

Effects of the low lying Dirac modes on the spectrum of QCD

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- C.B. Lang, M. Schröck: PRD 84 (2011), arXiv:1107.5195
- L.Ya. Glozman, C.B. Lang, M. Schröck: *in preparation*
- M. Schröck: PLB 711 (2012), arXiv:1112.5107



Outline

Motivation and introduction

Mesons

Baryons

Quark propagator

Conclusions

Why are the lowest Dirac eigenmodes interesting?

The Banks-Casher relation

$$\langle \bar{\psi}\psi \rangle = -\pi\rho(0)$$

directly relates the density of the Dirac modes near the origin $\rho(0)$ to the chiral condensate.

Reminder: chiral symmetry and its breaking

When neglecting the two lightest quark masses, the QCD Lagrangian becomes invariant under the symmetry group

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

The axial vector part of the $SU(2)_L \times SU(2)_R$ symmetry is broken spontaneously in the vacuum whereas the vector part is (approximately) preserved. The $U(1)$ axial symmetry is not only broken spontaneously but also explicitly (axial anomaly).

“Unbreaking” chiral symmetry

- Our goal is to construct hadron correlators out of *reduced* quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., [DeGrand, PRD 69 \(2004\)](#)).

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- Landau gauge quark propagator:
 - what happens to the dynamical mass generation of quarks as shown by the momentum space mass function?

Reducing quark propagators

- Consider the Hermitian Dirac operator $D_5 \equiv \gamma_5 D$ (real eigenvalues)
- Split the quark propagator $S \equiv D^{-1}$ into a low mode (lm) part and a *reduced* (red) part

$$\begin{aligned}
 S &= \sum_{i \leq k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5 + \sum_{i > k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5 \\
 &= S_{\text{lm}(k)} + S_{\text{red}(k)}
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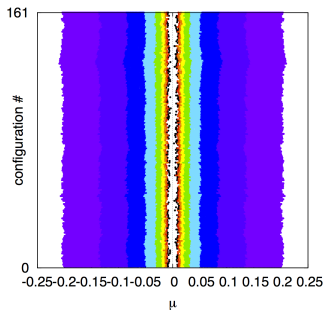
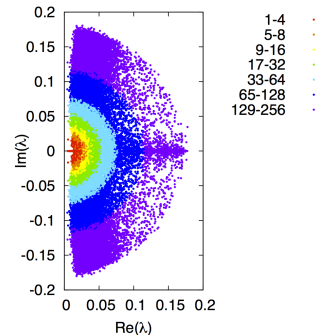
- In this work we investigate the *reduced (red)* part of the propagator

$$S_{\text{red}(k)} = S - S_{\text{lm}(k)}$$

The setup

- 161 configurations [Gattringer et al., PRD 79 \(2009\)](#)
- size $16^3 \times 32$
- two degenerate flavors of light fermions, $m_\pi = 322(5)$ MeV
- lattice spacing $a = 0.1440(12)$ fm
- Chirally Improved (CI) Dirac operator [Gattringer, PRD 63 \(2001\)](#)
(approximate solution of the Ginsparg-Wilson equation)
- three different kinds of quark sources: Jacobi smeared narrow (0.27 fm) and wide (0.55 fm) sources and a P wave like derivative source $\rightarrow$ serves a large operator basis for the variational method.

Eigenvalues

 D_5

 D


Mesons

We explore the following isovector mesons which would be related via the chiral symmetry [L.Ya. Glozman, Physics Reports 444 \(2007\)](#)

$U(1)_A$	$SU(2)_L \times SU(2)_R$ (axial)
$\pi \longleftrightarrow a_0$	$\rho \longleftrightarrow a_1$
$\rho \longleftrightarrow b_1$	

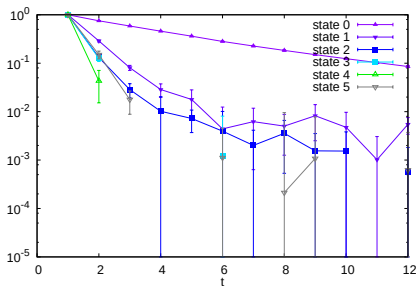
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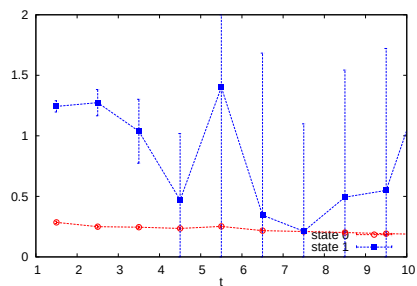
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$\rho \longleftrightarrow b_1$	$\pi \longleftrightarrow f_0$
	$a_0 \longleftrightarrow \eta$

$$\pi, J^{PC} = 0^{-+}, \text{red}(0)$$

all states: correlators

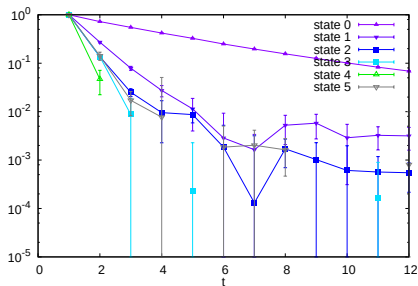


lowest state(s): eff. masses

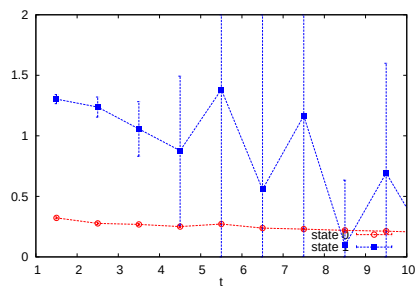


$\pi, J^{PC} = 0^{-+}, \text{red}(2)$

all states: correlators

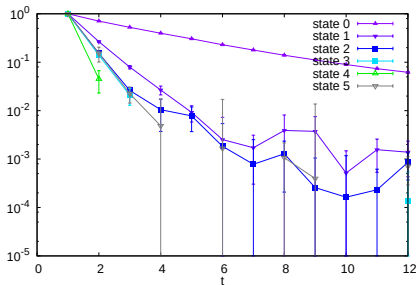


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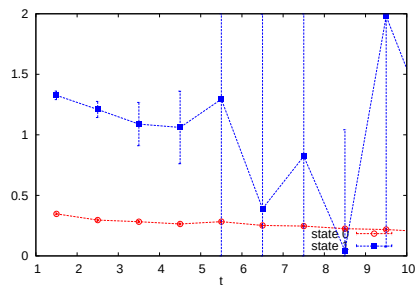


$\pi, J^{PC} = 0^{-+}, \text{red}(4)$

all states: correlators

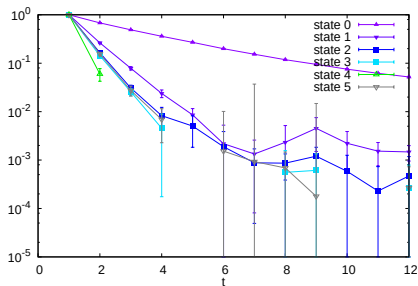


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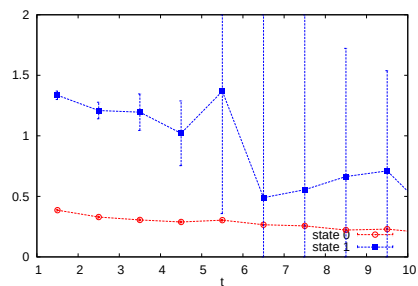


$\pi, J^{PC} = 0^{-+}, \text{red}(8)$

all states: correlators

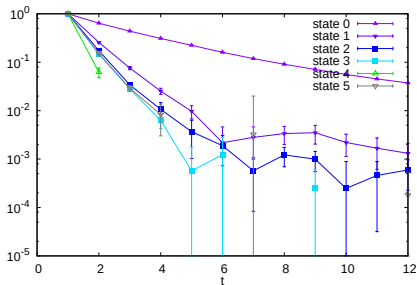


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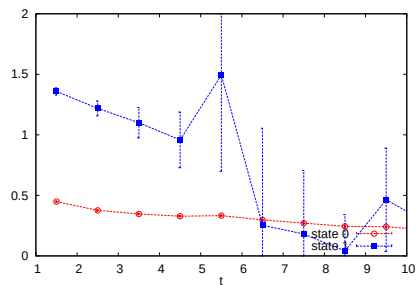


$\pi, J^{PC} = 0^{-+}, \text{red}(16)$

all states: correlators

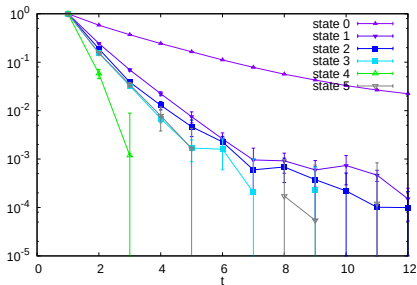


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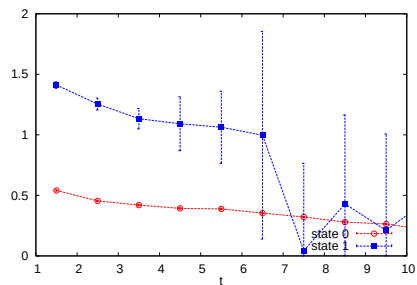


$\pi, J^{PC} = 0^{-+}, \text{red}(32)$

all states: correlators

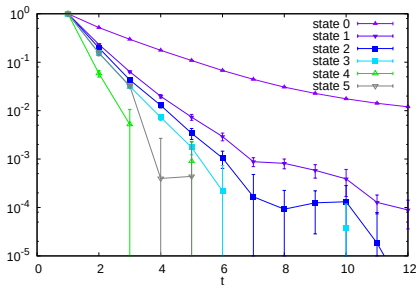


lowest state(s): eff. masses

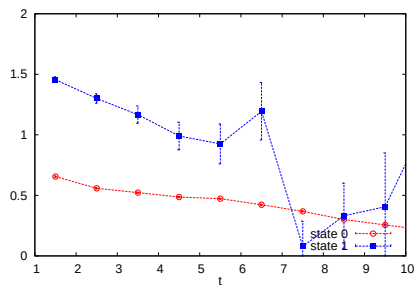


$\pi, J^{PC} = 0^{-+}, \text{red}(64)$

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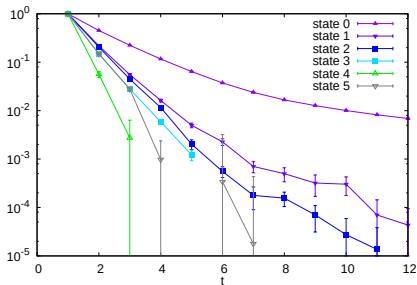


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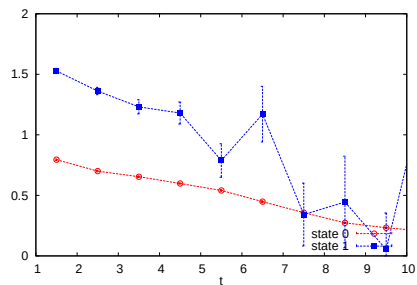


$\pi, J^{PC} = 0^{-+}, \text{red}(128)$

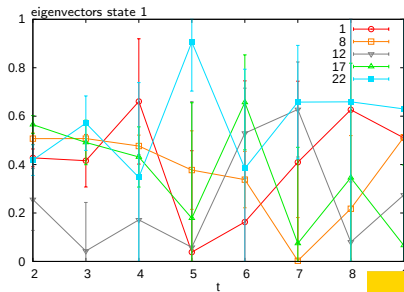
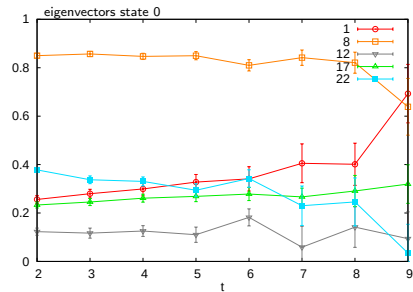
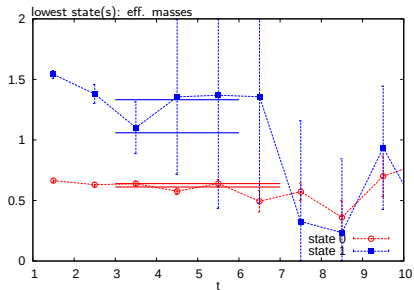
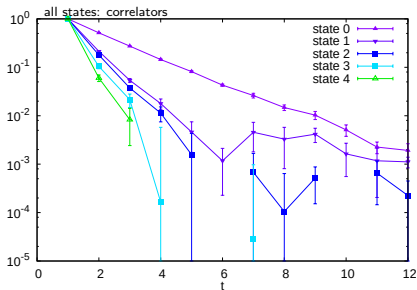
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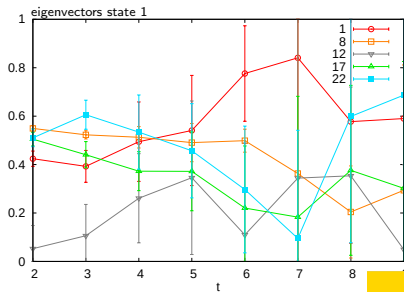
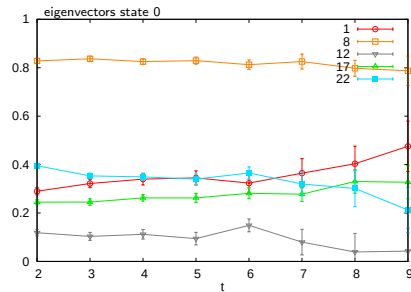
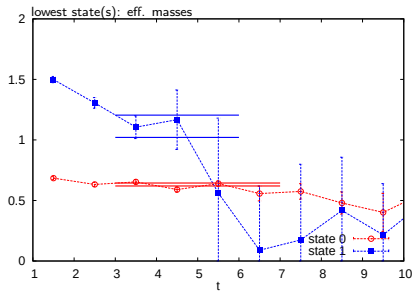
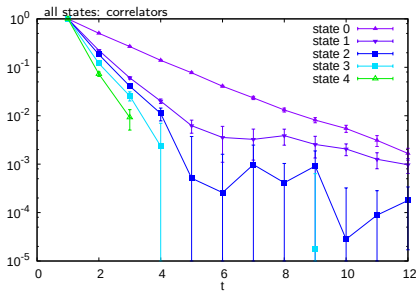


