Chiral restoration of the momentum space quark propagator through Dirac low-mode truncation

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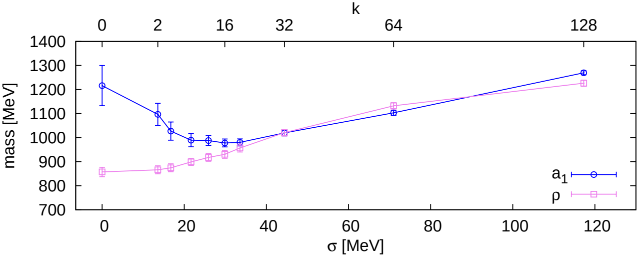
Confinement X
Munich, October 12, 2012

[Phys. Lett. B 711 (2012) 217-224; arXiv:1112.5107]

Outline

[Glozman, Lang, M.S., PRD 86 (2012) 014507; arXiv:1205.4887]

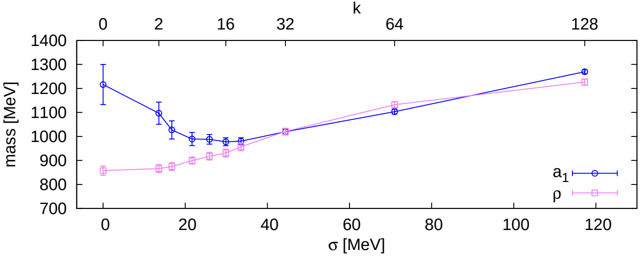


When gradually removing the chiral condensate from the vacuum in lattice QCD we found:

- degeneracy of ρ and a_1: restoration of the $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry
- heavy ρ and a_1 mesons: mass not due to dynamical chiral symmetry breaking

→ talk by L.Ya. Glozman, 11:30am, Section B: *Light Quarks*

Key questions to QCD



- How is the hadron mass generated in the light quark sector?
- How important is chiral symmetry breaking for the hadron mass?
- Are confinement and chiral symmetry breaking directly interrelated?

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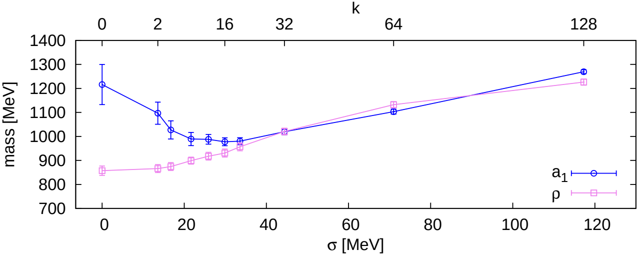

- How is the hadron mass generated in the light quark sector?
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$\rightarrow$ the momentum space quark propagator under Dirac low-mode truncation may give new insights here

The Banks–Casher relation

The lowest eigenmodes of the Dirac operator are related to the quark condensate of the vacuum:

$$\langle \overline{\psi}\psi \rangle = -\pi \rho(0)$$

- $\rho(0)$: density of the lowest quasi-zero eigenmodes of the Dirac operator
- here the sequence of limits is important: $V \to \infty$ then $m_q \to 0$

"Unbreaking" chiral symmetry

- we construct *reduced* quark propagators which exclude a variable number of the lowest Dirac eigenmodes

- we use the Hermitian Dirac operator $D_5 \equiv \gamma_5 D$ (real eigenvalues)
- split the quark propagator $S \equiv D^{-1}$ into a low mode (lm) part and a *reduced* (red) part

$$
\begin{aligned}
S &= \sum_{i \leq k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 + \sum_{i > k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 \\
&= S_{\mathrm{lm}(k)} + S_{\mathrm{red}(k)}
\end{aligned}
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$$S = \sum_{i \le k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5 + \sum_{i > k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5$$
$$= S_{\mathrm{lm}(k)} + S_{\mathrm{red}(k)}$$

