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Unbreaking chiral symmetry

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Motivation and Introduction

Dynamical chiral symmetry breaking in QCD is associated with the low lying spectral modes of the Dirac operator [1]. In a series of papers [2, 3, 4] it was emphasized that low modes saturate the pseudoscalar and axial vector correlators at large distances and do not affect the part where high-lying states appear. In [4, 5] low mode saturation and also effects of low mode removal for mesons were studied for quenched configurations with the overlap Dirac operator [6, 7].

A complementary question is how important the low mode sector is for confinement and mass generation of hadrons. Here we study what happens, if one removes up to 512 low lying modes from the valence quark sector. We compute propagators of the pion and other mesons and determine the effect of this removal on the mass spectrum. This way we want to shed light on the role of the condensate related to the spectral part of the Dirac operator in confinement and chiral symmetry breaking.

Reduced Dirac operator

We want to construct meson correlators from quark propagators which exclude the lowest part of the Dirac spectrum. There are two alternative definitions of reduction: based on eigenmodes of D or based on eigenmodes of the hermitian Dirac operator $D_5 \equiv \gamma_5 D$. We introduce the reduced quark propagator via the spectral representation of D_5,

$$S_{\mathrm{red5}(k)} \equiv S - \sum_{i \le k} \mu_i^{-1} |v_i\rangle\langle v_i| \gamma_5$$

and in this study we will concentrate on our results from the above definition of the reduced quark propagator.

Chiral symmetry and its breaking

Neglecting the masses of the u and d quarks the QCD Lagrangian is invariant under the symmetry group

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_V \times \mathrm{U}(1)_A \, .$$

The chiral symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ consists of independent transformations in the isospin space for the left- and right-handed quark fields and can be represented equivalently by independent isospin and axial rotations for the combined quark fields.

The non-degenerate masses of parity partners indicate the dynamical (spontaneous) breaking of this chiral symmetry with the order parameter $\langle \overline{\psi}\psi \rangle$, the chiral condensate.

The flavor singlet axial transformation symmetry $\mathrm{U}(1)_A$ is broken explicitly due to the non-invariance of the fermion integration measure, the so-called axial anomaly. In addition to the anomaly also the chiral condensate breaks this symmetry.

Both symmetry breaking signals are related to low lying modes of the Dirac operator. The axial anomaly involves the topological charge of the gauge configuration, which is proportional to the net number of exactly chiral (zero-) modes via the Atiyah-Singer index theorem [9]. The chiral condensate is associated with the density of the Dirac operator's low lying (but non-zero) modes [1]. The non-vanishing quark condensate indicates breaking of both symmetries.

Gauge configurations

The following setup has been used: 161 configurations [10, 11] of size $16^3 \times 32$ (lattice spacing $a = 0.144(1)\,\mathrm{fm}$) including two degenerate flavors of light fermions with a corresponding pion mass of $m_\pi = 322(5)\,\mathrm{MeV}$.

For the dynamical quarks of the configurations as well as for the valence quarks the so-called Chirally Improved Dirac operator [12, 13] has been used. This operator is an approximate solution to the Ginsparg–Wilson equation and therefore exhibits better chiral properties than the simpler Wilson Dirac operator while being less expensive by an order of magnitude in comparison to the chirally exact overlap operator.

Mesons

We restrict ourselves to the study of isovectors, in particular the chiral partners:

- The vector mesons ρ ($J^{PC} = 1^{--}$) with interpolating fields $\overline{u}(x)\gamma_i d(x)$ and $\overline{u}(x)\gamma_4\gamma_i d(x)$ and a_1 ($J^{PC} = 1^{++}$) with interpolating field $\overline{u}(x)\gamma_i\gamma_5 d(x)$; in a chirally symmetric world the vector and the axial vector interpolator get mixed via the isospin axial transformations.
- The pseudoscalar π ($J^{PC} = 0^{-+}$) with interpolating fields $\overline{u}(x)\,(\gamma_4)\,\gamma_5 d(x)$ and the scalar a_0 ($J^{PC} = 0^{++}$) with $\overline{u}(x)d(x)$.

We compute from the quark propagators meson propagators, projected to vanishing momentum and determine the hadron masses from a range of Euclidean time values where the correlation function exhibits exponential decay.

Results: Removing the low mode sector

Fig. 1 shows the meson propagators for various stages of low mode *removal*, always in comparison with the full propagator and Fig. 2 combines the corresponding mass fits to the regions of exponential behavior.