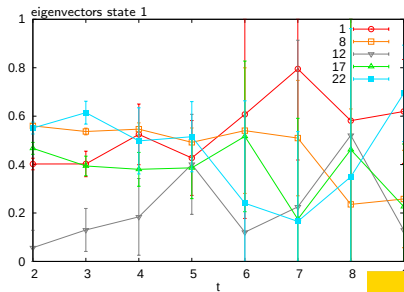
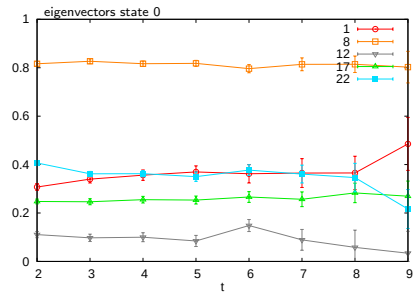
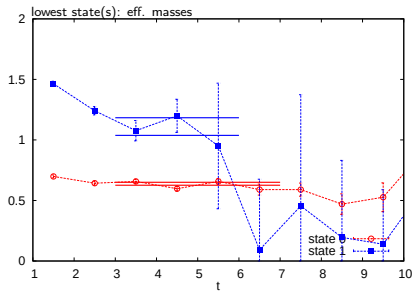
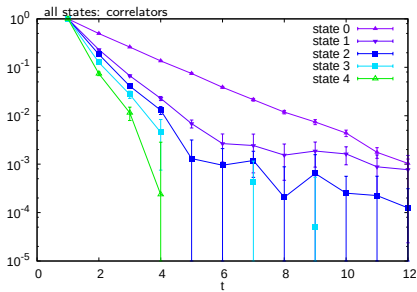
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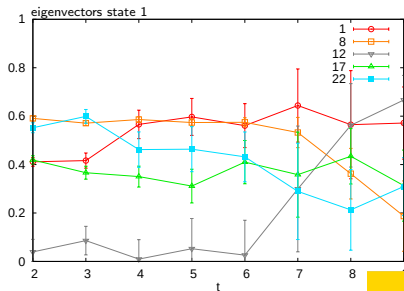
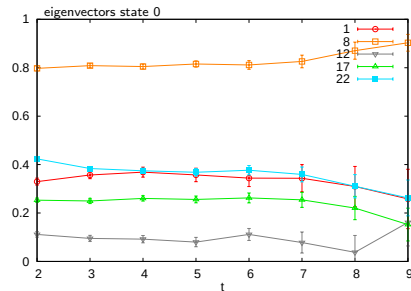
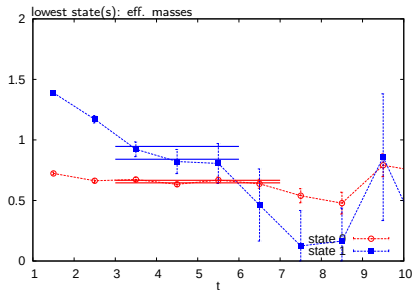
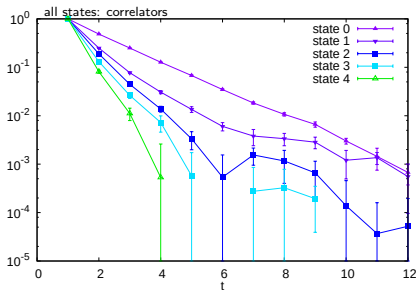
ρ , $J^{PC} = 1^{--}$, red(0)

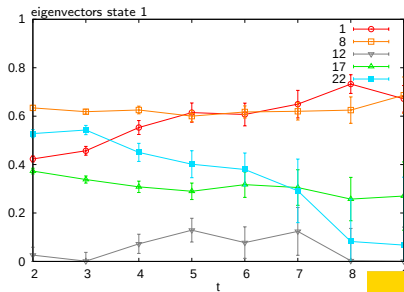
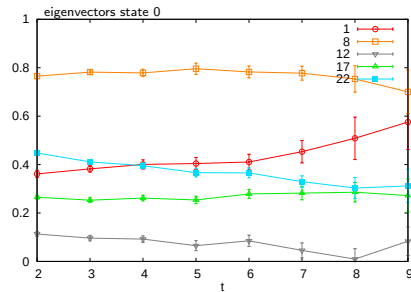
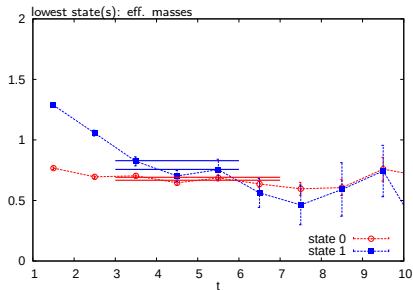
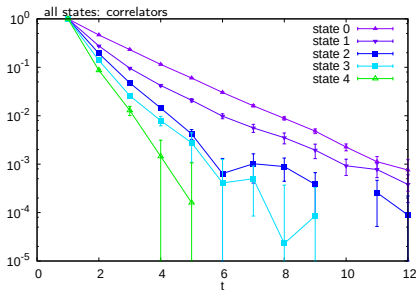


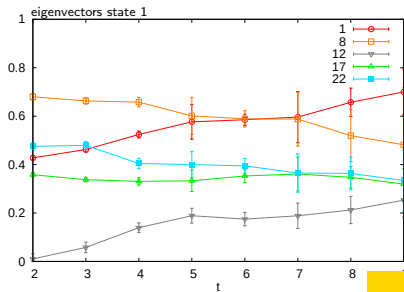
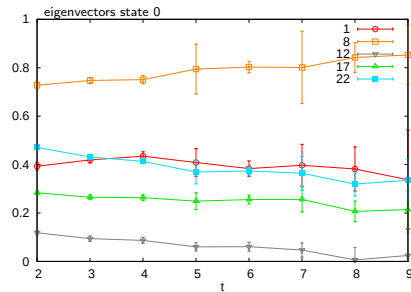
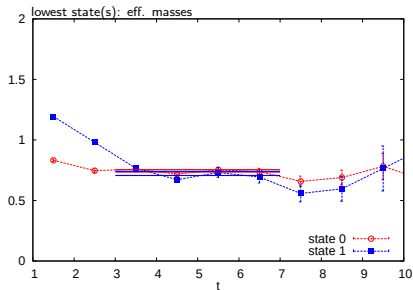
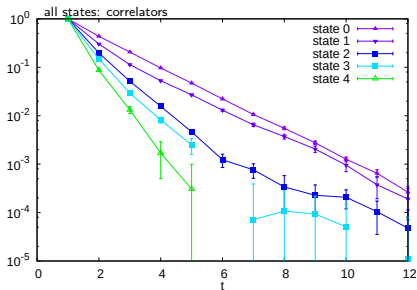
$\rho, J^{PC} = 1^{--}, \text{red}(2)$ 

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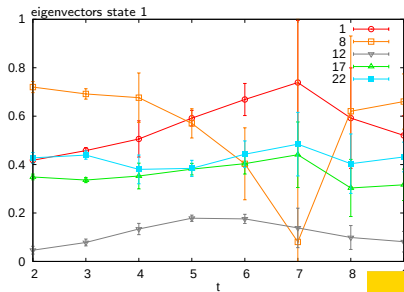
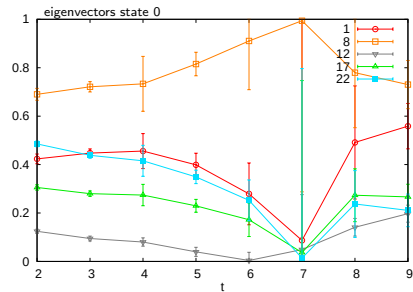
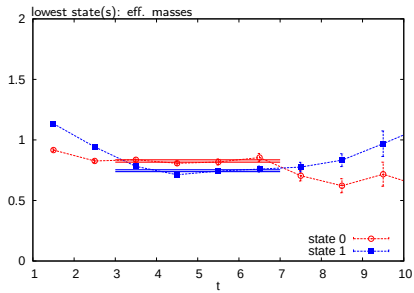
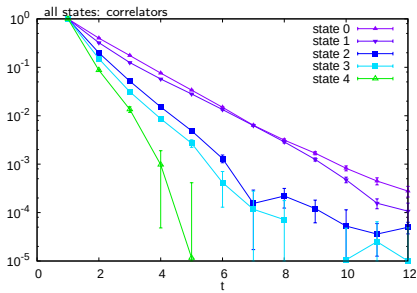
ρ , $J^{PC} = 1^{--}$, red(8)

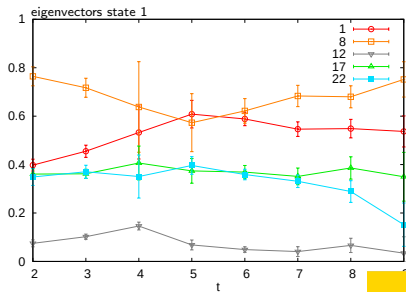
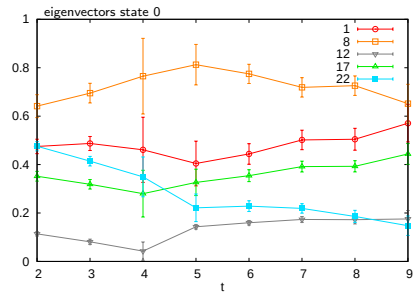
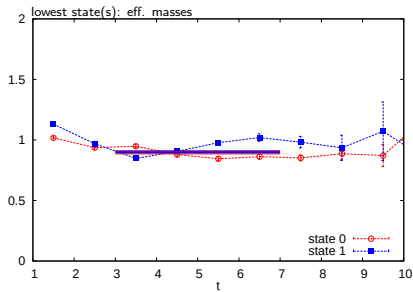
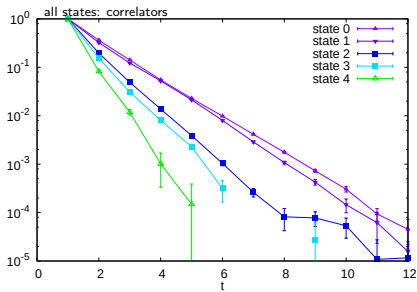


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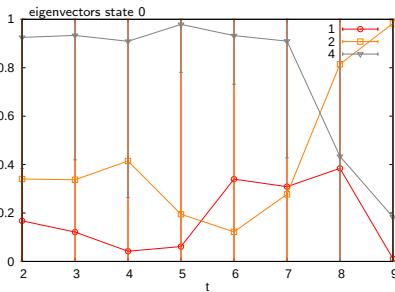
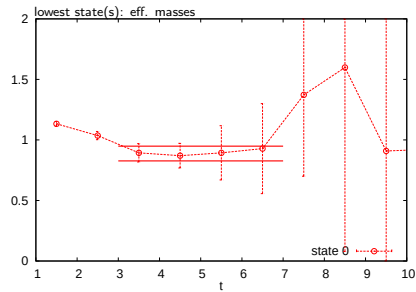
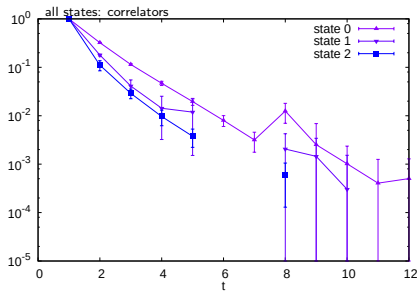
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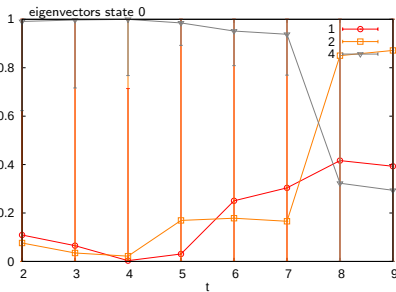
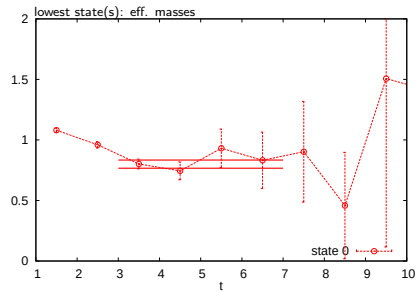
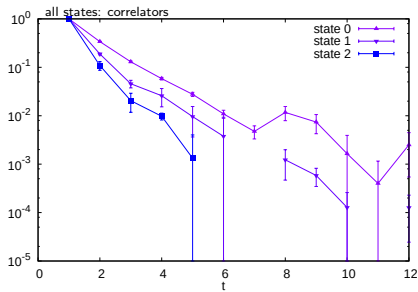


$\rho, J^{PC} = 1^{--}, \text{red}(128)$ 

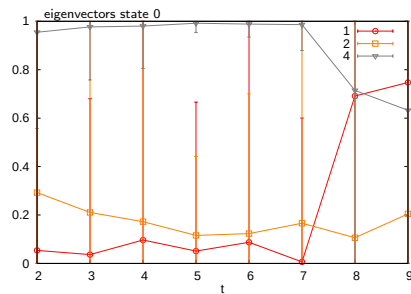
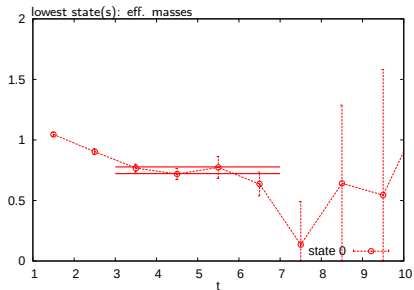
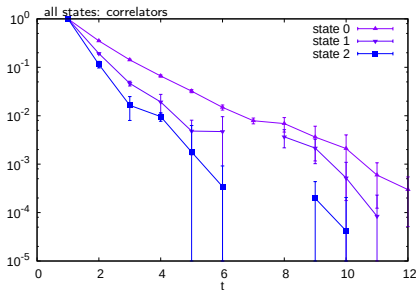
$$a_1, J^{PC} = 1^{++}, \text{red}(0)$$