- in this work we investigate the *reduced* (red) part of the propagator

$$S_{\mathrm{red}(k)} = S - S_{\mathrm{lm}(k)}$$

The chirally improved (CI) Dirac operator

[Gattringer, PRD 63 (2001) 114501], [Gattringer et al., Nucl. Phys. 597 (2001) B451]

- approximate solution to the Ginsparg–Wilson (GW) equation
- constructed by expanding the most general Dirac operator in a basis of simple operators,

$$D_{\mathrm{CI}}(x,y) = \sum_{i=1}^{16} c_{xy}^{(i)}(U)\Gamma_i + m_0 \mathbb{1},$$

  sum runs over all elements Γ_i of the Clifford algebra. The coefficients $c_{xy}^{(i)}(U)$ consist of path ordered products of the link variables U (here we use paths up to length four).
- Inserting this expansion into the GW equation then turns into a system of coupled quadratic equations for the expansion coefficients
- That expansion provides for a natural cutoff which turns the quadratic equations into a simple finite system.

The setup

- 125 configurations [Gattringer et al., PRD 79 (2009)]
- size $16^3 \times 32$
- two light degenerate dynamical CI quark flavors with the mass parameter set to $m_0 = -0.077$ and a resulting bare AWI-mass of $m = 15.3(3)\,\mathrm{MeV}$
- lattice spacing $a = 0.1440(12)\,\mathrm{fm}$

Nonperturbative quark propagator

The tree-level quark propagator is

$$S_0(p) = \frac{1}{i\not{p} + m}$$

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$$S_0(p) \to S_{\mathrm{bare}}(a;p) = Z_2(\mu;a)S(\mu;p)$$

the renormalized quark propagator

$$S(\mu;p) = \frac{1}{i\not{p}A(\mu;p^2) + B(\mu;p^2)} = \frac{Z(\mu;p^2)}{i\not{p} + M(p^2)}.$$

We calculate $S_{\mathrm{bare}}(a;p)$ in minimal Landau gauge on the lattice and therefrom extract

- the renormalization function $Z(\mu;p^2)$
- the renormalization point independent mass function $M(p^2)$

The CI quark propagator at tree-level

- The lattice quark propagator at tree-level differs from the continuum case due to discretization artifacts

$$S_L^{(0)}(p) = \left(ia\not{k} + aM_L^{(0)}(p) \right)^{-1} .$$

- we extract the lattice momenta $ak(p)$ and the tree-level mass function $aM_L^{(0)}(p)$ and compare it to its analytic expressions

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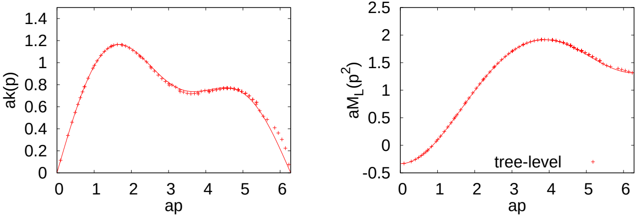
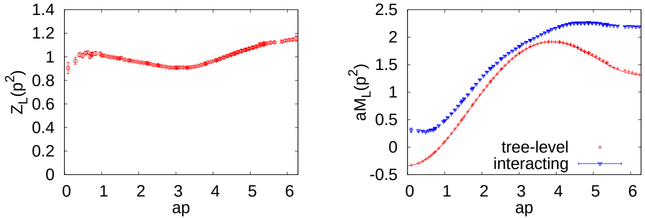


The interacting CI quark propagator



[1] [Skullerud et al., PRD 64 (2001) 074508]

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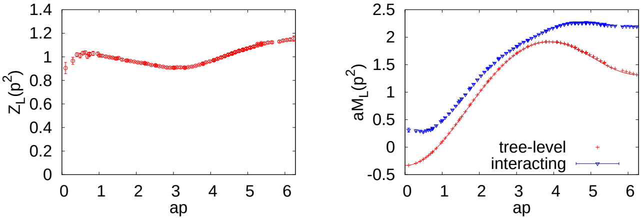

Improvement:

