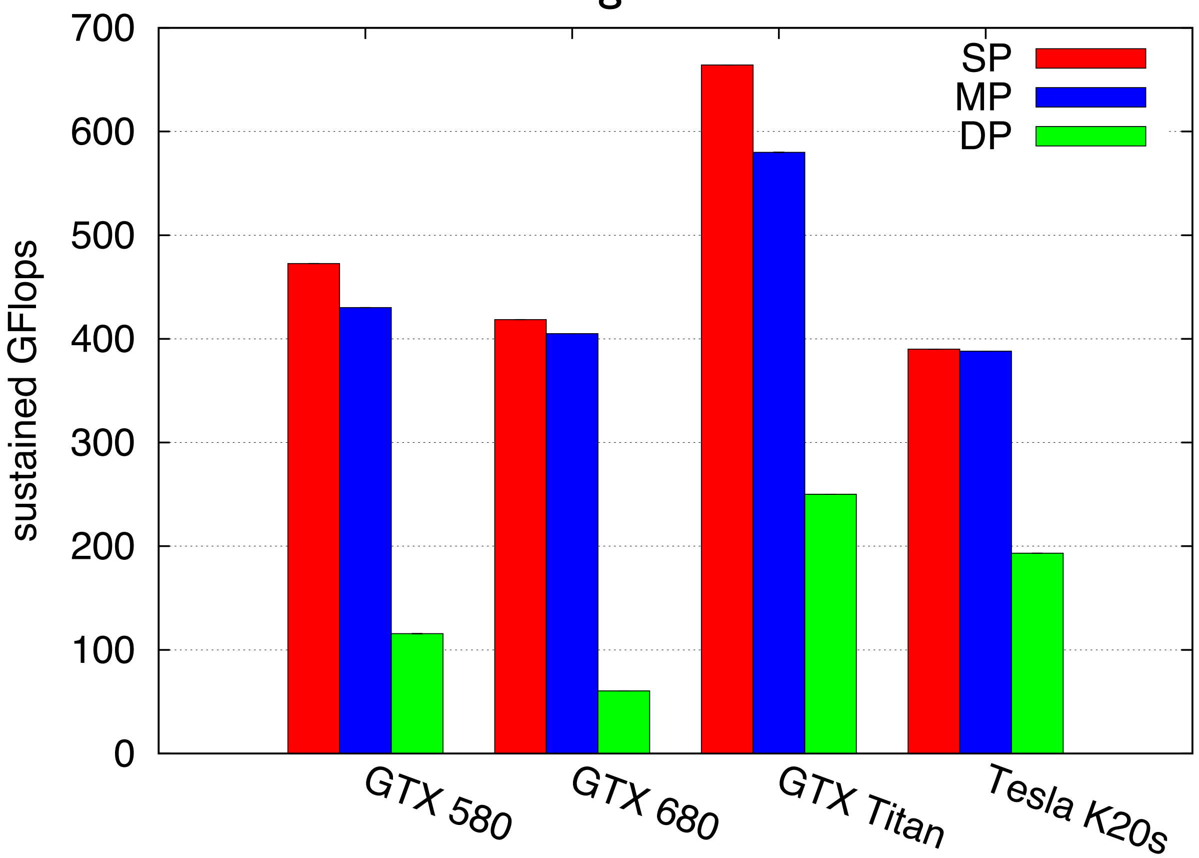
- Gauge configurations fixed to **Coulomb gauge**; residual gauge freedom fixed to Integrated Polyakov gauge

- Wilson-Dirac mass parameter $\rho = 1.6$

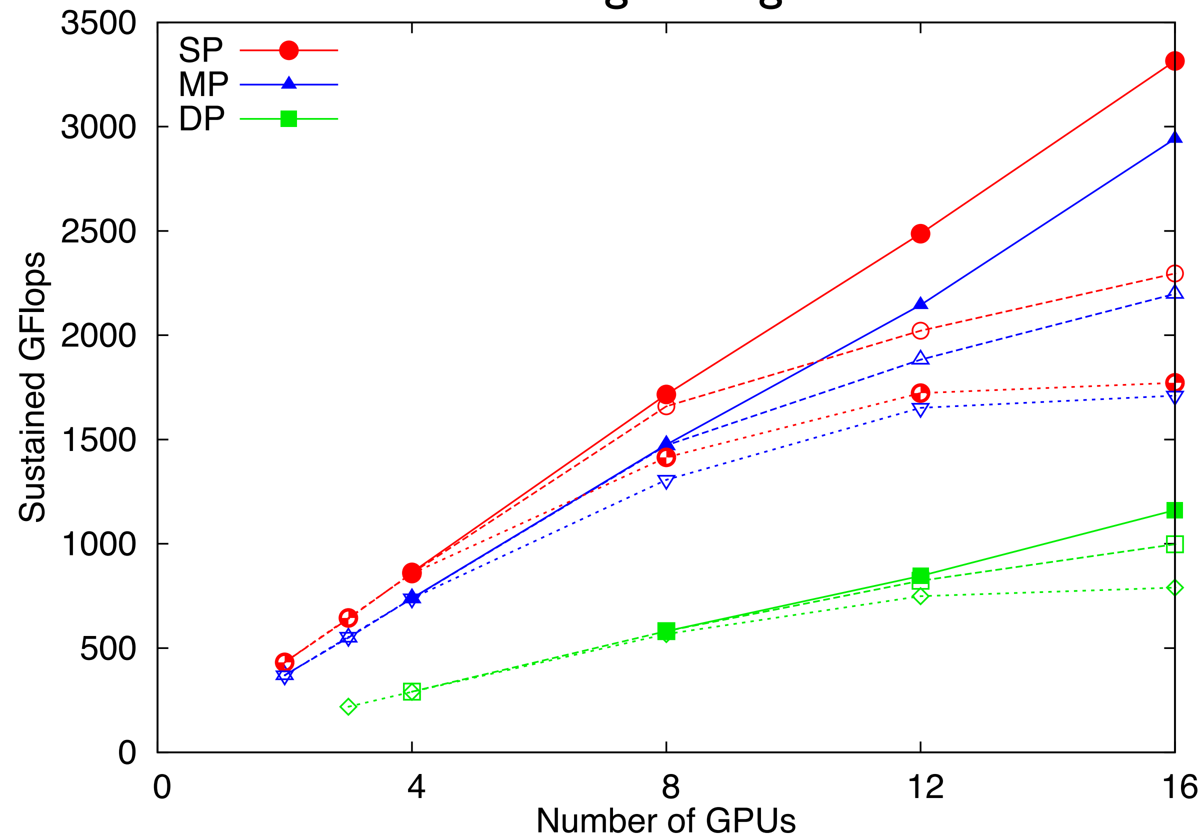
- To improve condition number, 140 low modes of kernel operator computed exactly

cuLGT: gauge fixing on GPUs

[M.S., H.Vogt, Comp. Phys. Commun. **184** (2013) 1907-1919]