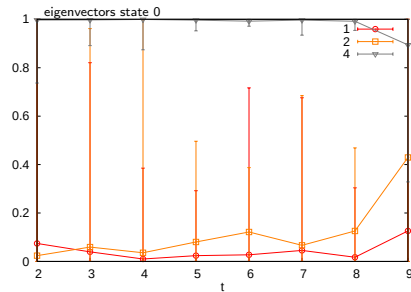
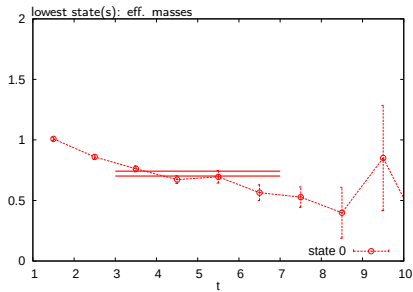
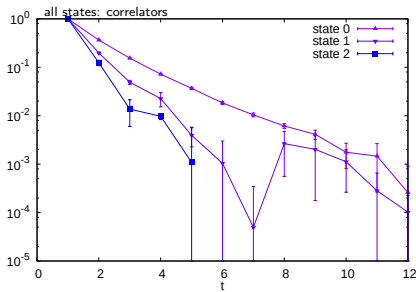
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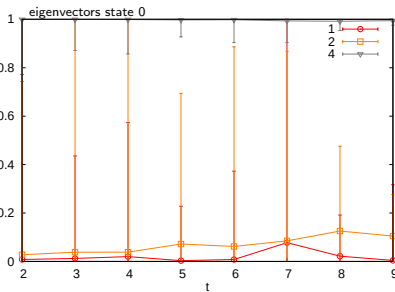
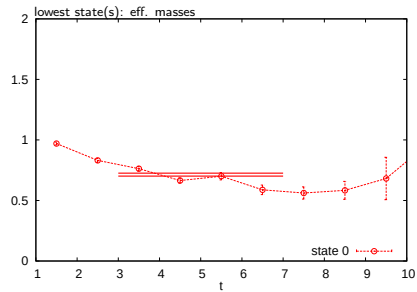
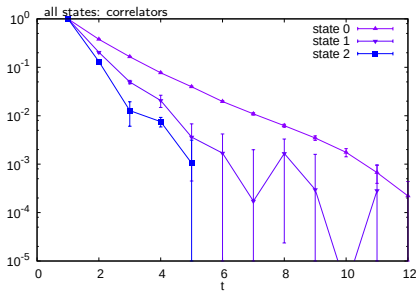
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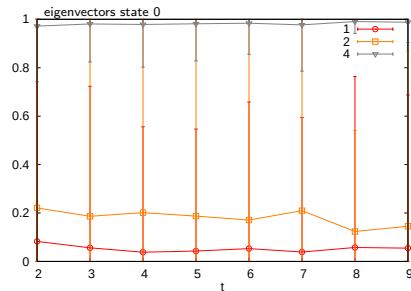
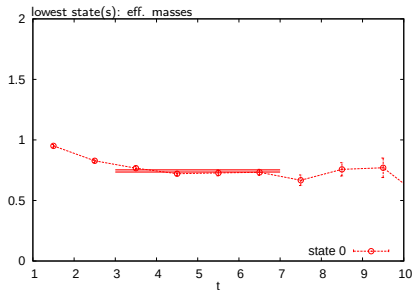
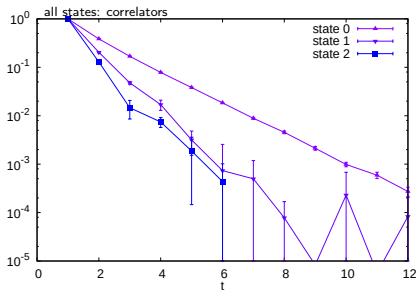
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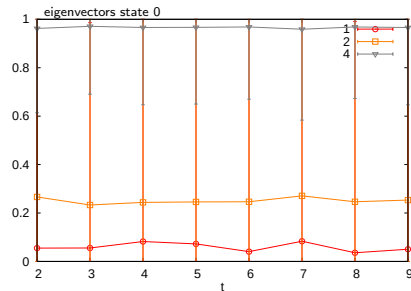
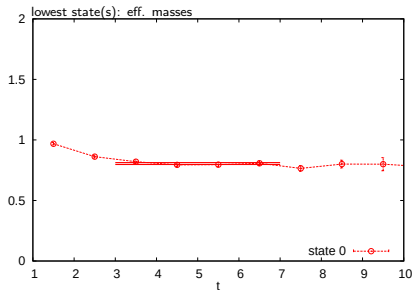
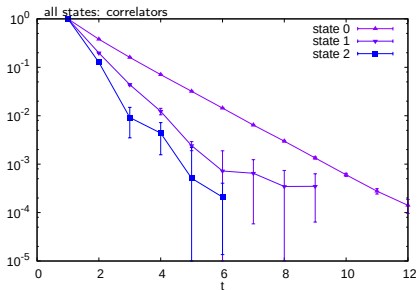
$$a_1, J^{PC} = 1^{++}, \text{red}(16)$$



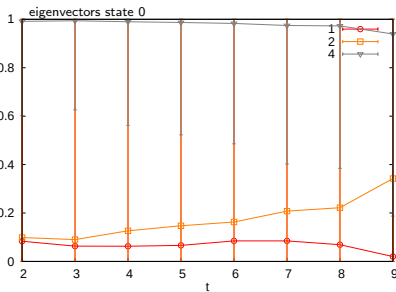
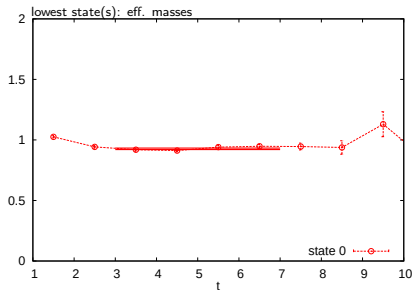
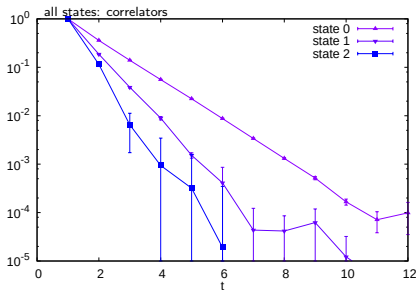
$a_1, J^{PC} = 1^{++}, \text{red}(32)$



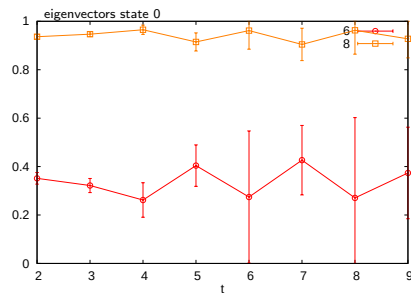
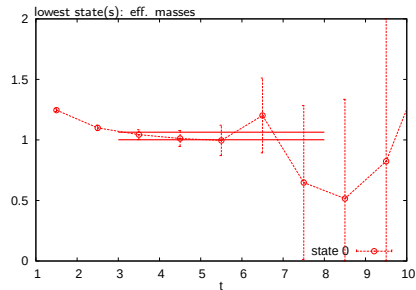
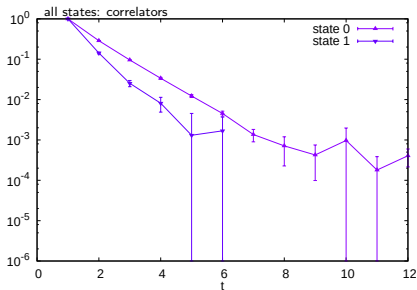
$a_1, J^{PC} = 1^{++}, \text{red}(64)$



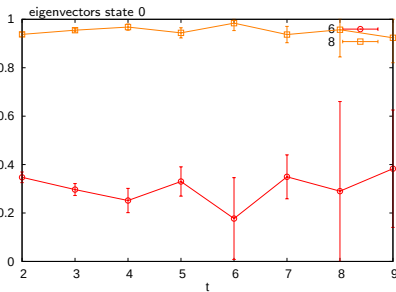
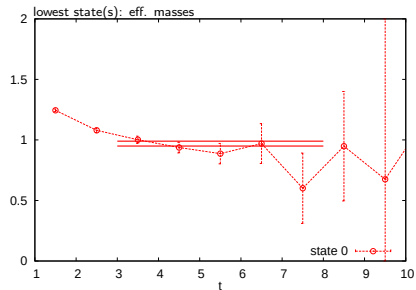
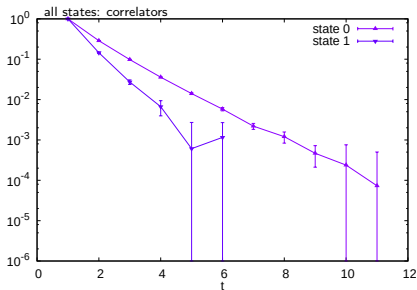
$a_1, J^{PC} = 1^{++}, \text{red}(128)$



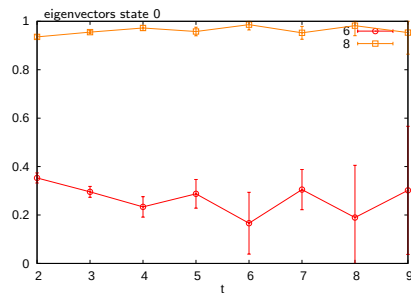
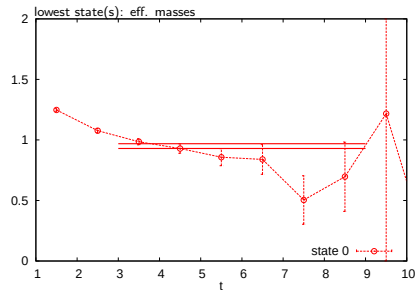
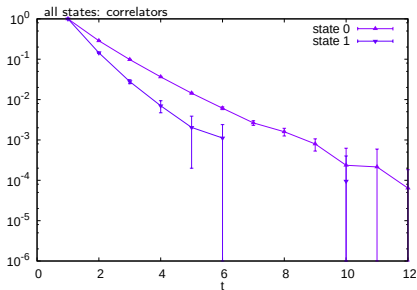
$$b_1, J^{PC} = 1^{+-}, \text{red}(0)$$



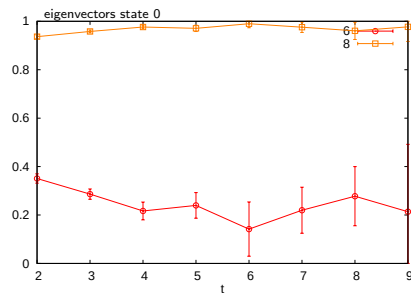
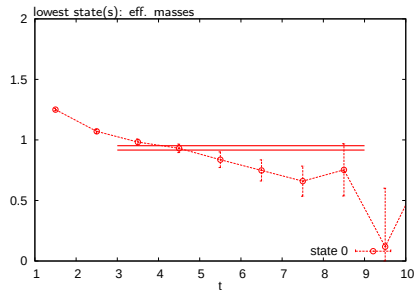
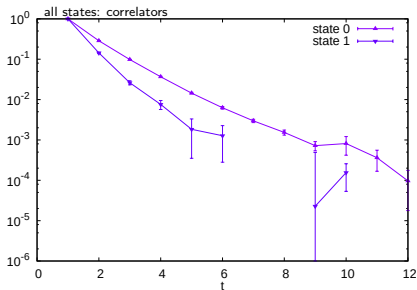
$$b_1, J^{PC} = 1^{+-}, \text{red}(2)$$



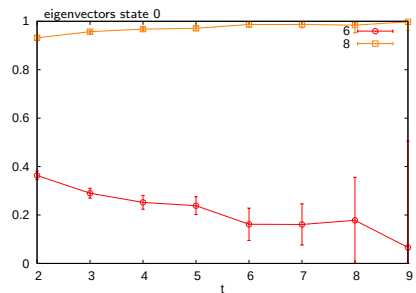
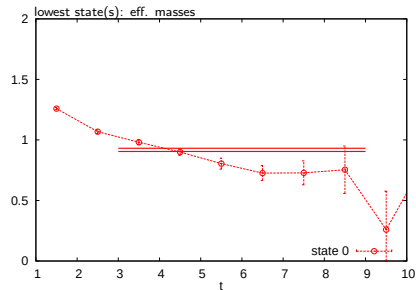
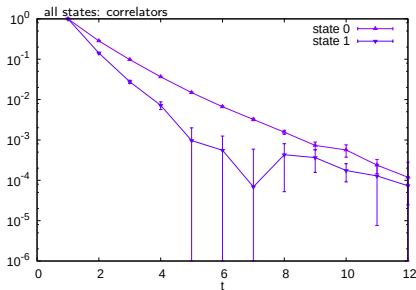
$$b_1, J^{PC} = 1^{+-}, \text{red}(4)$$



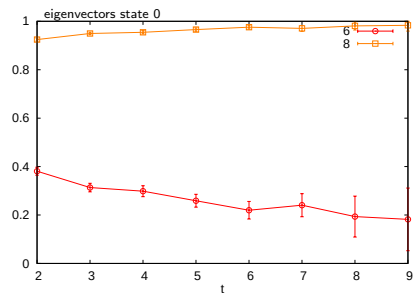
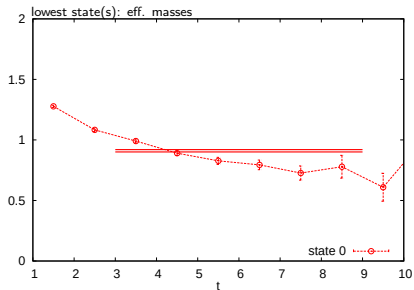
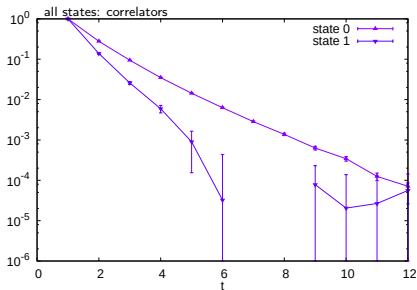
$$b_1, J^{PC} = 1^{+-}, \text{red}(8)$$



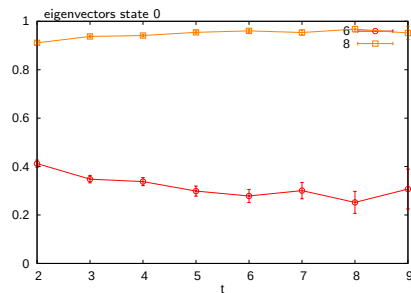
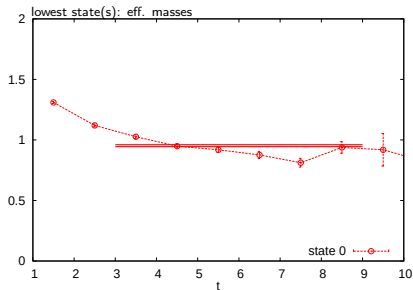
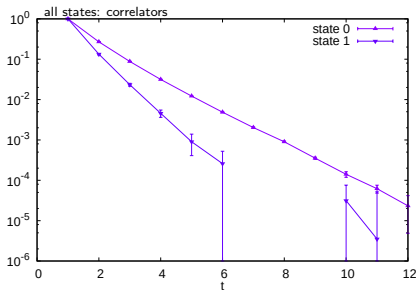
$$b_1, J^{PC} = 1^{+-}, \text{red}(16)$$



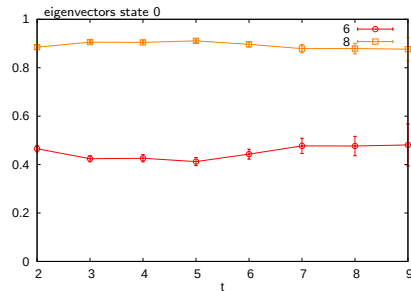
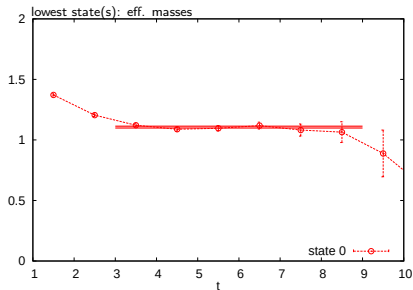
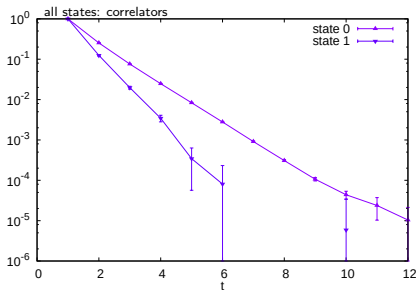
$$b_1, J^{PC} = 1^{+-}, \text{red}(32)$$



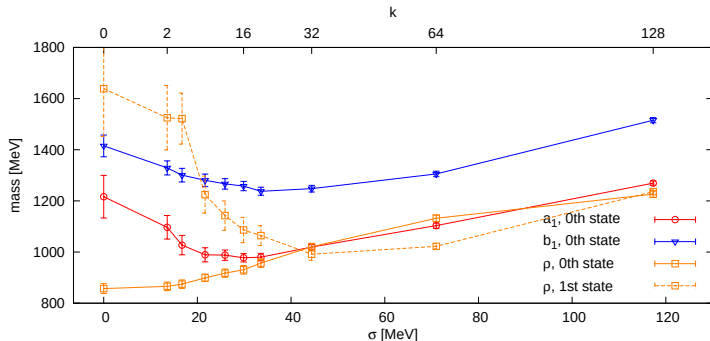
$$b_1, J^{PC} = 1^{+-}, \text{red}(64)$$