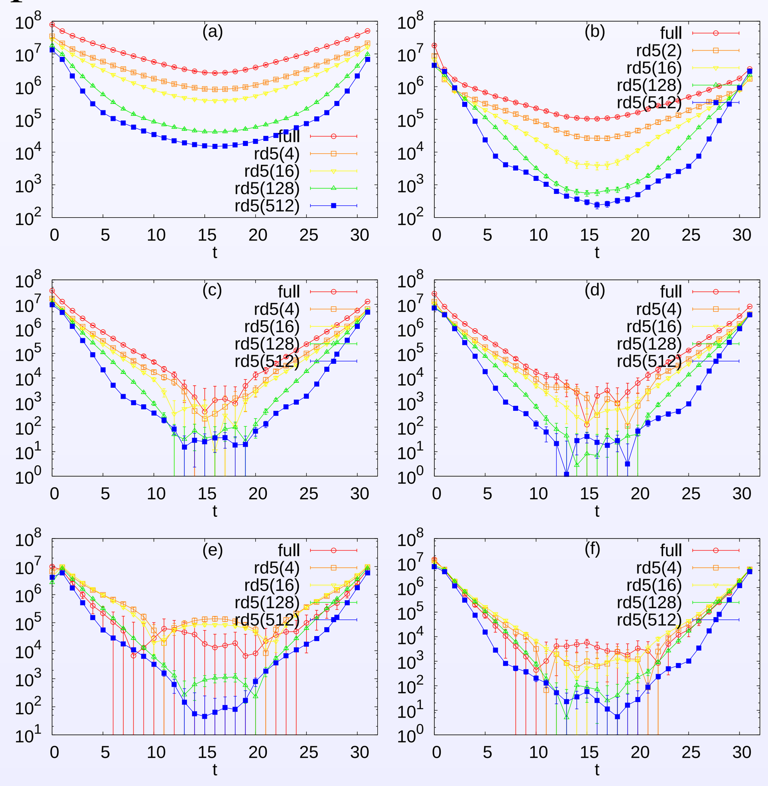

Figure 1: Correlation functions for the *reduced* interpolators as compared to the correlators from full propagators. Top: $J^{PC} = 0^{-+}$ with interpolators (a) $\overline{u}\gamma_5 d$, (b) $\overline{u}\gamma_4\gamma_5 d$. Middle: $J^{PC} = 1^{--}$ with (c) $\overline{u}\gamma_i d$, (d) $\overline{u}\gamma_4\gamma_i d$. Bottom: Reduced (e) $J^{PC} = 0^{++}$ ($\overline{u}d$) and (f) $J^{PC} = 1^{++}$ ($\overline{u}\gamma_i\gamma_5 d$).

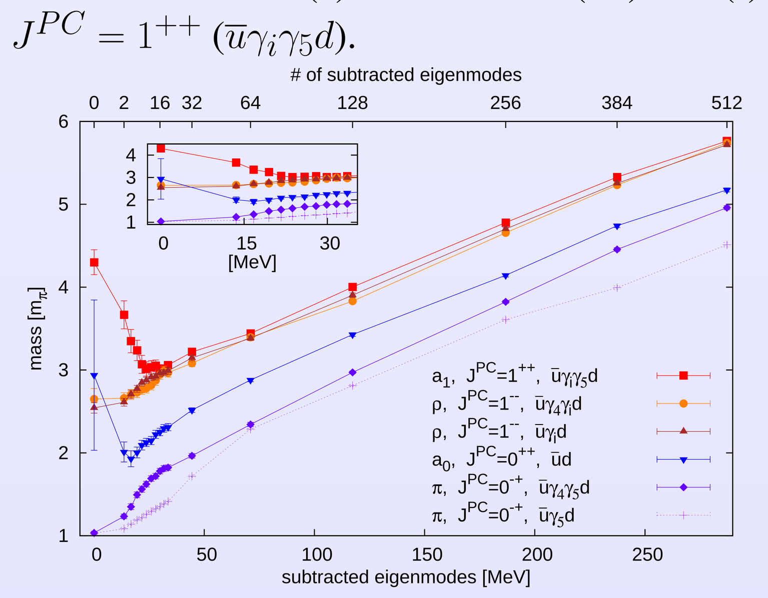

Figure 2: The masses of all considered mesons as a function of the reduced spectrum, subtracting the 0–512 lowest modes of D_5.