> 80% of max. bandwidth throughput

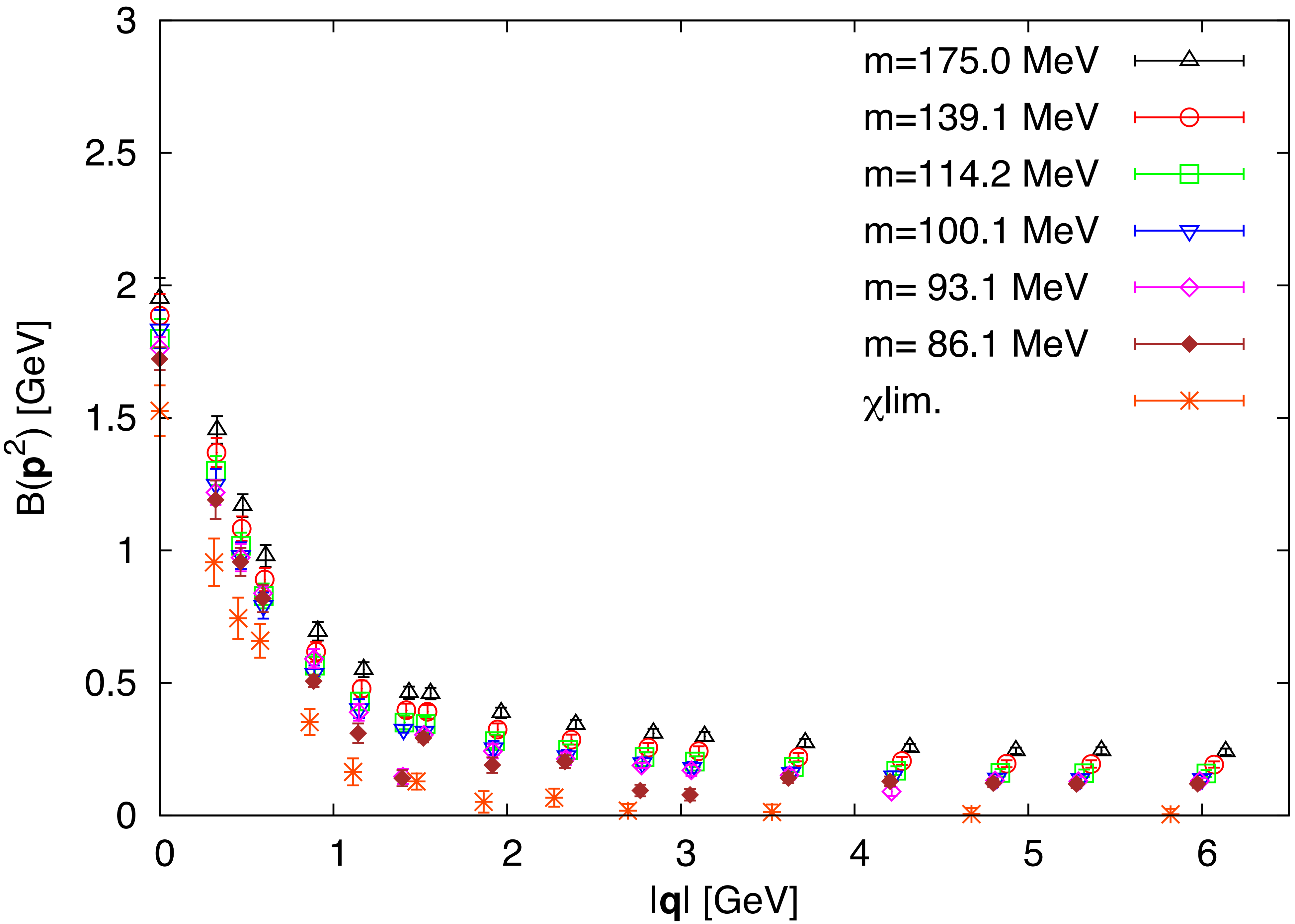


on Tesla C2070: $64^3 \times N_t$ with $N_t = 256, 128, 96$ (top to bottom)