$b_1, J^{PC} = 1^{+-}, \text{red}(128)$



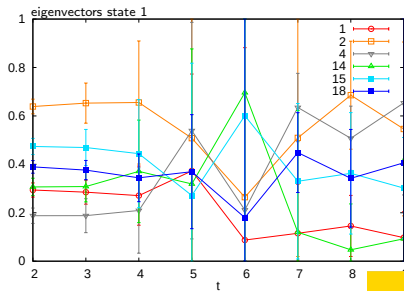
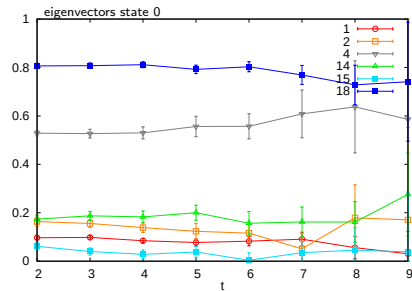
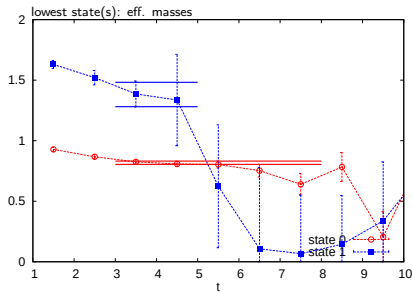
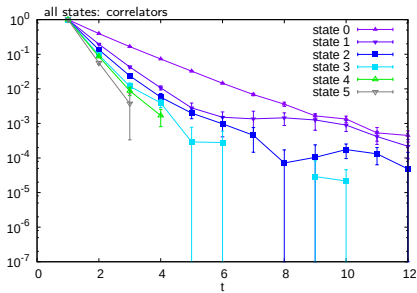
Meson masses vs. Dirac eigenmode reduction level

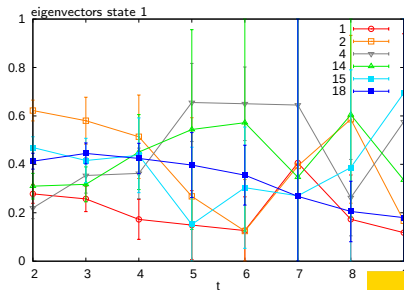
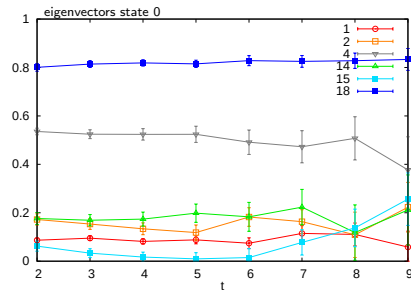
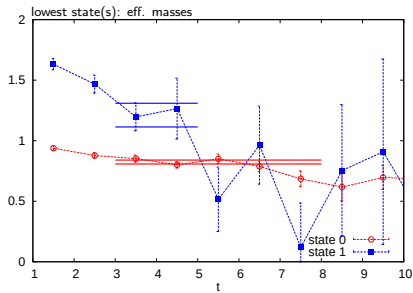
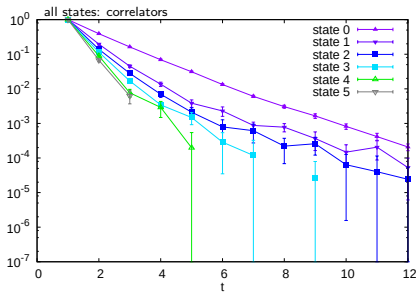


Baryons

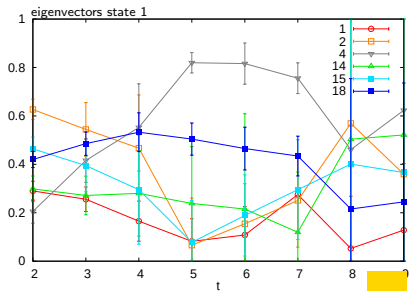
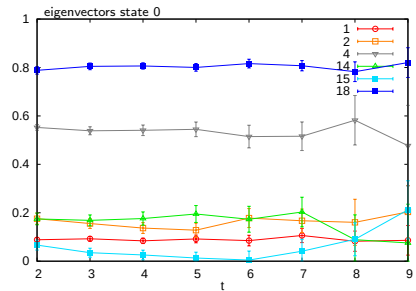
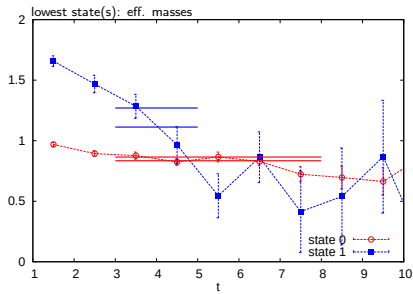
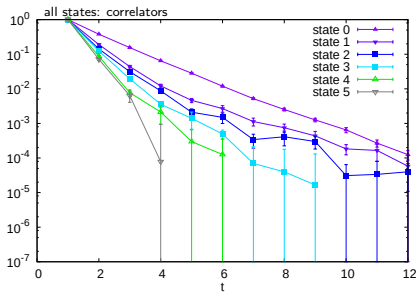
We investigate the nucleon and the Δ both with negative and positive parity

- in the constituent quark model, the splitting between the Δ and the nucleon is thought to be (at least partly) due to a Goldstone boson exchange interaction: what happens to the splitting when we restore the chiral symmetry?
- do the masses of the nucleon and the $N(1535)$ meet?

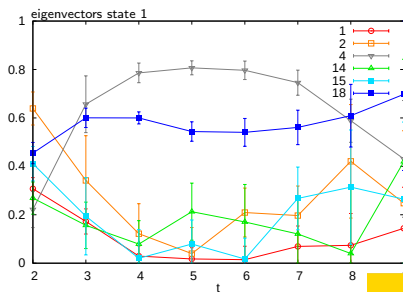
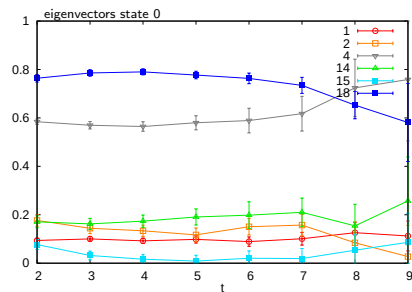
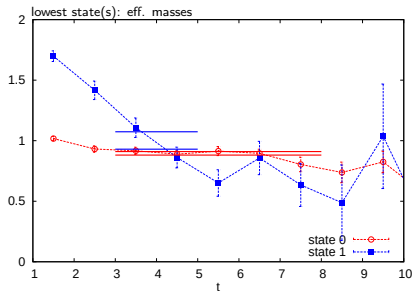
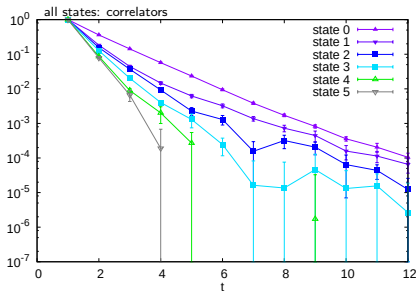
$N(+)$, red(0)

$N(+)$, red(2)

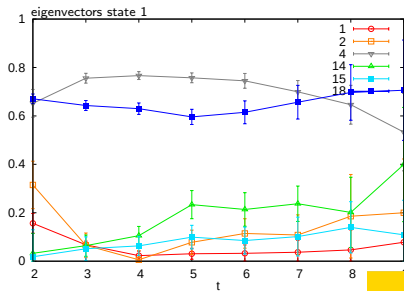
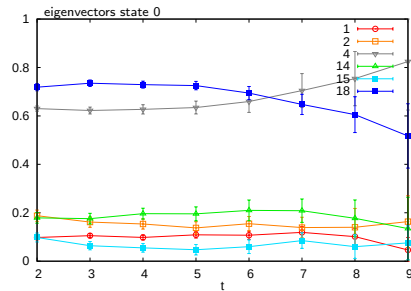
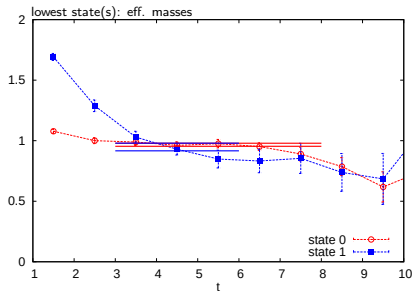
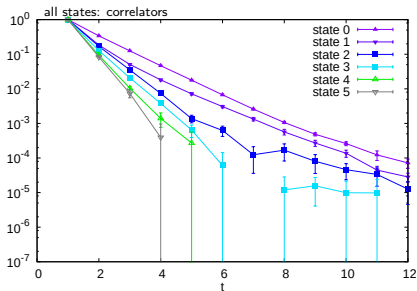
$N(+)$, red(4)

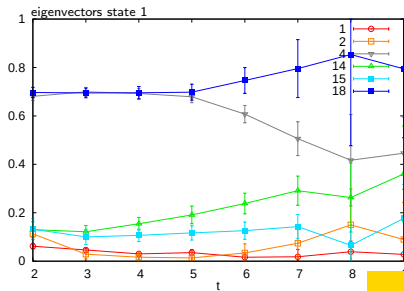
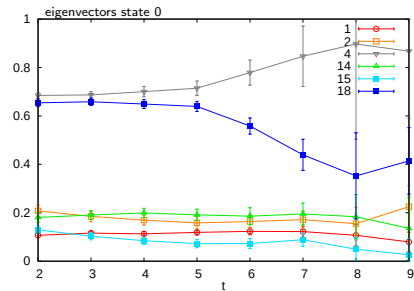
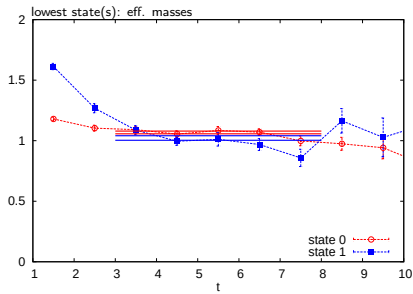
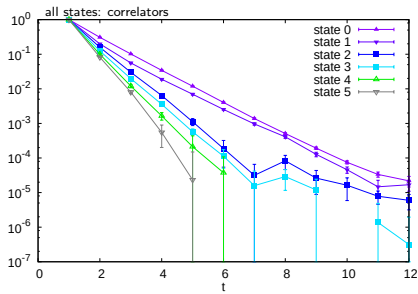


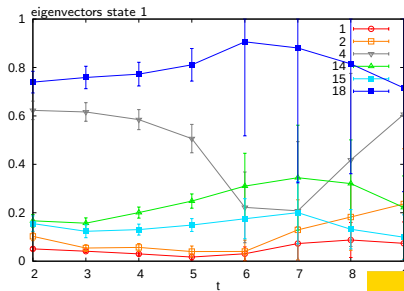
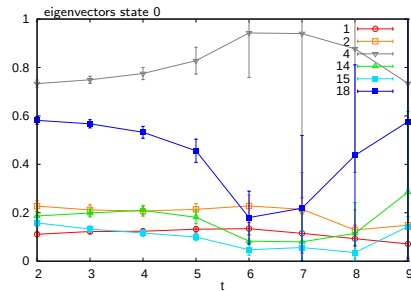
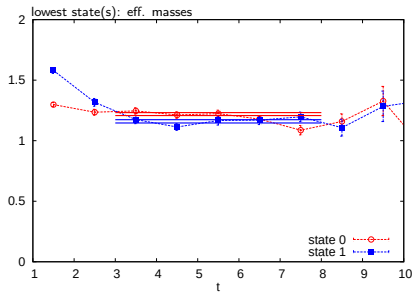
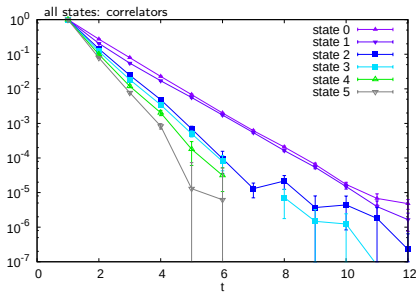
$N(+)$, red(8)

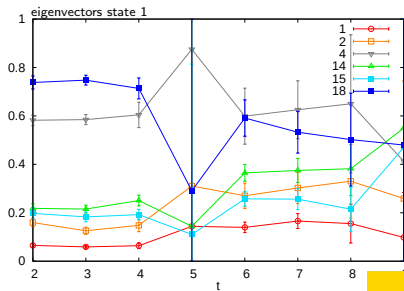
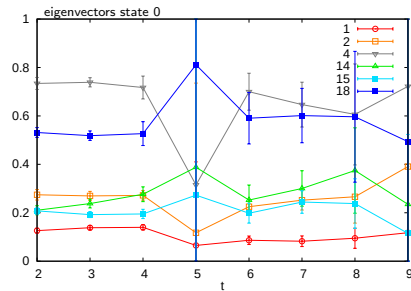
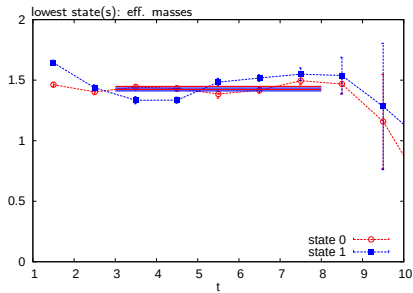
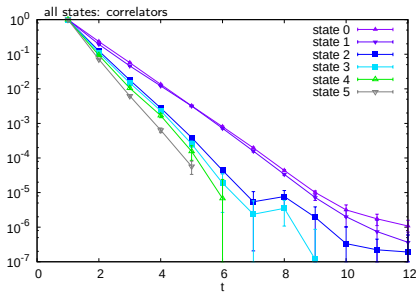


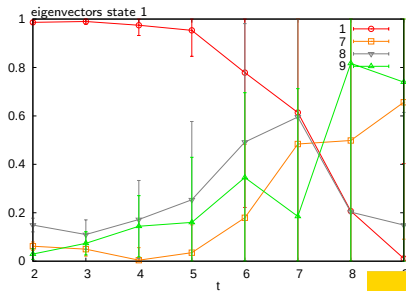
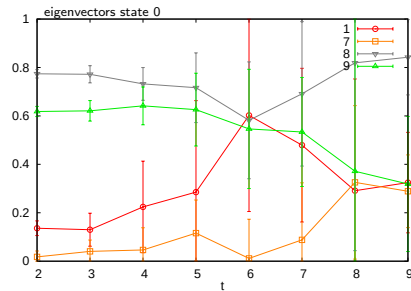
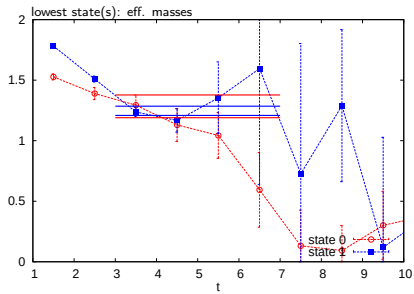
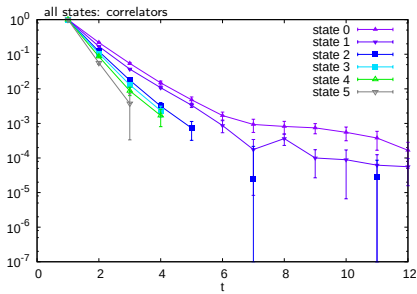
$N(+)$, red(16)