The range of exponential behavior of the correlators shrinks, as can be seen in the log-plots in Fig. 1.

In [14] the parity-chiral group and the effect of symmetry breaking on the meson spectrum is discussed. E.g., whereas the $\mathrm{U}(1)_A$ breaking lifts the degeneracy between pion and a_0, the breaking of the chiral $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry is related to the mass differences of pion and f_0 (and a_0 and η).

At low truncation levels the a_0-mass rapidly drops; it does not drop down to the pion mass value. This might indicate some remnant of the anomaly breaking for the $J = 0$ states.

The pion interpolators exhibit a puzzling behavior. The classical pion interpolator $\overline{u}\gamma_5 d$ quickly looses its exponential behavior at larger (Euclidean) distances; only a more massive decay signal is observed at smaller distances (Fig. 1) which is the mass shown in Fig. 2. That mass approaches the mass value from the second interpolator $\overline{u}\gamma_4\gamma_5 d$ with the pion quantum numbers. Its mass signal becomes separated from the other "state". Only at higher truncation levels the mass values approach each other.

For the $J^{PC} = 1^{--}$ vector meson ρ there are two chiral representations, which correspond to the vector interpolator $\overline{u}\gamma_i d$ and (Dirac-)tensor interpolator $\overline{u}\gamma_4\gamma_i d$. Their chiral partners [14] are the a_1 and the h_1 mesons, respectively. There is no noticeable splitting between the two ρ-interpolators for all stages of truncation. We do find, however intriguing behavior comparing the ρ-mass with the a_1 result. The masses approach each other and are compatible with each other from truncation level 8 onwards. This indicates restoration of the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry for $J = 1$ states.

Conclusions

We have computed hadron propagators while removing increasingly more of the low lying eigenmodes of the Dirac operator. This allows us to study their influence on certain hadron masses. Due to the relationship of the low eigensector with chiral symmetry breaking, this amounts to partially restoring chiral symmetry (in the valence quarks).

We find drastic behavior for some meson interpolators when starting to remove low eigenmodes. At truncation level 16 the behavior saturates and then the mass values rise uniformly with roughly parallel slopes. The confinement properties remain intact, i.e., we still observe clear bound states for most of the studied isovector (scalar, vector and axial vector) mesons. An exception is the pion, where no clear exponential decay of the correlation function is seen in the $\overline{u}\gamma_5 d$ interpolator, but a massive state is seen in the $\overline{u}\gamma_4\gamma_5 d$ interpolator. The mass values of the vector meson chiral partners a_1 and ρ approach each other rapidly when 8 or more low modes are removed.

We conclude that essential confinement properties remain intact, even when the low eigenmodes of the Dirac operator are removed in the valence sector. Restoration of chiral symmetry is observed in that approximation.

[1] T. Banks and A. Casher, Nucl. Phys. B **169**, 103 (1980).

[2] T. A. DeGrand and A. Hasenfratz, Phys.Rev. **D64**, 034512 (2001), hep-lat/0012021.

[3] T. DeGrand, Phys. Rev. **D64**, 094508 (2001), hep-lat/0106001.

[4] T. A. DeGrand, Phys.Rev. **D69**, 074024 (2004), hep-ph/0310303.

[5] T. DeGrand and S. Schaefer, Comput. Phys. Commun. **159**, 185 (2004), hep-lat/0401011.

[6] H. Neuberger, Phys. Lett. B **417**, 141 (1998), hep-lat/9707022.

[7] H. Neuberger, Phys. Lett. B **427**, 353 (1998), hep-lat/9801031.

[8] G. Bali, L. Castagnini, and S. Collins, PoS **LATTICE2010**, 096 (2010), 1011.1353.

[9] M. F. Atiyah and I. M. Singer, Ann. Math. **93**, 139 (1971).

[10] C. Gattringer *et al.*, Phys. Rev. **D79**, 054501 (2009), 0812.1681.

[11] BGR [Bern-Graz-Regensburg], G. P. Engel, C. B. Lang, M. Limmer, D. Mohler, and A. Schäfer, Phys. Rev. D **82**, 034505 (2010), 1005.1748.

[12] C. Gattringer, Phys. Rev. D **63**, 114501 (2001), hep-lat/0003005.

[13] C. Gattringer, I. Hip, and C. B. Lang, Nucl. Phys. B **597**, 451 (2001), hep-lat/0007042.

[14] L. Y. Glozman, Phys. Rep. **444**, 1 (2007), hep-ph/0701081.