code available: www.culgt.com and github.com/culgt

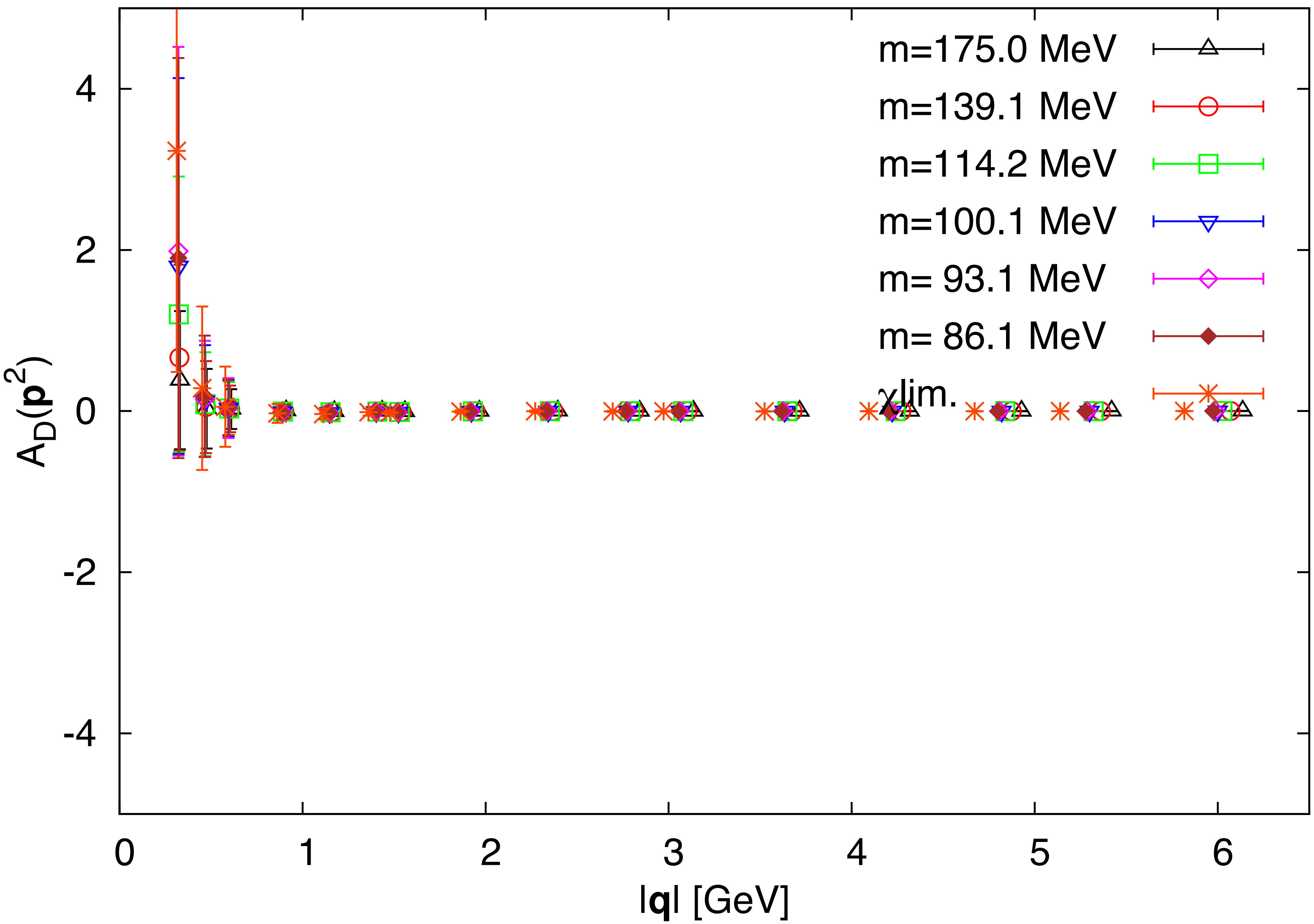
Scalar component

$$S^{-1}(\boldsymbol{p}, p_4) = i\gamma_i p_i A_s(\boldsymbol{p}) + i\gamma_4 p_4 A_t(\boldsymbol{p}) + \gamma_4 p_4 \gamma_i p_i A_d(\boldsymbol{p}) + B(\boldsymbol{p})$$



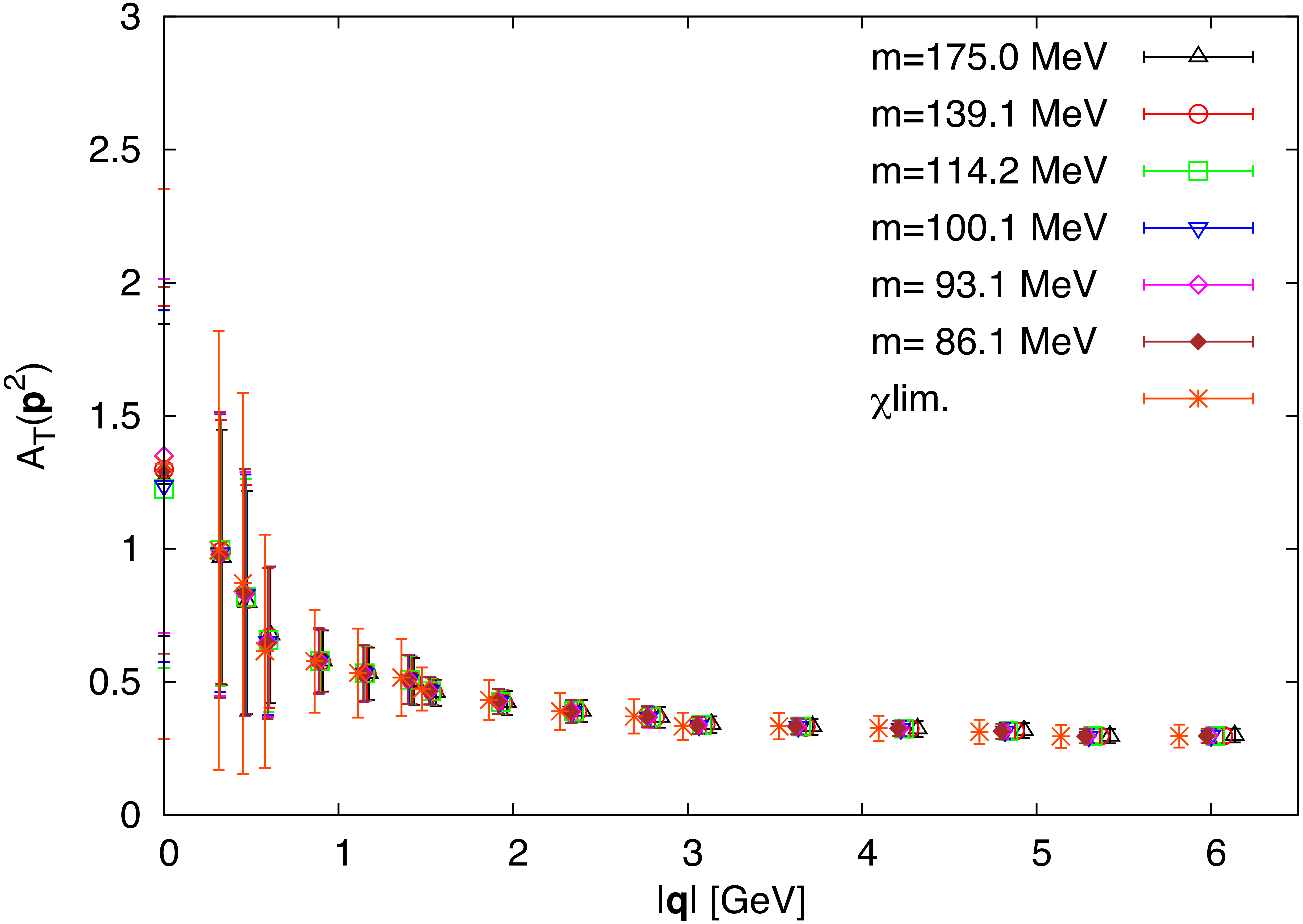
Mixed component

$$S^{-1}(\boldsymbol{p}, p_4) = i\gamma_i p_i A_s(\boldsymbol{p}) + i\gamma_4 p_4 A_t(\boldsymbol{p}) + \gamma_4 p_4 \gamma_i p_i A_d(\boldsymbol{p}) + B(\boldsymbol{p})$$