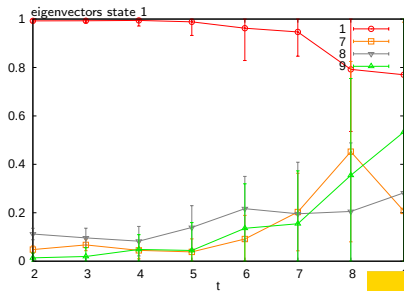
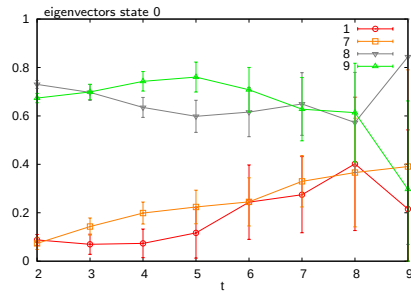
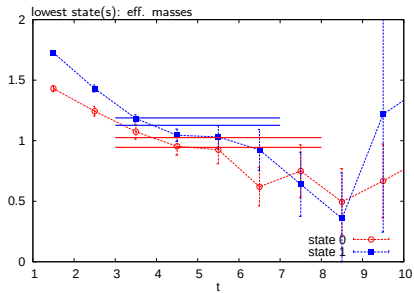
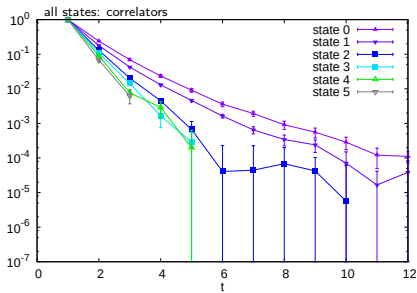


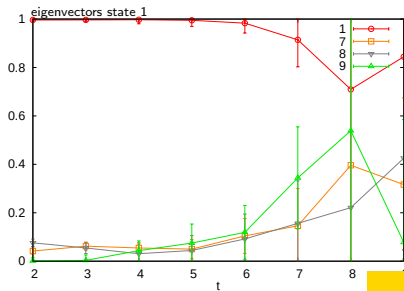
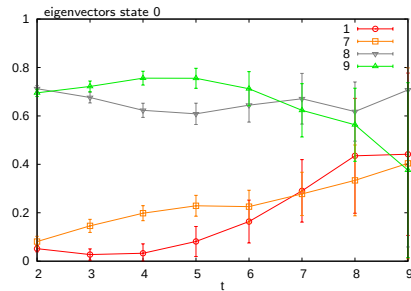
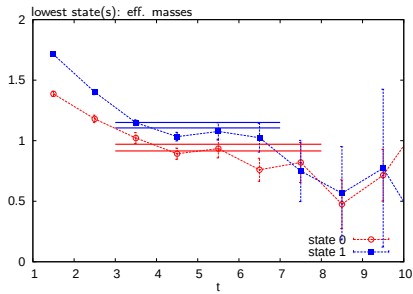
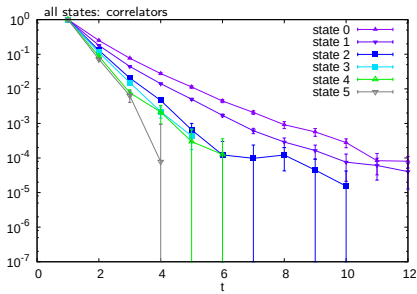
$N(+)$, red(32)

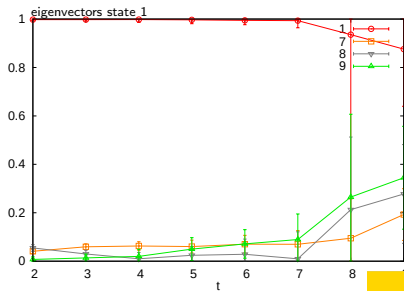
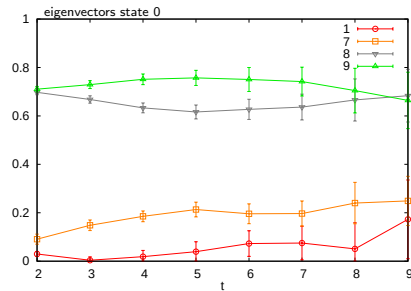
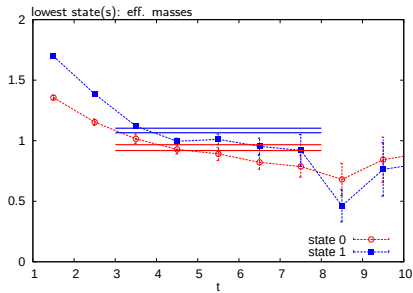
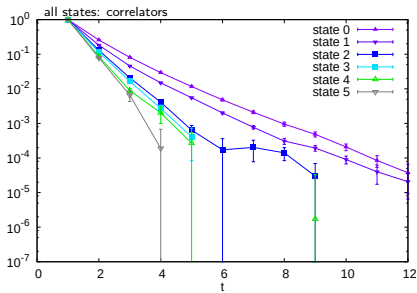
$N(+)$, red(64)

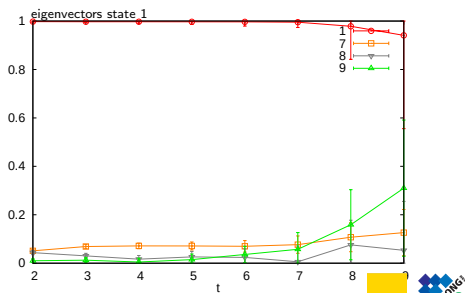
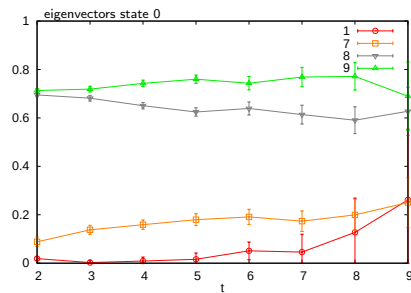
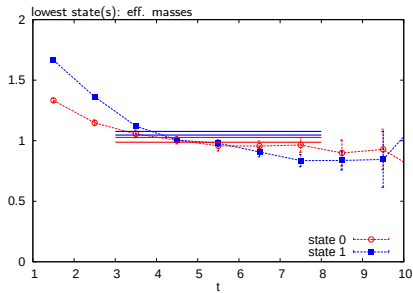
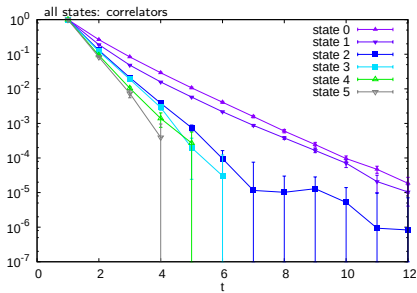
$N(+)$, red(128)

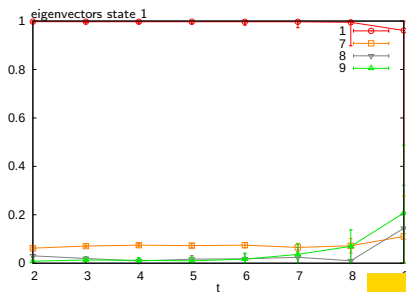
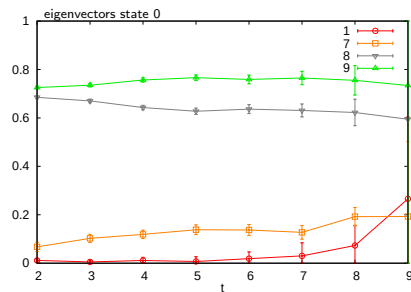
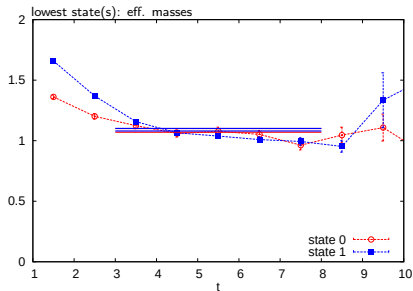
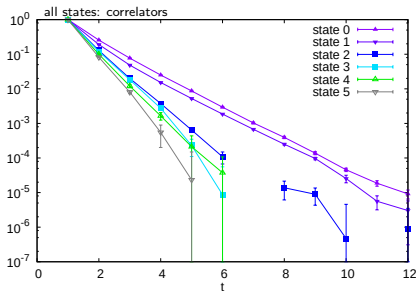
$N(-)$, red(0)

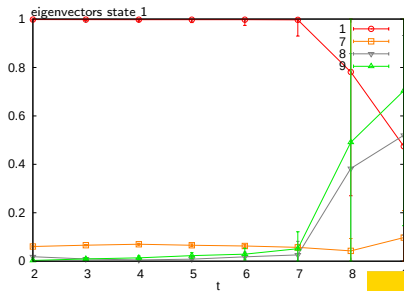
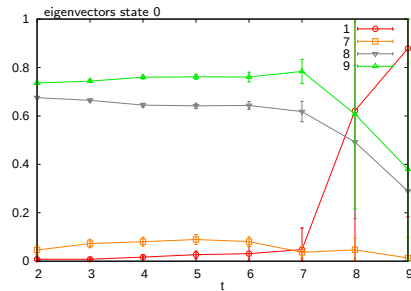
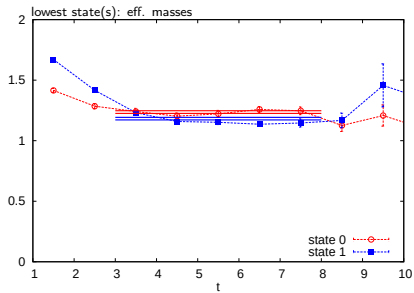
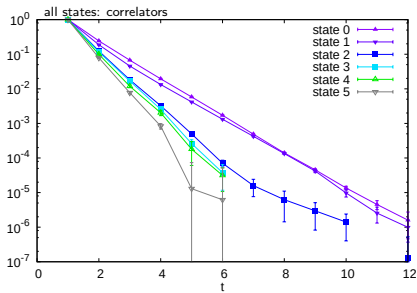
$N(-)$, red(2)

$N(-)$, red(4)

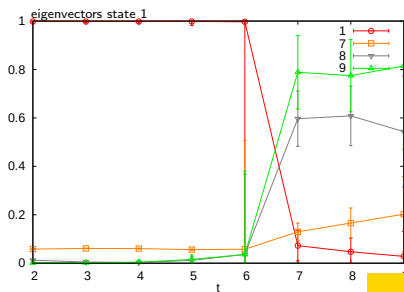
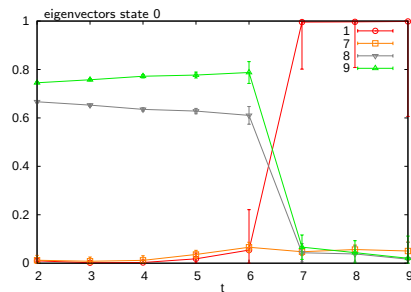
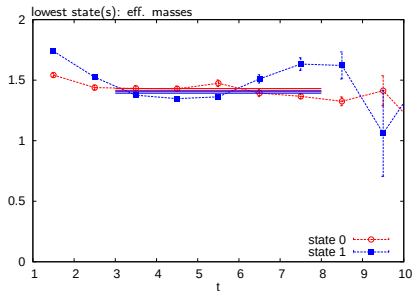
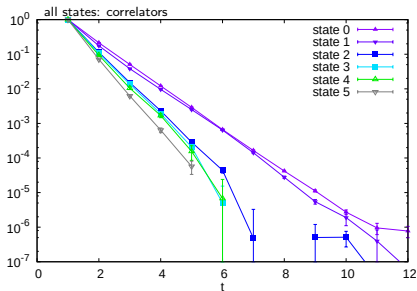
$N(-)$, red(8)

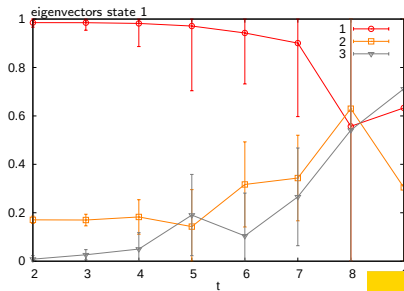
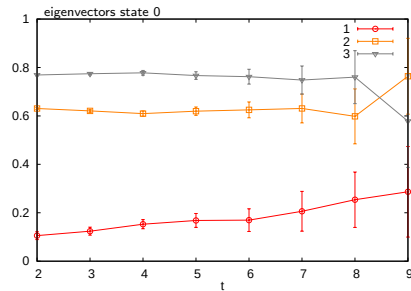
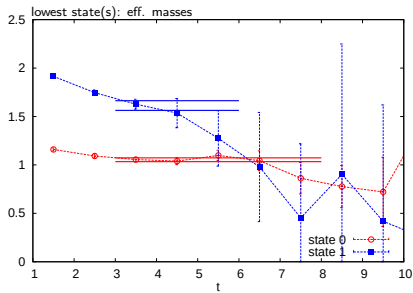
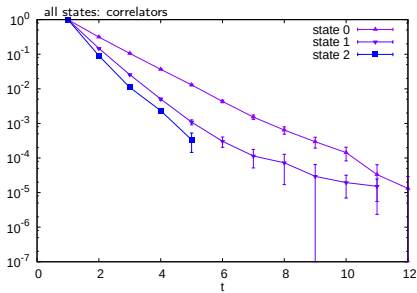
$N(-)$, red(16)

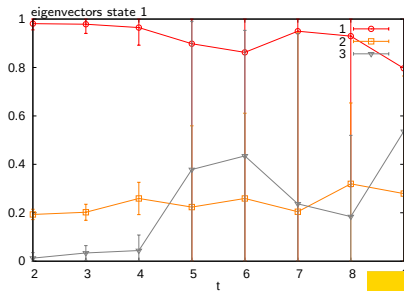
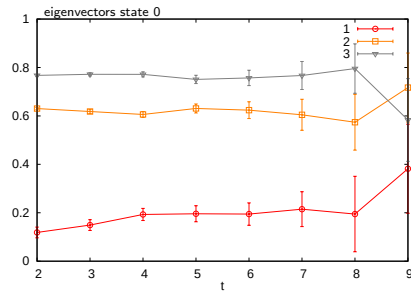
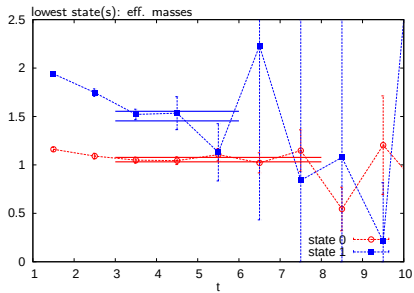
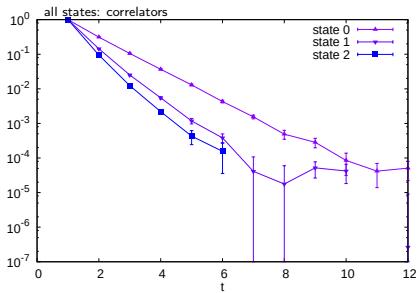
$N(-)$, red(32)