- tree-level improvement to reduce $\mathcal{O}(a)$ errors of off-shell quantities
- tree-level correction to blank out tree-level discretization artifacts[1]:

$$Z_L(p) \to \frac{Z_L(p)}{Z_L^{(0)}(p)} \,, \quad aM_L(p) \to \frac{M_L(p) A_L^{(0)}(p)}{B_L^{(0)}(p) + m_{\mathrm{add}}} am$$

  with am_{add} such that $B_L^{(0)}(0) = m$

- a data cut at $4\,\mathrm{GeV}$

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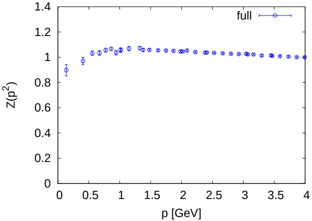

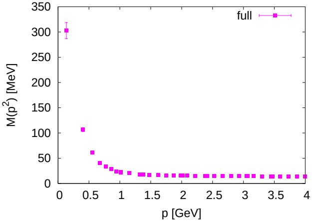

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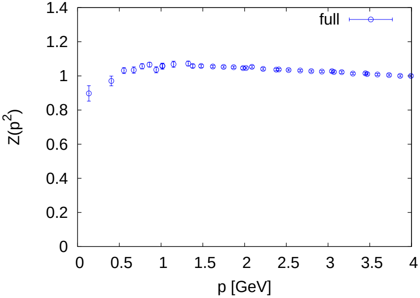

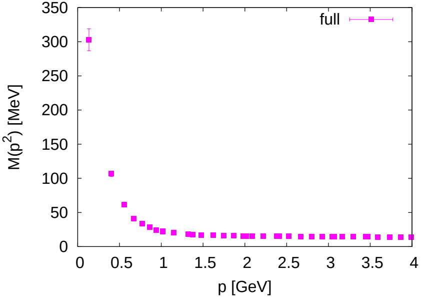

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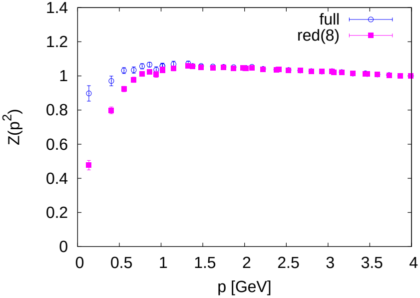

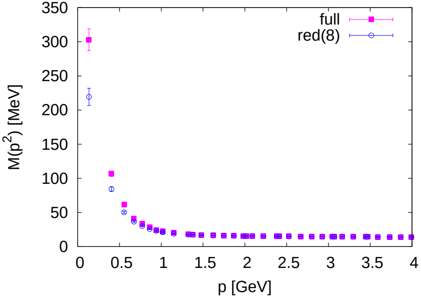

- the dynamically generated mass decreases with the truncation level
- $Z(p^2)$ gets suppressed in the IR

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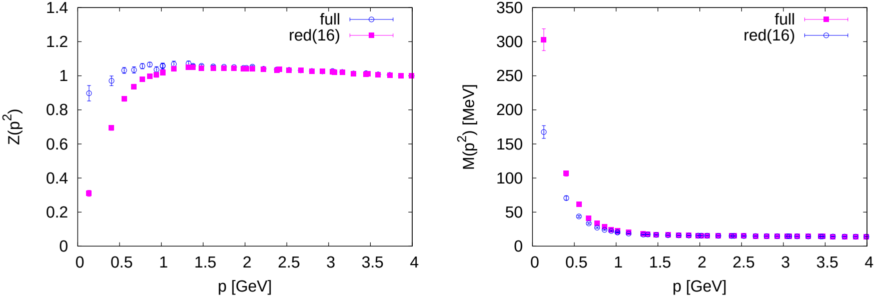