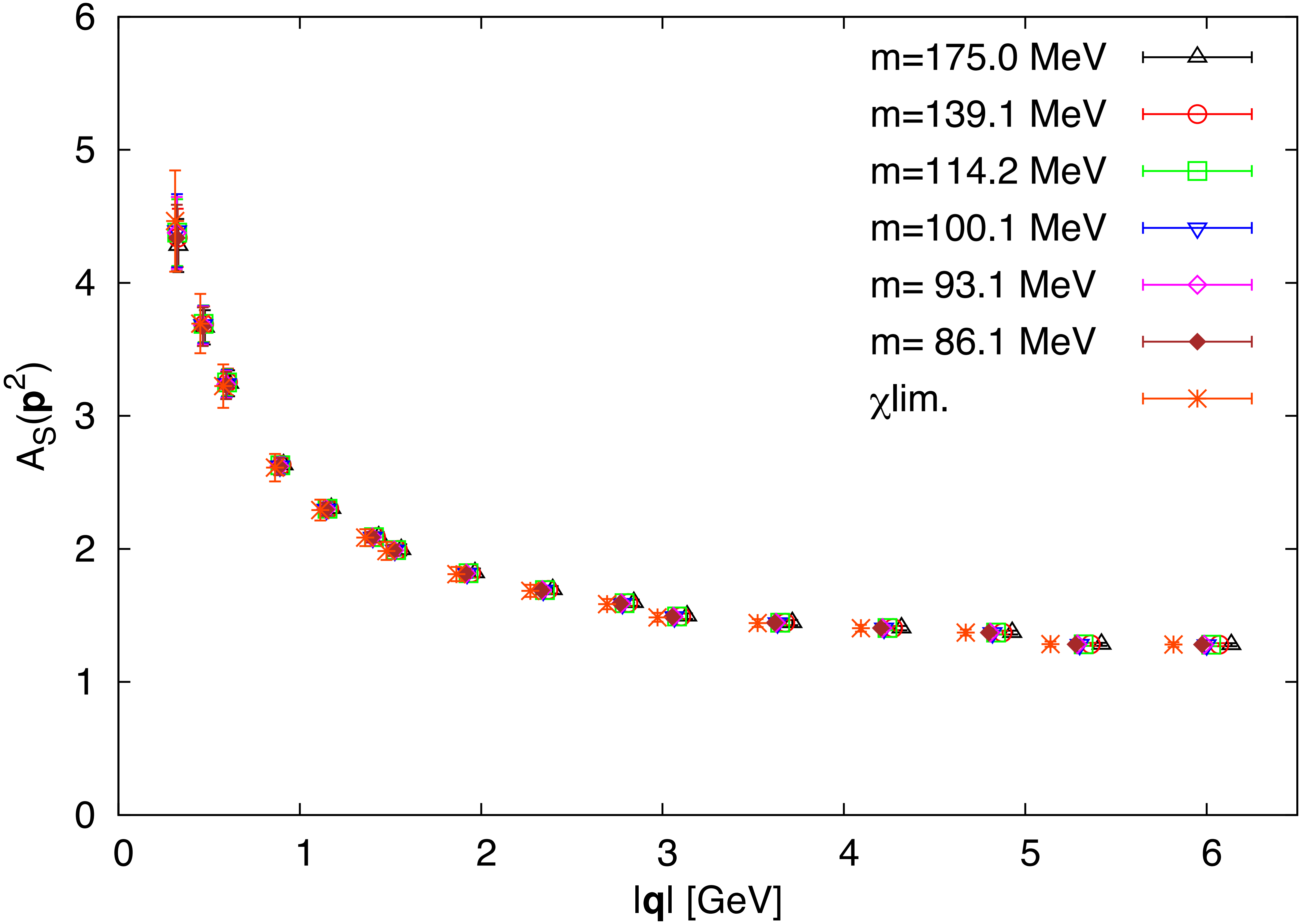
Temporal component

$$S^{-1}(\boldsymbol{p}, p_4) = i\gamma_i p_i A_s(\boldsymbol{p}) + i\gamma_4 p_4 A_t(\boldsymbol{p}) + \gamma_4 p_4 \gamma_i p_i A_d(\boldsymbol{p}) + B(\boldsymbol{p})$$