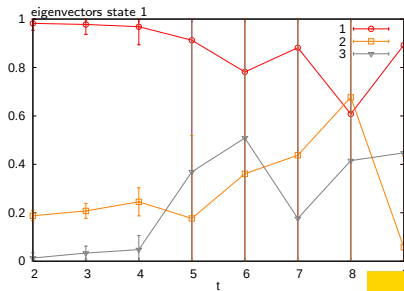
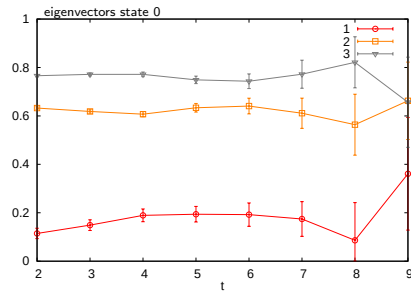
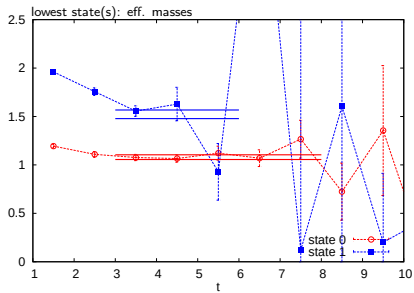
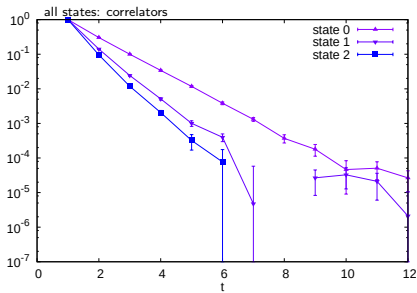
$N(-)$, red(64)

$N(-)$, red(128)

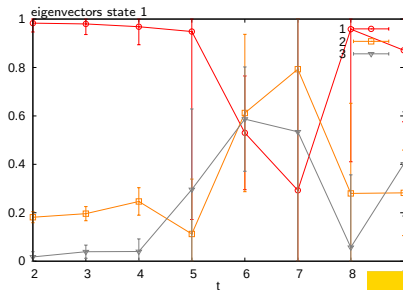
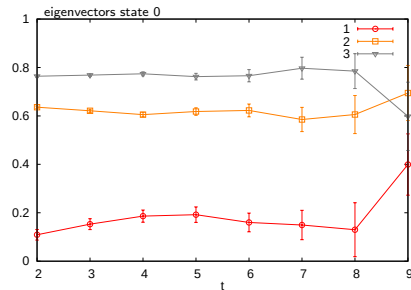
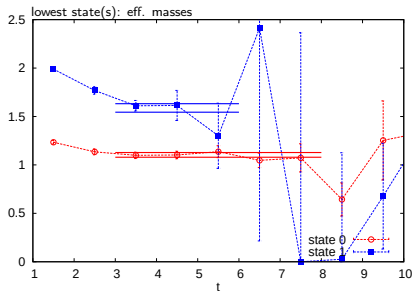
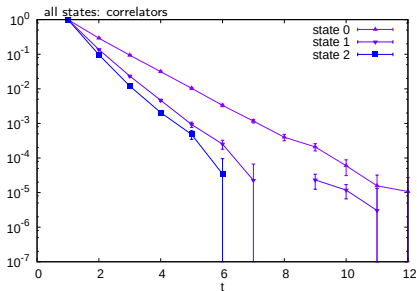


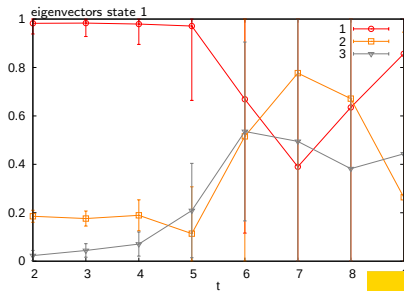
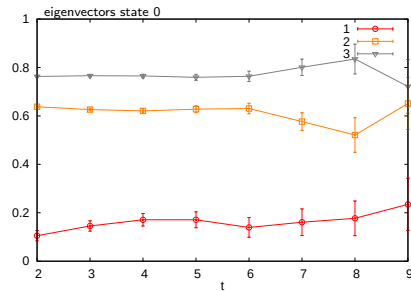
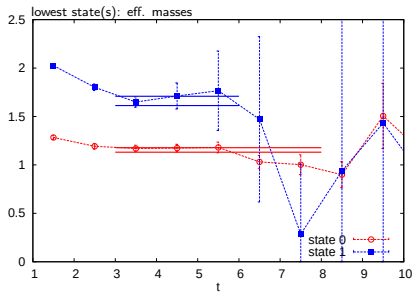
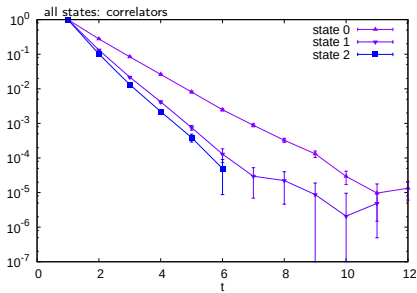
$\Delta(+)$, red(0)

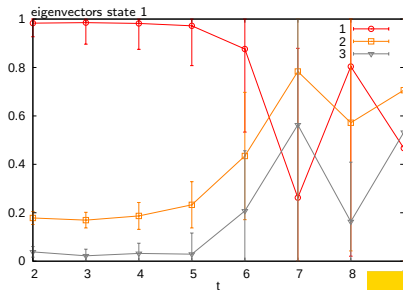
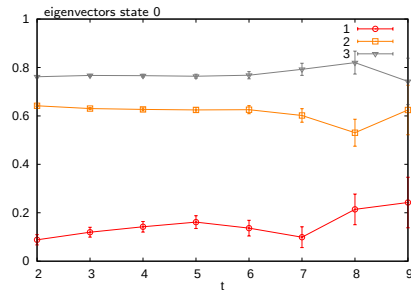
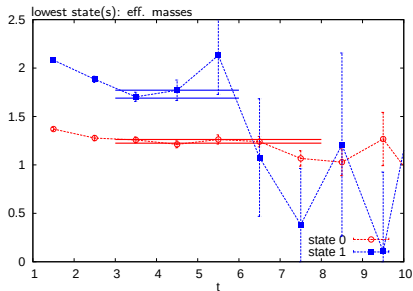
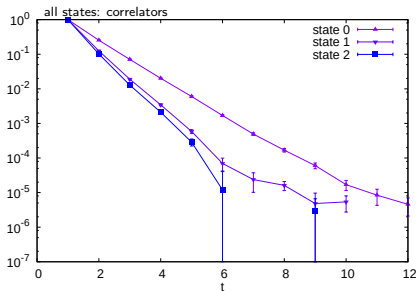
$\Delta(+)$, red(2)

$\Delta(+)$, red(4)

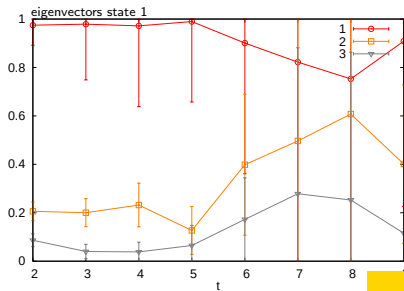
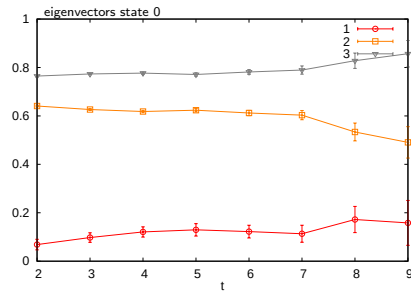
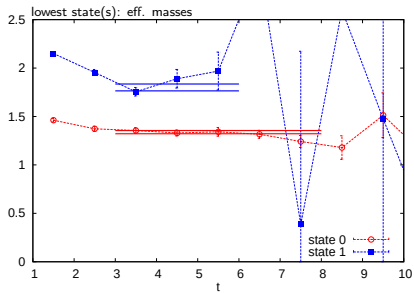
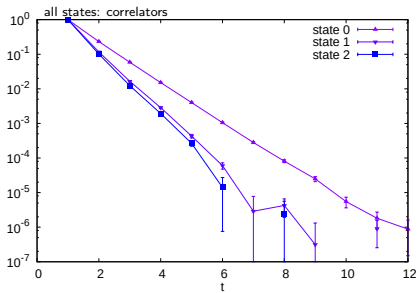
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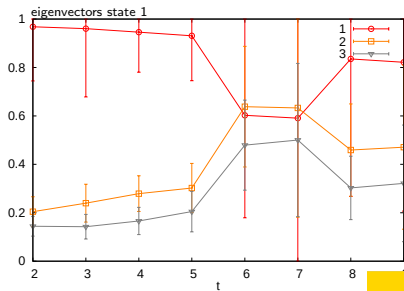
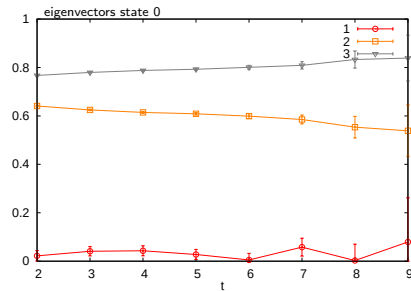
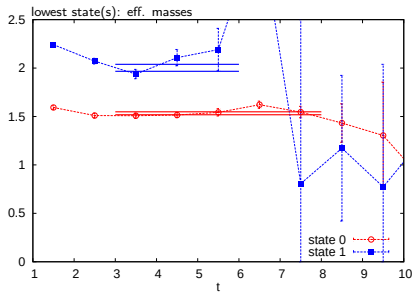
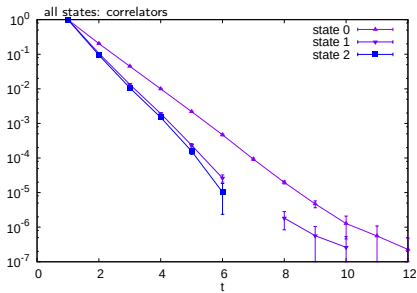


$\Delta(+)$, red(16)

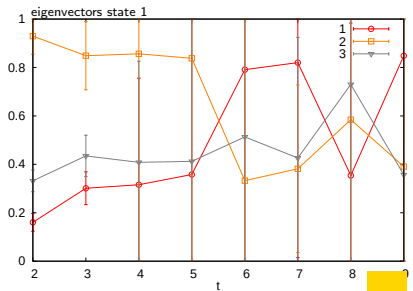
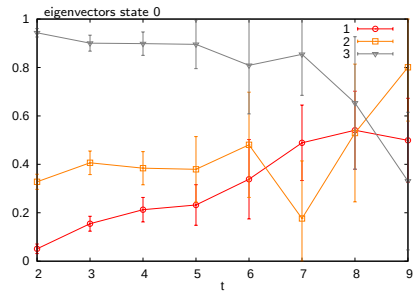
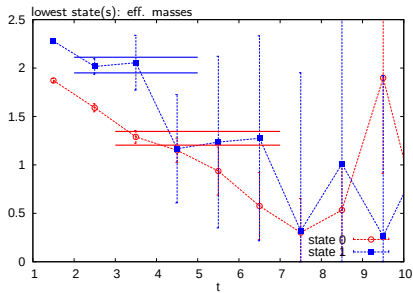
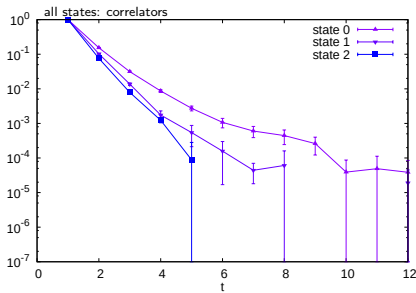
$\Delta(+)$, red(32)

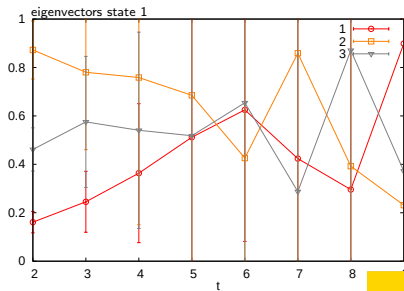
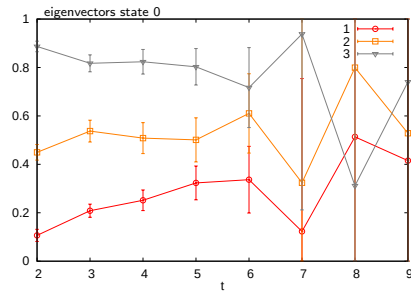
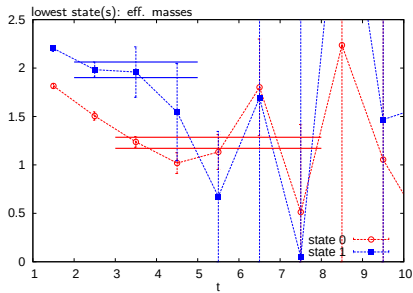
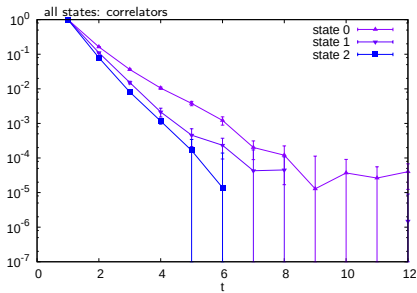
$\Delta(+)$, red(64)

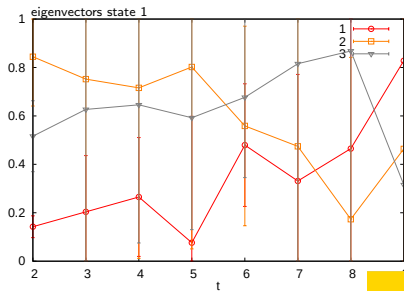
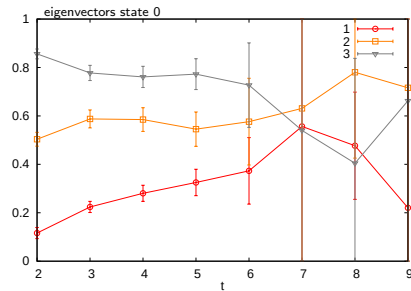
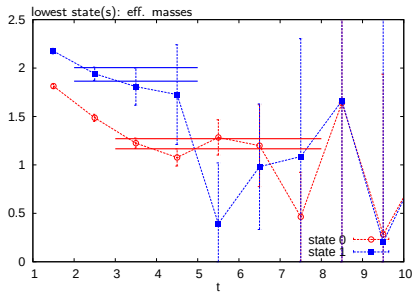
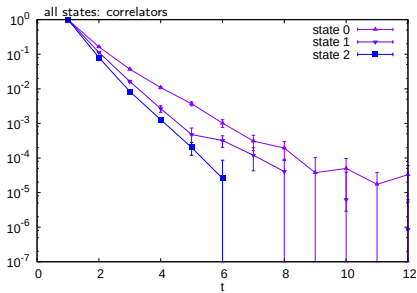