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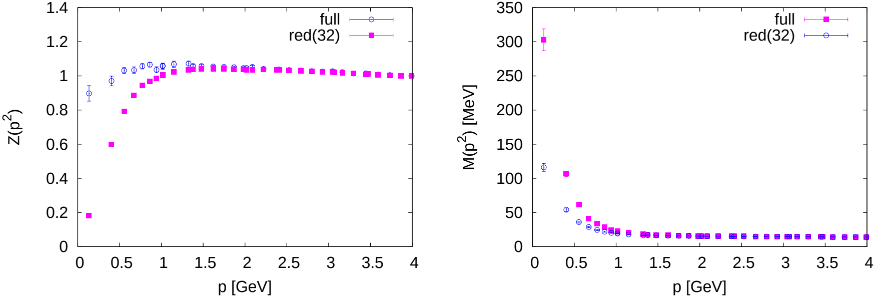

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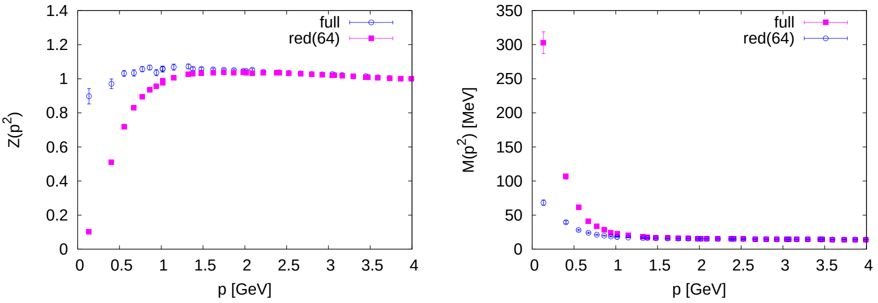

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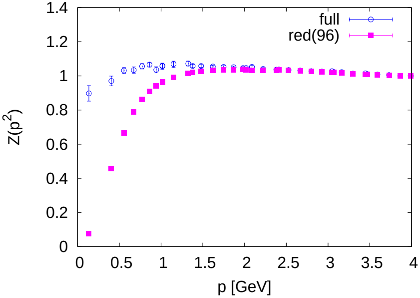

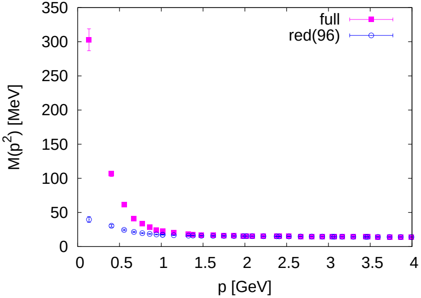

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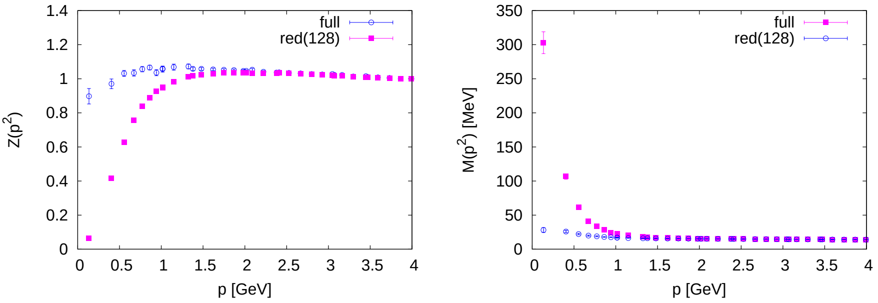

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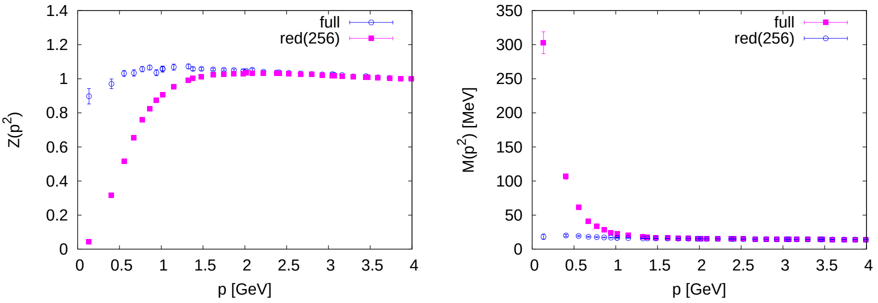