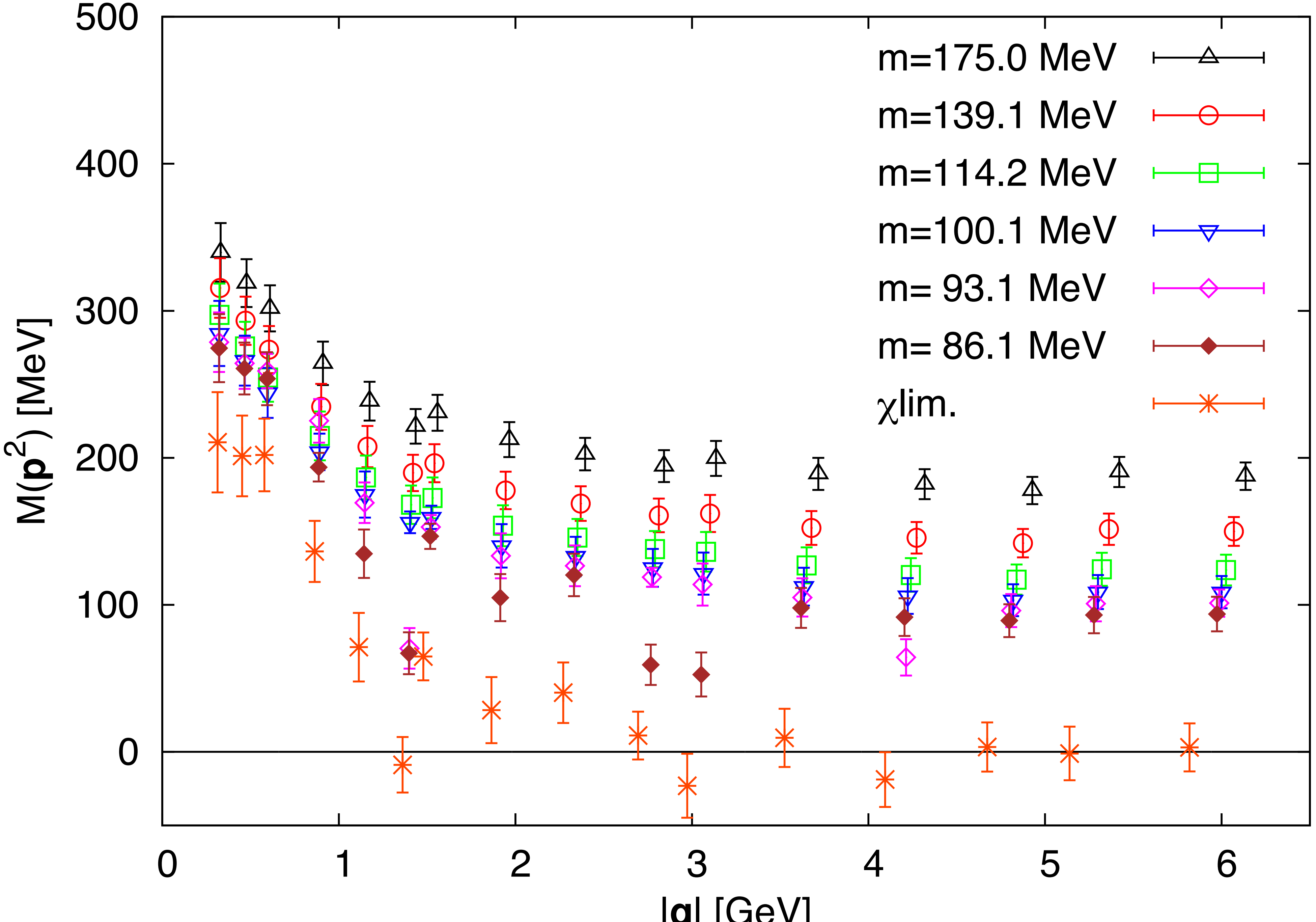
Spatial component

$$S^{-1}(\boldsymbol{p}, p_4) = i\gamma_i p_i A_s(\boldsymbol{p}) + i\gamma_4 p_4 A_t(\boldsymbol{p}) + \gamma_4 p_4 \gamma_i p_i A_d(\boldsymbol{p}) + B(\boldsymbol{p})$$