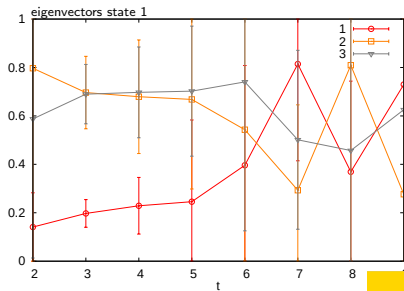
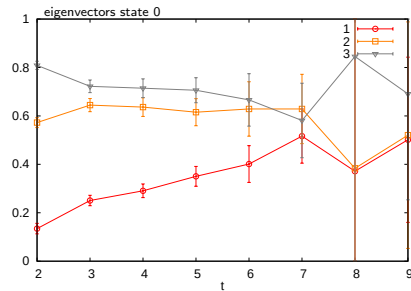
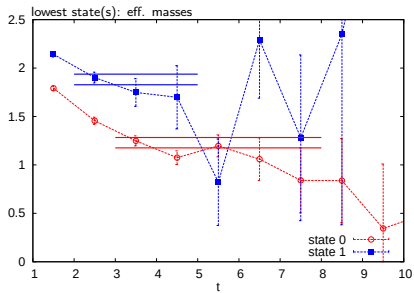
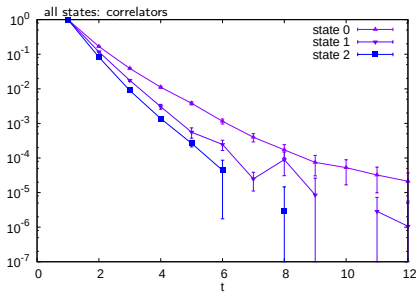
$\Delta(+)$, red(128)

$\Delta(-)$, red(0)

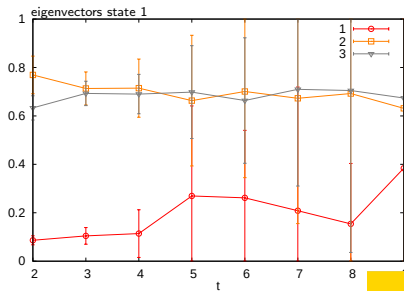
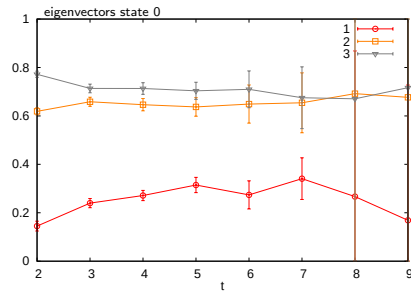
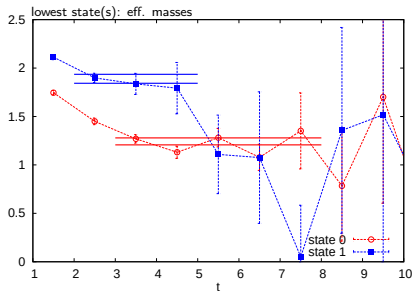
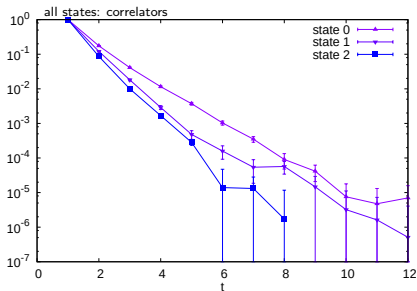


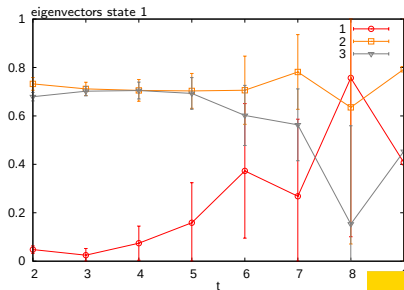
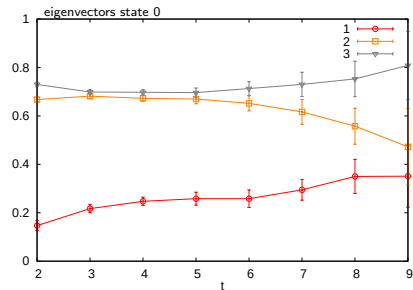
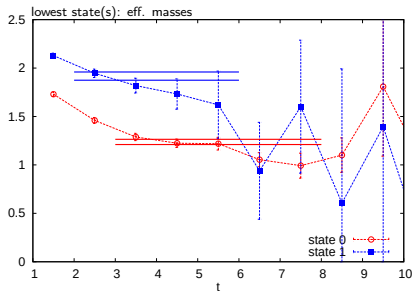
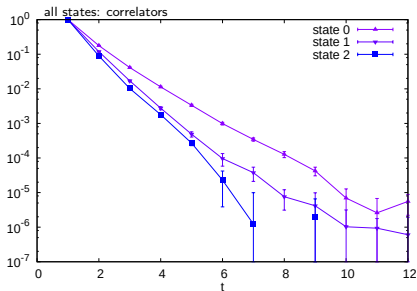
$\Delta(-)$, red(2)

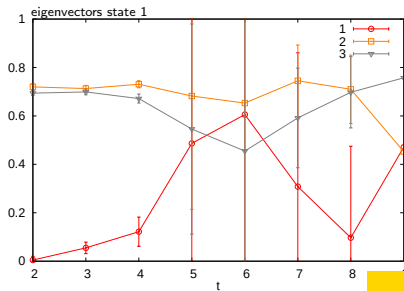
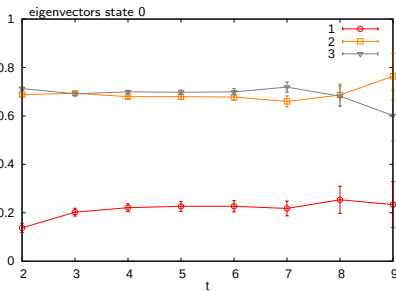
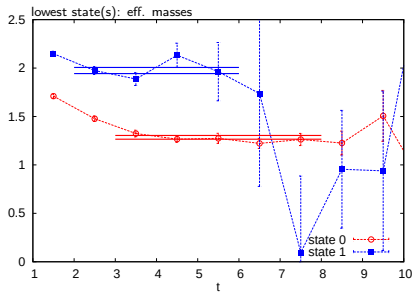
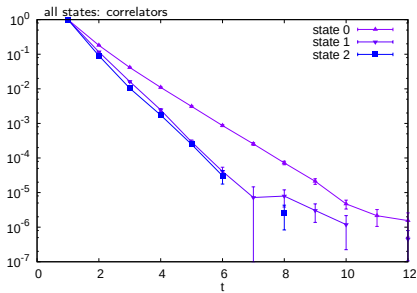
$\Delta(-)$, red(4)

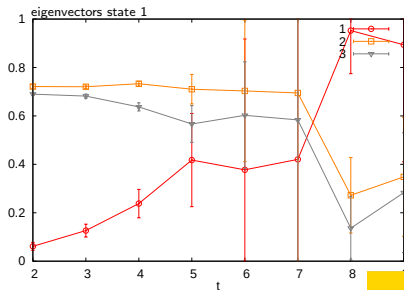
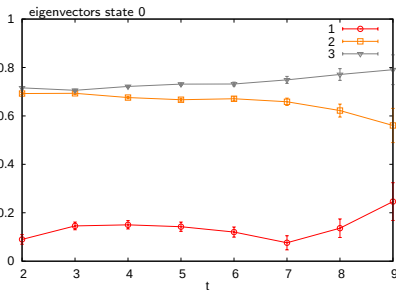
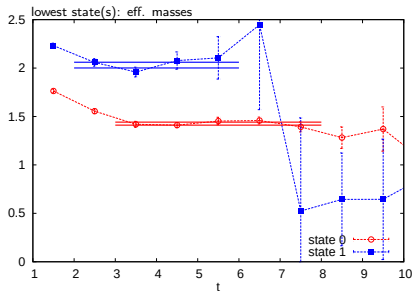
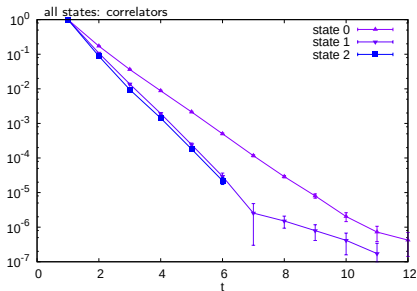
$\Delta(-)$, red(8)

$\Delta(-)$, red(16)

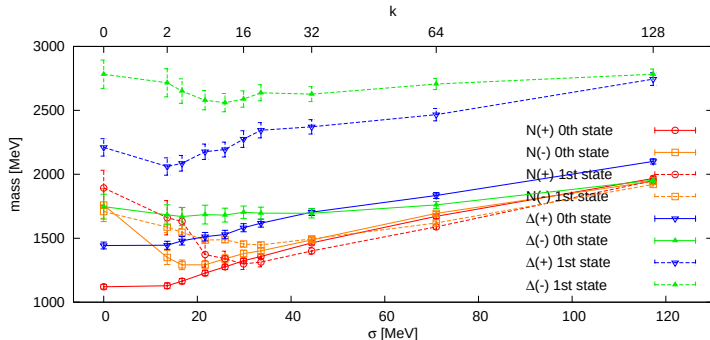


$\Delta(-)$, red(32)

$\Delta(-)$, red(64)

$\Delta(-)$, red(128)

Baryon masses vs. Dirac eigenmode reduction level



The lattice quark propagator

The tree-level quark propagator is

$$S_0(p) = \frac{1}{i\not{p} + m_0}$$

The lattice quark propagator

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$$S_0(p) = \frac{1}{i\not{p} + m_0}$$

$$S_0(p) \rightarrow S_{\text{bare}}(a; p) = Z_2(\mu; a)S(\mu; p)$$

The lattice quark propagator

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the renormalized quark propagator

$$S(\mu; p) = \frac{1}{i\not{p}A(\mu; p^2) + B(\mu; p^2)} = \frac{Z(\mu; p^2)}{i\not{p} + M(p^2)}.$$

The lattice quark propagator

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$$S_0(p) \rightarrow S_{\text{bare}}(a; p) = Z_2(\mu; a) S(\mu; p)$$

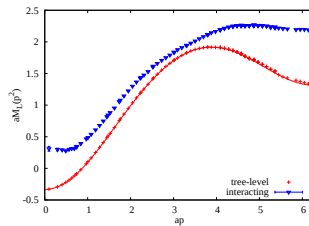
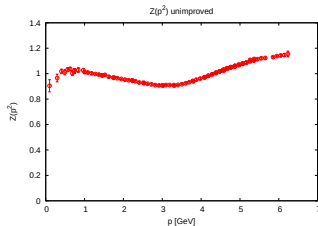
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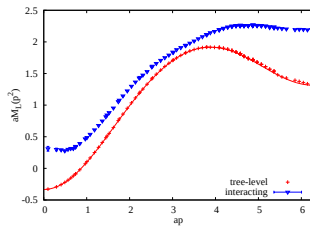
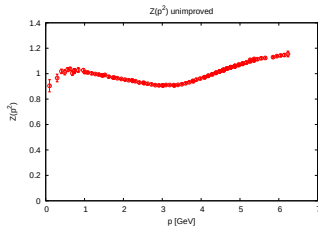
We calculate $S_{\text{bare}}(a; p)$ in Landau gauge on the lattice and therefrom extract

- the renormalization function $Z(\mu; p^2)$
- the renormalization point independent mass function $M(p^2)$

Improving the lattice quark propagator



Improving the lattice quark propagator



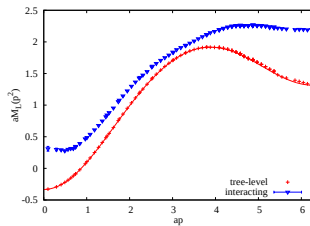
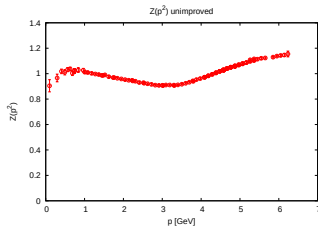
To improve the functions we perform

- tree-level improvement to reduce $\mathcal{O}(a)$ errors of off-shell quantities

Skullerud, Williams (2001) hep-lat/0007028, Heatlie et al. (1991)

$$S_I(x, y) = \langle S_I(x, y; U) \rangle \equiv \langle (1 + b_q am) S(x, y; U) - a\lambda \delta(x - y) \rangle$$

Improving the lattice quark propagator



To improve the functions we perform

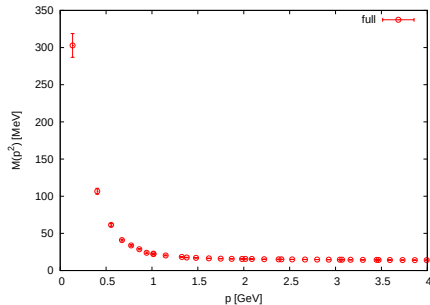
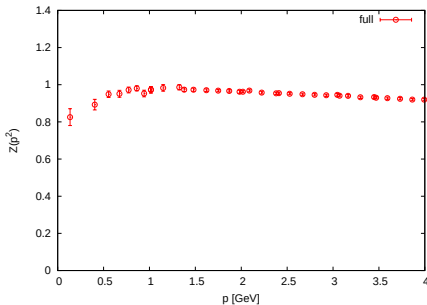
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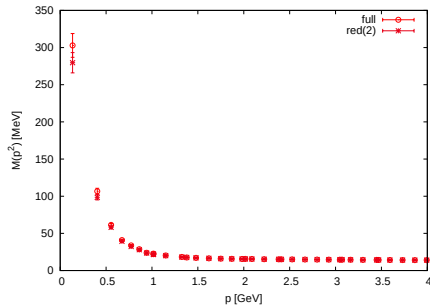
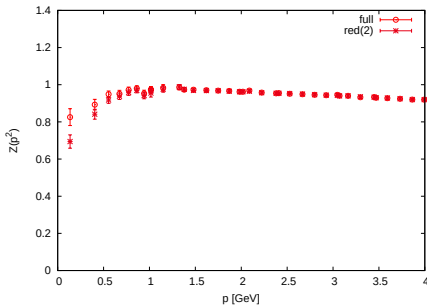
$$S_I(x, y) = \langle S_I(x, y; U) \rangle \equiv \langle (1 + b_q am) S(x, y; U) - a\lambda \delta(x - y) \rangle$$

- tree-level correction: $Z \rightarrow Z/Z_0$ and $M \rightarrow M/(M_0 + m_{\text{ren}})$
- a data cut at 4 GeV since for larger momenta lattice artifacts dominate

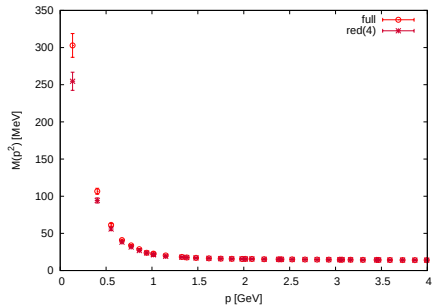
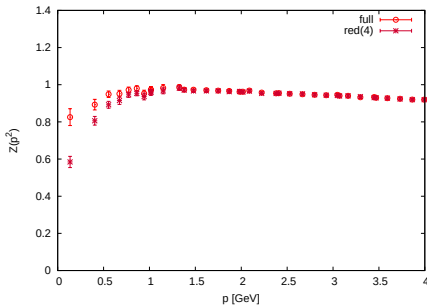
The quark propagator under eigenmode reduction