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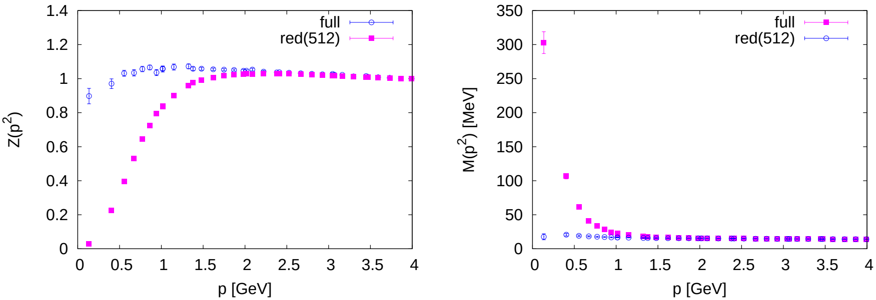

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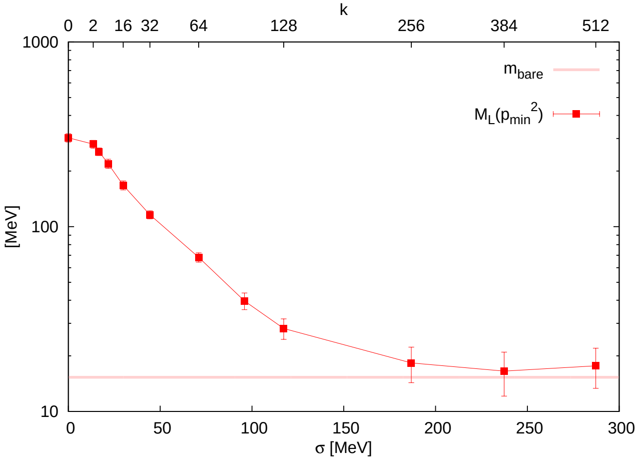
Dynamical quark mass generation vs. truncation level

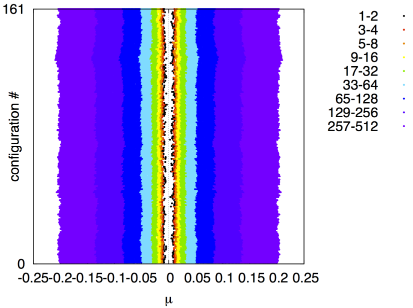


Summary

- low lying eigenvalues of the Dirac operator are associated with chiral symmetry breaking
- we gradually restore the chiral symmetry by Dirac low-mode truncation
- the dynamical mass generation of quarks dissolves with the truncation level
- in our scenario of artificially restored chiral symmetry the chiral partner hadrons have degenerate masses of the same order as in full QCD → talk by L.Ya. Glozman, 11:30am, Section B: *Light Quarks*
- we have shown that this mass is not generated by the constituent quarks themselves but must be generated solely by their interactions
- moreover we have demonstrated that far traveling quarks are suppressed in a chirally symmetric world

Eigenvalues

D_5



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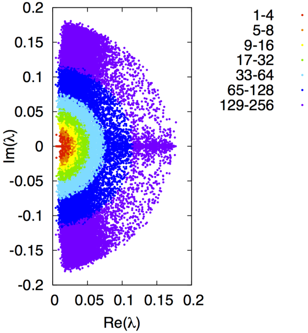