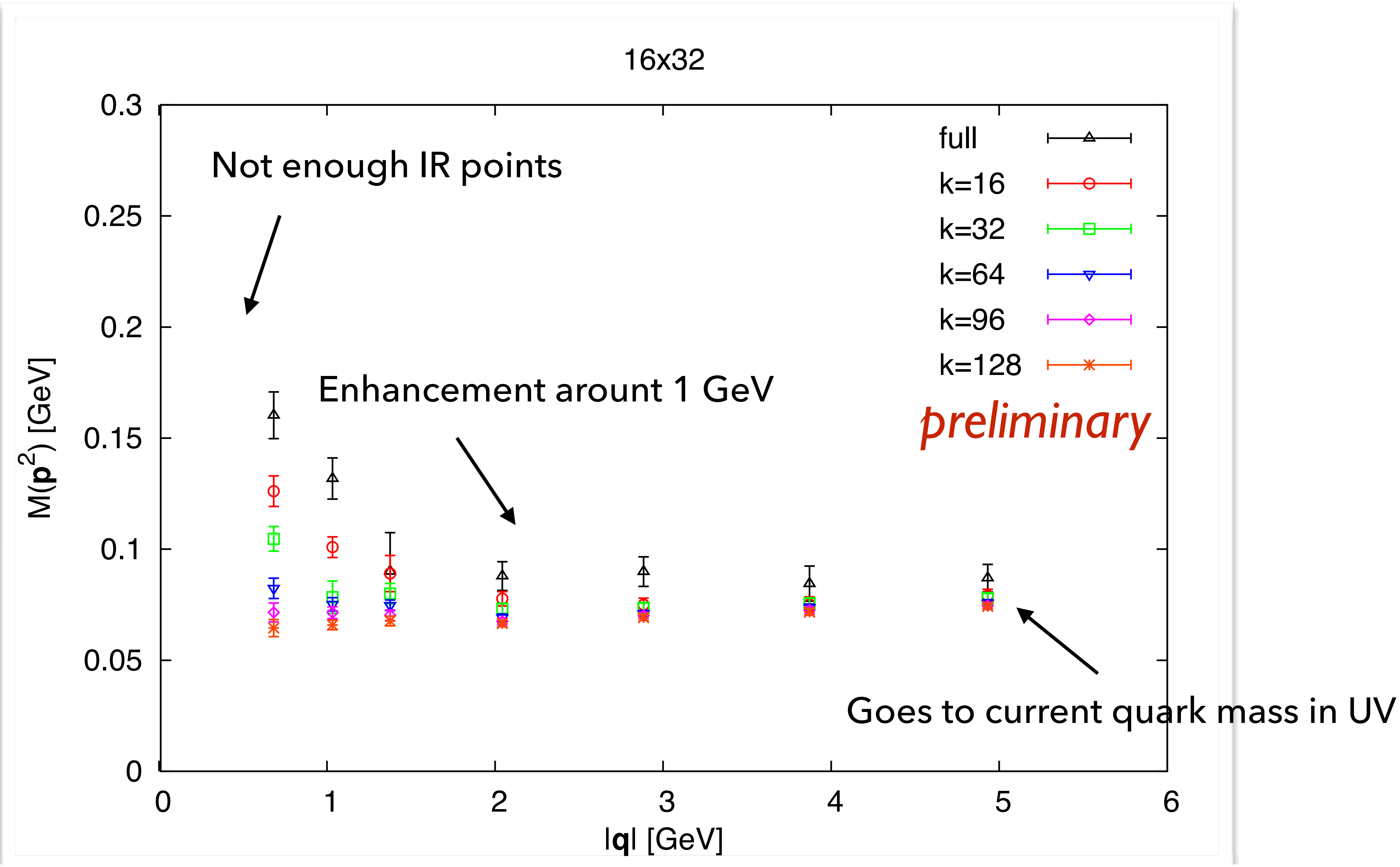
Mass function

$$M(\boldsymbol{p}) = \frac{B(\boldsymbol{p})}{A(\boldsymbol{p})}$$



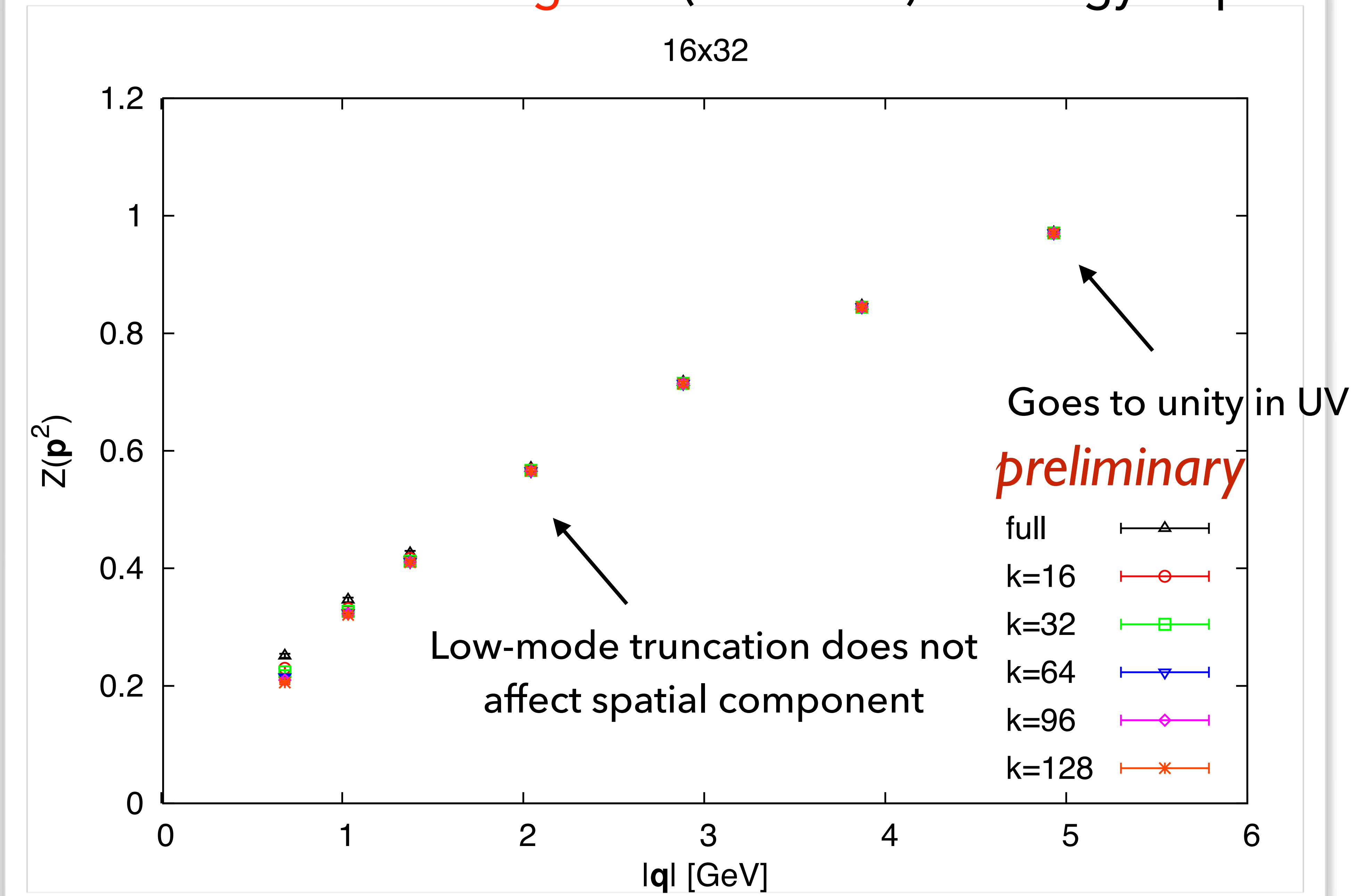
Dynamical mass & effect of low mode removal

- Dynamical quark mass approaches current quark mass in IR after removing enough modes



Spatial component & effect of low mode

- Here $Z = 1/A_s$ shown
- Spatial component does not change its shape after low-mode truncation
- It is an indication that divergence (in IR-limit) in energy dispersion still holds