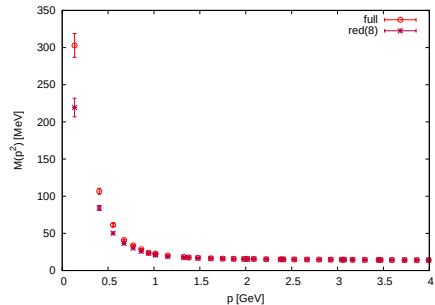
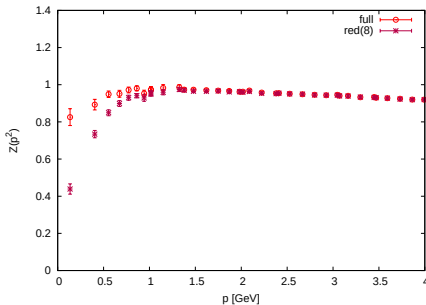
The quark propagator under eigenmode reduction



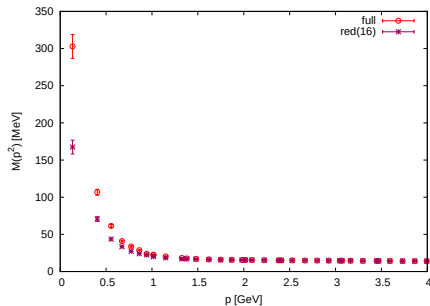
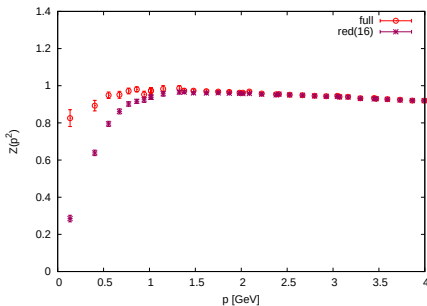
The quark propagator under eigenmode reduction



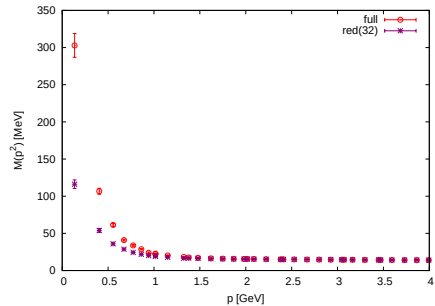
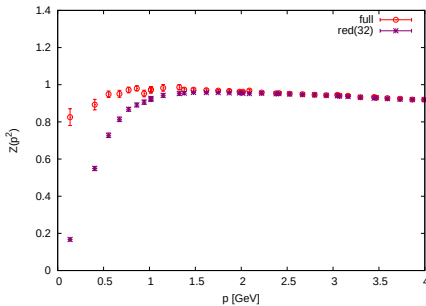
The quark propagator under eigenmode reduction



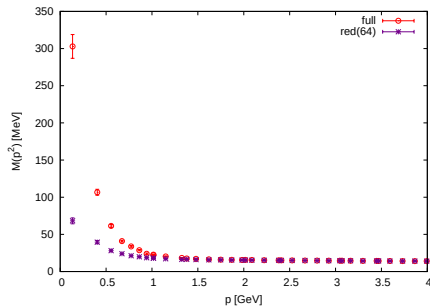
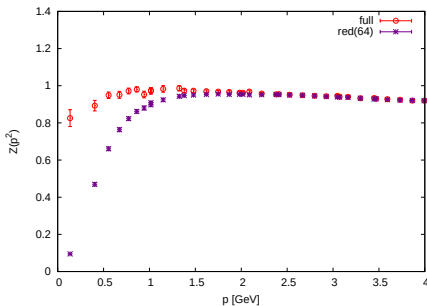
The quark propagator under eigenmode reduction



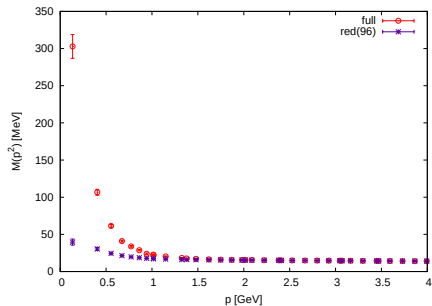
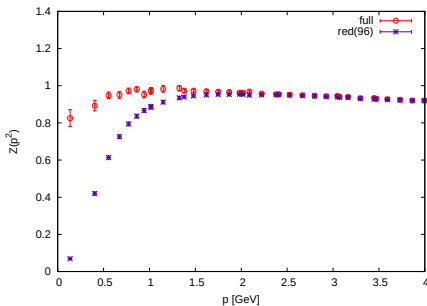
The quark propagator under eigenmode reduction



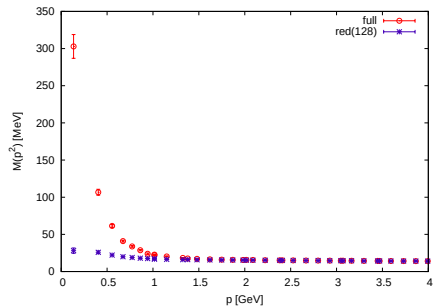
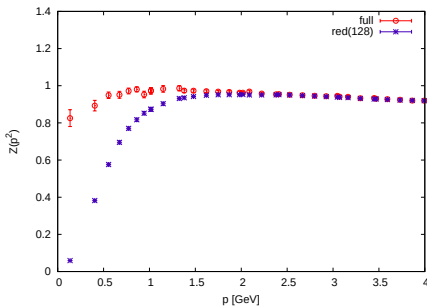
The quark propagator under eigenmode reduction



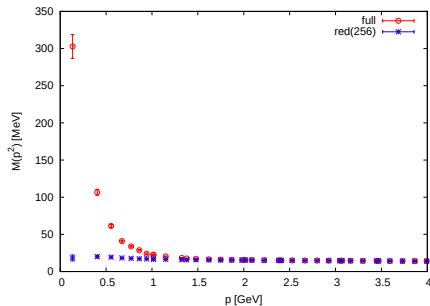
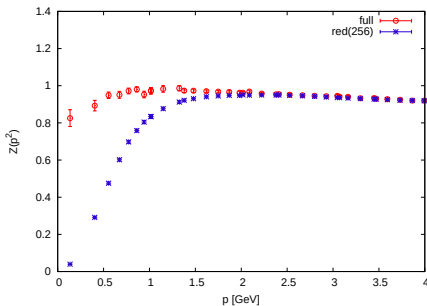
The quark propagator under eigenmode reduction



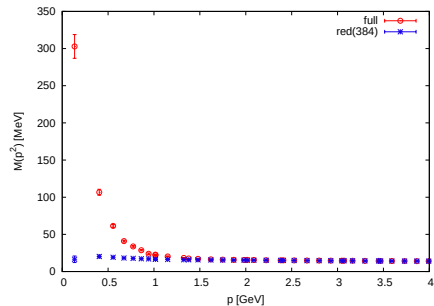
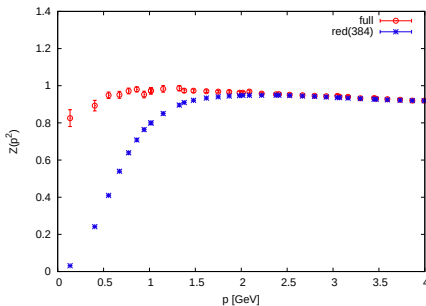
The quark propagator under eigenmode reduction



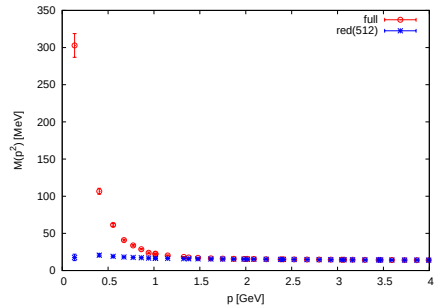
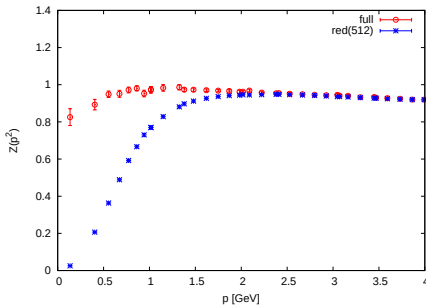
The quark propagator under eigenmode reduction



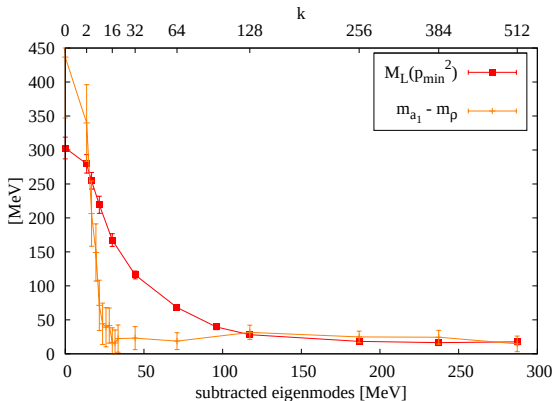
The quark propagator under eigenmode reduction



The quark propagator under eigenmode reduction



“Constituent quark mass” vs. eigenmode reduction level

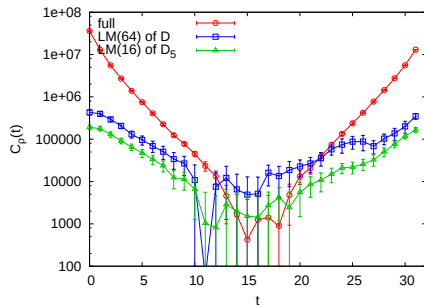
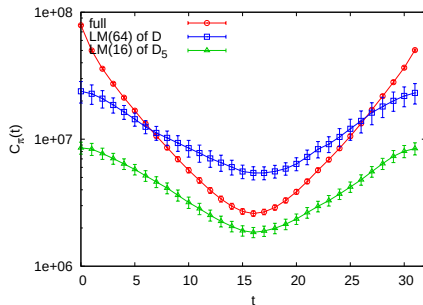


We find a reduction level window where chiral symmetry is restored (as seen by the $a_1 - \rho$ splitting) but dynamical mass is still generated, see also [O'Malley, Kamleh, Leinweber, Moran: arXiv:1112.2490](#)

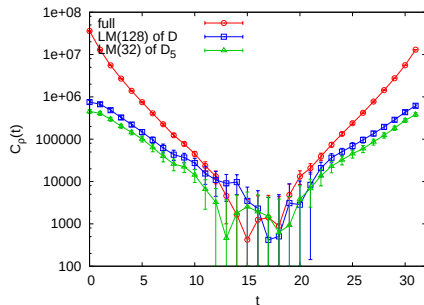
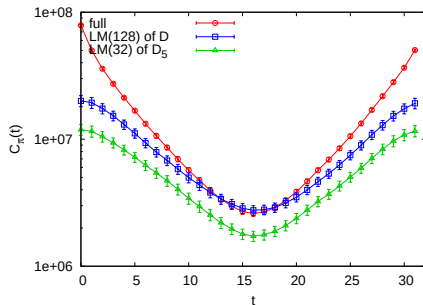
Conclusions

- low lying eigenvalues of the Dirac operator are associated with chiral symmetry breaking
- we have computed hadron propagators while removing increasingly more of the low lying eigenmodes of the Dirac operator
- the confinement properties remain intact, i.e., we still observe clear bound states for most of the studied hadrons
- an exception is the pion, where no clear exponential decay of the correlation function is seen
- the mass values of the vector meson chiral partners a_1 and ρ approach each other
- the nucleon and the $N(1535)$ become degenerate
- the splitting between N and Δ decreases
- the dynamical mass generation of quarks as seen from the IR behavior of $M(p^2)$ decreases towards zero, although it is still at roughly 55% when removing 16 eigenmodes of D_5 which is where we found chiral symmetry to be restored

Low-mode contribution of D and D_5 to the π and ρ correlators



Low-mode contribution of D and D_5 to the π and ρ correlators



ρ interpolators

$\#_\rho$	interpolator(s)
1	$\bar{a}_n \gamma_k b_n$
8	$\bar{a}_w \gamma_k \gamma_t b_w$
12	$\bar{a}_{\partial_k} b_w - \bar{a}_w b_{\partial_k}$
17	$\bar{a}_{\partial_i} \gamma_k b_{\partial_i}$
22	$\bar{a}_{\partial_k} \epsilon_{ijk} \gamma_j \gamma_5 b_w - \bar{a}_w \epsilon_{ijk} \gamma_j \gamma_5 b_{\partial_k}$

Interpolators for the ρ -meson, $J^{PC} = 1^{--}$. The first column shows the number, the second shows the explicit form of the interpolator. cf. [Engel et al., PRD 82 \(2010\), arXiv:1005.1748](#)

a_1 interpolators

# a_1	interpolator(s)
1	$\bar{a}_n \gamma_k \gamma_5 b_n$
2	$\bar{a}_n \gamma_k \gamma_5 b_w + \bar{a}_w \gamma_k \gamma_5 b_n$
4	$\bar{a}_w \gamma_k \gamma_5 b_w$

a_1 -meson, $J^{PC} = 1^{++}$, cf. [Engel et al., PRD 82 \(2010\), arXiv:1005.1748](#)

b_1 interpolators

$\#_{b_1}$	interpolator(s)
6	$\bar{a}_{\partial_k} \gamma_5 b_n - \bar{a}_n \gamma_5 b_{\partial_k}$
8	$\bar{a}_{\partial_k} \gamma_5 b_w - \bar{a}_w \gamma_5 b_{\partial_k}$

b_1 -meson, $J^{PC} = 1^{+-}$, cf. [Engel et al., PRD 82 \(2010\), arXiv:1005.1748](#)

N interpolators

- $N^{(i)} = \epsilon_{abc} \Gamma_1^{(i)} u_a (u_b^T \Gamma_2^{(i)} d_c - d_b^T \Gamma_2^{(i)} u_c)$
- $N(+)$: 1, 2, 4, 14, 15, 18
- $N(-)$: 1, 7, 8, 9

$\chi^{(i)}$	$\Gamma_1^{(i)}$	$\Gamma_2^{(i)}$	smearing	# N
$\chi^{(1)}$	1	$C \gamma_5$	$(nn)n$	1
			$(nn)w$	2
			$(nw)n$	3
			$(nw)w$	4
			$(ww)n$	5
			$(ww)w$	6
$\chi^{(2)}$	γ_5	C	$(nn)n$	7
			$(nn)w$	8
			$(nw)n$	9
			$(nw)w$	10
			$(ww)n$	11
			$(ww)w$	12
$\chi^{(3)}$	$i \mathbb{1}$	$C \gamma_t \gamma_5$	$(nn)n$	13
			$(nn)w$	14
			$(nw)n$	15
			$(nw)w$	16
			$(ww)n$	17
			$(ww)w$	18

cf. Engel et al., PRD 82 (2010), arXiv:1005.1748

Δ interpolators

- $\epsilon_{abc} u_a (u_b^T C \gamma_k u_c)$
- $\Delta(+)$: 1, 2, 3
- $\Delta(-)$: 1, 2, 3

smearing	$\#_{\Delta}$
$(nn)n$	1
$(nn)w$	2
$(nw)n$	3
$(nw)w$	4
$(ww)n$	5
$(ww)w$	6

cf. Engel et al., PRD 82 (2010), arXiv:1005.1748