Analytical expressions for the tree-level CI Dirac operator I

$$
\begin{aligned}
M_L^{(0)}(p) &= s_1 + 48 s_{13} \\
&+ (2s_2 + 12 s_8)(\cos(p_0) + \cos(p_1) + \cos(p_2) + \cos(p_3)) \\
&+ (8s_3 + 64 s_{11})(\cos(p_0)\cos(p_1) + \cos(p_0)\cos(p_2) \\
&+ \cos(p_0)\cos(p_3) + \cos(p_1)\cos(p_2) + \cos(p_1)\cos(p_3) \\
&+ \cos(p_2)\cos(p_3)) \\
&+ 48 s_5(\cos(p_0)\cos(p_1)\cos(p_2) + \cos(p_0)\cos(p_1)\cos(p_3) \\
&+ \cos(p_0)\cos(p_2)\cos(p_3) + \cos(p_1)\cos(p_2)\cos(p_3)) \\
&+ 8 s_6(\cos(p_0)\cos(2p_1) + \cos(p_0)\cos(2p_2) \\
&+ \cos(p_0)\cos(2p_3) + \cos(p_1)\cos(2p_2) \\
&+ \cos(p_1)\cos(2p_3) + \cos(p_2)\cos(2p_3) \\
&+ \cos(2p_0)\cos(p_1) + \cos(2p_0)\cos(p_2) \\
&+ \cos(2p_0)\cos(p_3) + \cos(2p_1)\cos(p_2) \\
&+ \cos(2p_1)\cos(p_3) + \cos(2p_2)\cos(p_3)) \\
&+ 384 s_{10}\cos(p_0)\cos(p_1)\cos(p_2)\cos(p_3) \\
&+ m_0
\end{aligned}
$$

Analytical expressions for the tree-level CI Dirac operator II

$$
\begin{aligned}
k_0 &= 2v_1 \sin(p_0) + 8v_2 \sin(p_0)(\cos(p_1) + \cos(p_2) + \cos(p_3)) \\
&\quad + (32v_4 + 16v_5) \sin(p_0)(\cos(p_1)\cos(p_2) + \cos(p_1)\cos(p_3) \\
&\quad + \cos(p_2)\cos(p_3)), \\
k_1 &= 2v_1 \sin(p_1) + 8v_2 \sin(p_1)(\cos(p_0) + \cos(p_2) + \cos(p_3)) \\
&\quad + (32v_4 + 16v_5) \sin(p_1)(\cos(p_0)\cos(p_2) + \cos(p_0)\cos(p_3) \\
&\quad + \cos(p_2)\cos(p_3)), \\
k_2 &= 2v_1 \sin(p_2) + 8v_2 \sin(p_2)(\cos(p_0) + \cos(p_1) + \cos(p_3)) \\
&\quad + (32v_4 + 16v_5) \sin(p_2)(\cos(p_0)\cos(p_1) + \cos(p_0)\cos(p_3) \\
&\quad + \cos(p_1)\cos(p_3)), \\
k_3 &= 2v_1 \sin(p_3) + 8v_2 \sin(p_3)(\cos(p_0) + \cos(p_1) + \cos(p_2)) \\
&\quad + (32v_4 + 16v_5) \sin(p_3)(\cos(p_0)\cos(p_1) + \cos(p_0)\cos(p_2) \\
&\quad + \cos(p_1)\cos(p_2))
\end{aligned}
$$

The relevant D_{CI} coefficients

s_1	$0.1481599252 \times 10^{1}$
s_2	$-0.5218251439 \times 10^{-1}$
s_3	$-0.1473643847 \times 10^{-1}$
s_5	$-0.2186103421 \times 10^{-2}$
s_6	$0.2133989696 \times 10^{-2}$
s_8	$-0.3997001821 \times 10^{-2}$
s_{10}	$-0.4951673735 \times 10^{-3}$
s_{11}	$-0.9836500799 \times 10^{-3}$
s_{13}	$0.7529838581 \times 10^{-2}$
v_1	$0.1972229309 \times 10^{0}$
v_2	$0.8252157565 \times 10^{-2}$
v_4	$0.5113056314 \times 10^{-2}$
v_5	$0.1736609425 \times 10^{-2}$
m_0	-0.077