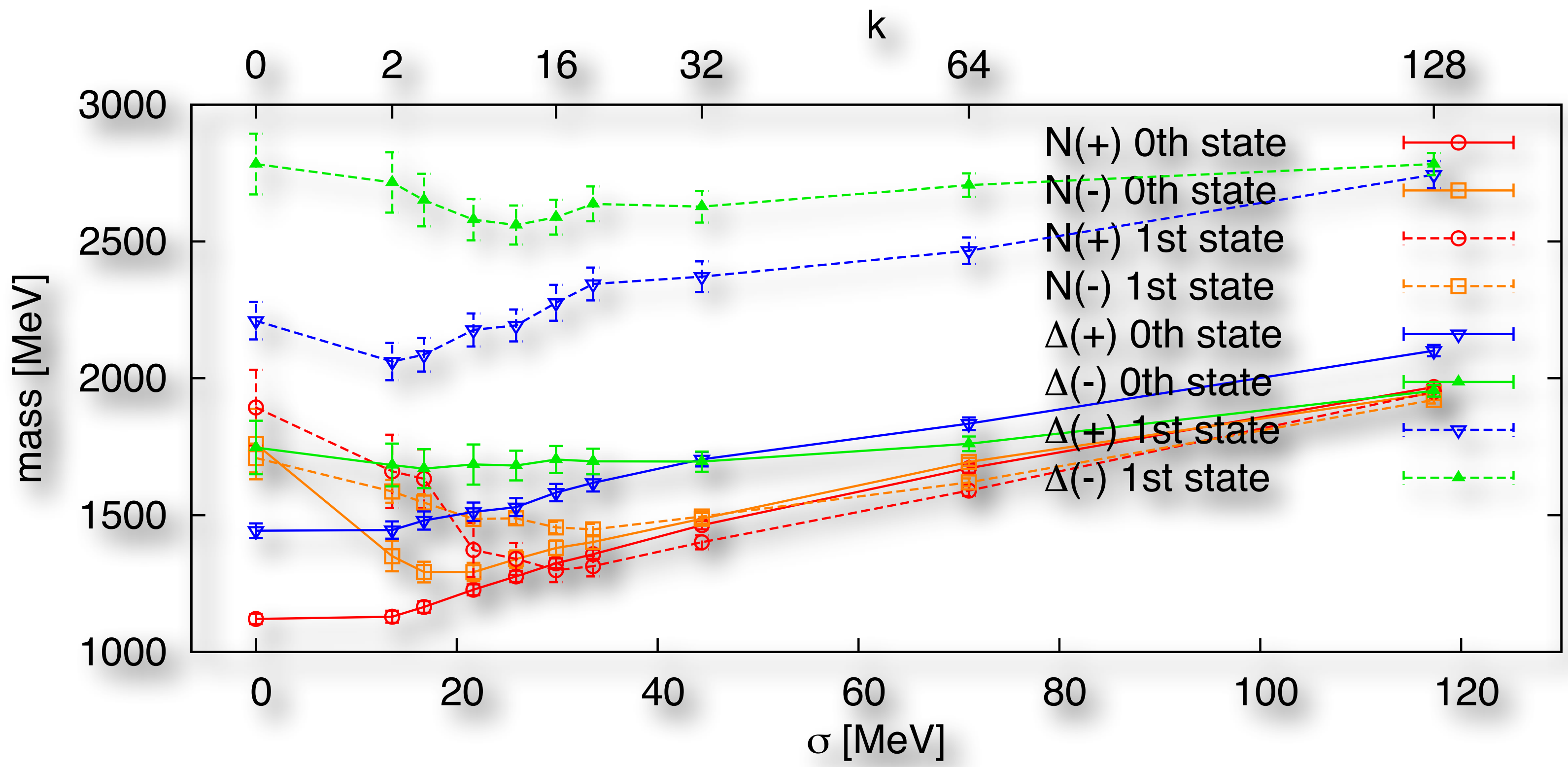


Summary & Conclusions

- we removed the lowest Dirac eigenmodes from valence quark propagators
- the meson spectrum and the quark mass function show that chiral symmetry gets restored
- the quark energy dispersion relation seems to remain IR divergent
- we have strong hints that confinement survives the restoration of chiral symmetry.

Appendix

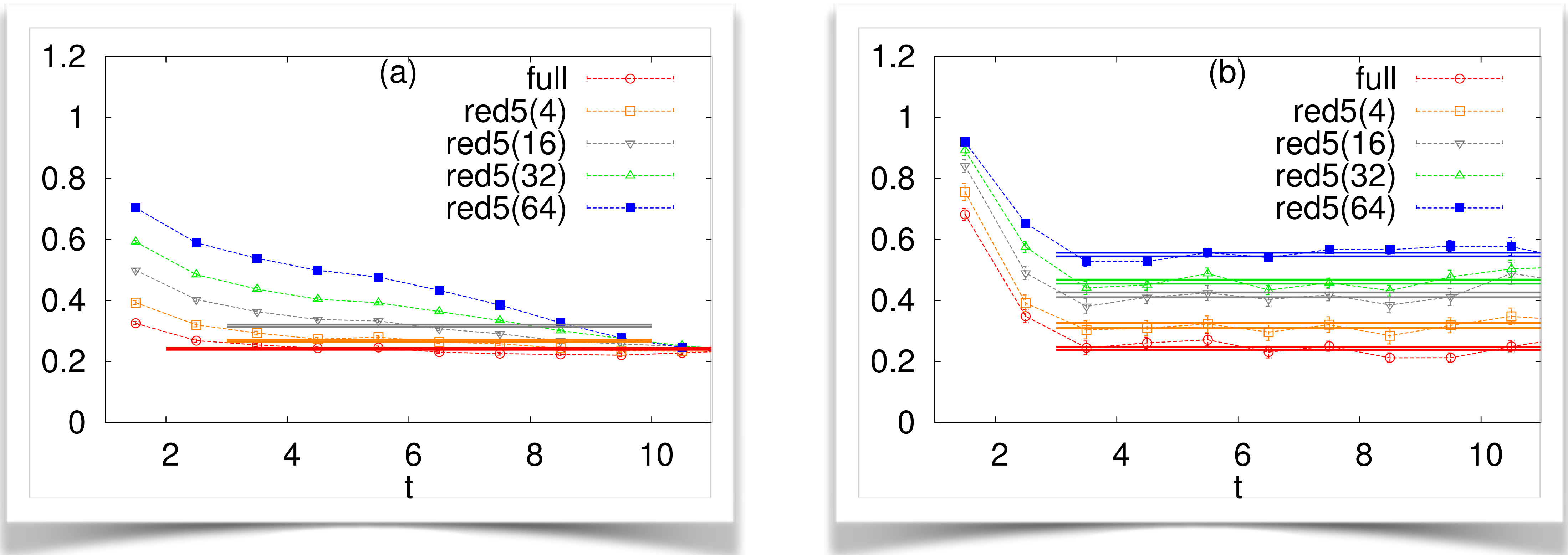
Baryon mass evolution



[Glozman, Lang, M.S., Phys. Rev. D **86** (2012) 014507]

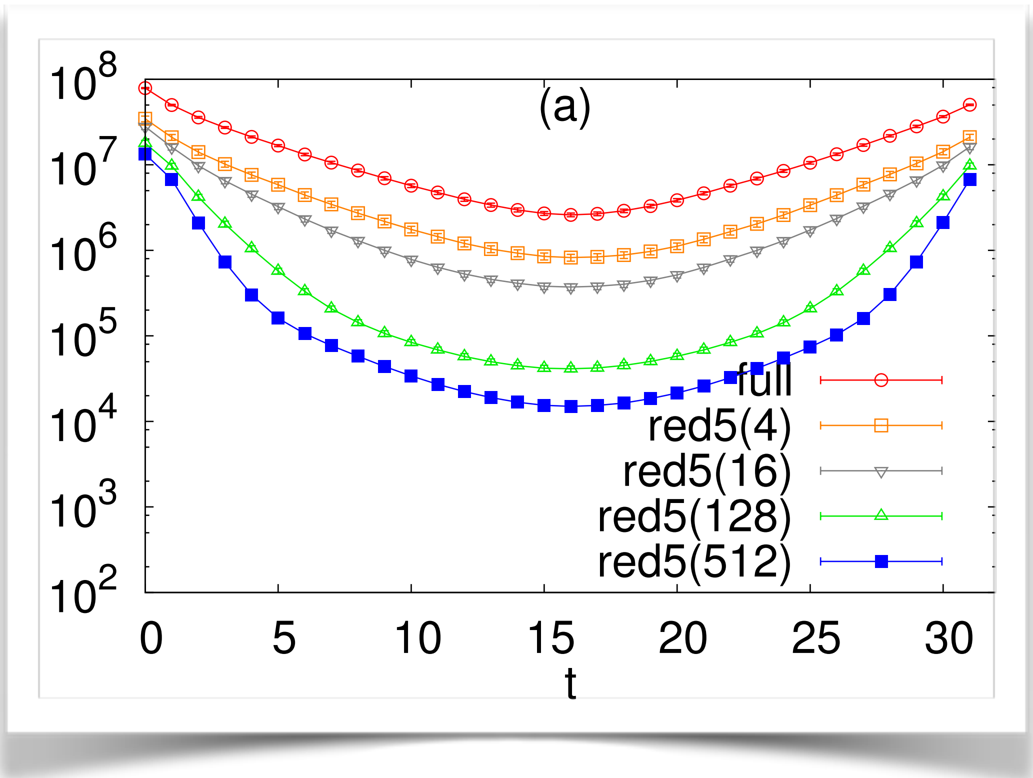
- parity doubling in the $J = 1/2$ and $J = 3/2$ channels
- degeneracy of nucleon ground and exited states
- splitting of Δ ground vs. excited state remains: persistence of confinement

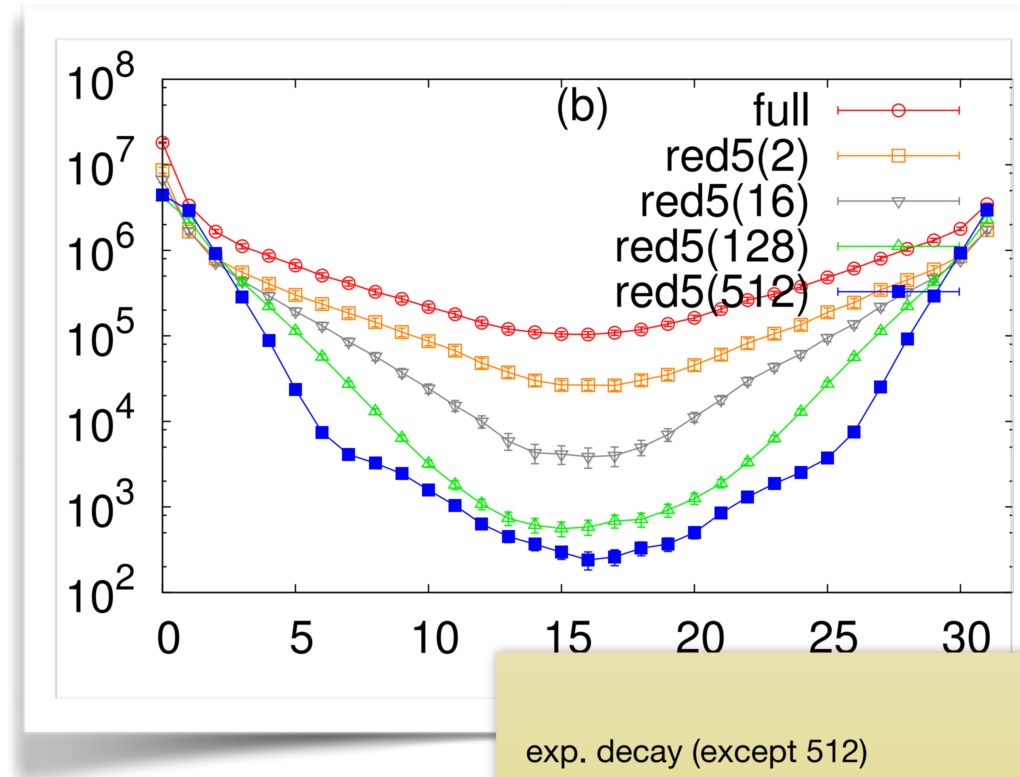
Pion without low-modes



- Low-mode truncated effective masses of the $J^{PC} = 0^{-+}$ sector in comparison to the eff. masses from full propagators
- interpolators: (a) $\bar{u}\gamma_5 d$ (b) $\bar{u}\gamma_4\gamma_5 d$

Pion without low-modes

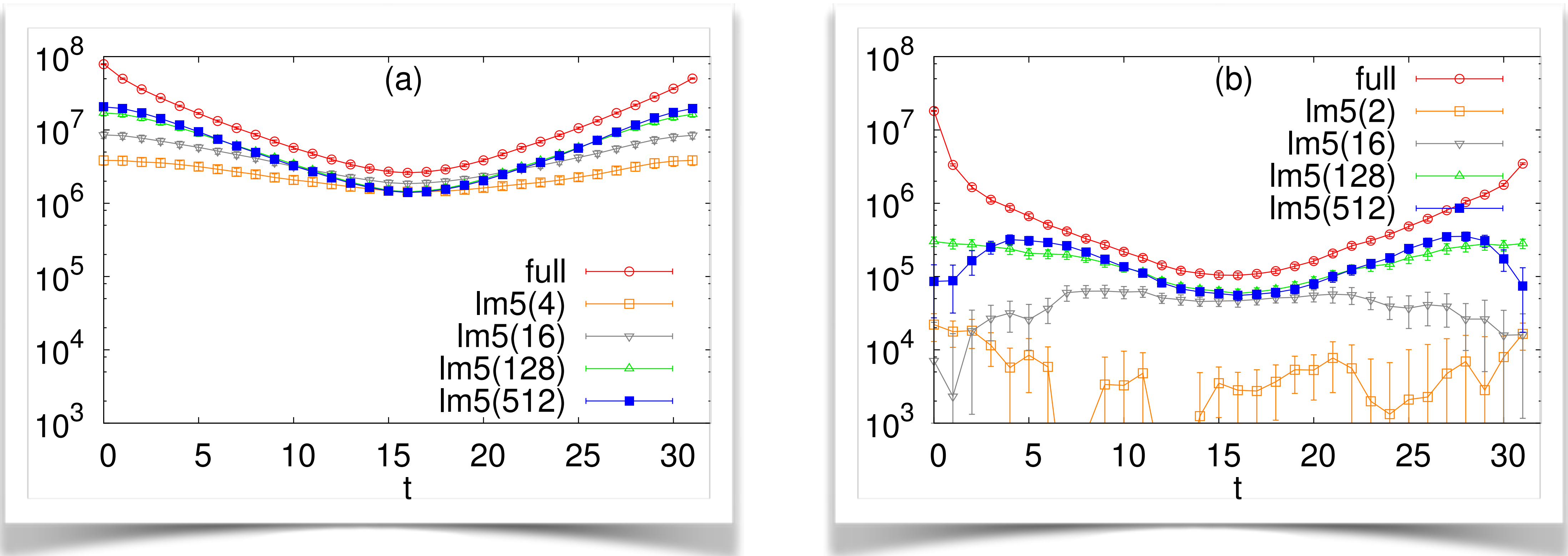




exp. decay (except 512)

- Low-mode truncated correlators of the $J^{PC} = 0^{-+}$ sector in comparison to the correlators from full propagators
- interpolators: (a) $\bar{u}\gamma_5 d$ (b) $\bar{u}\gamma_4\gamma_5 d$

Pion low-modes only



- Low-mode contribution to the correlators for the $J^{PC} = 0^{-+}$ sector in comparison to the correlators from full propagators
- interpolators: (a) $\bar{u}\gamma_5 d$ (b) $\bar{u}\gamma_4\gamma_5 d$

pion strongly dominated by low-modes